T.Williamson on KK principle

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content

1.Background —— KK principle

In his 1951, G.H. von Wright suggested that epistemic logic— the logic of the term "knows" — is a branch of modal logic— that is to say, the logic of possibility and necessity.

Von Wright′s suggestion was taken up by Jaakko Hintikka, who developed one of the first modal systems of epistemic logic in his book *Knowledge and Belief: An Introduction to the Logic of the two notions*(1962).

$Np \rightarrow NNp$ ⇒ ? $Kp \rightarrow KKp$

1.Background —— KK principle

Suppose we say that evidence for a proposition, P, is **conclusive** iff it is so strong that, once one discovers it, further inquiry cannot give one reason to stop believing P. The concept of knowledge used by many philosophers seems to be a strong one on which one knows P only if one′ s evidence for P is conclusive in this sense. It is plausible that the KK principle holds for this strong concept of knowledge.

To see this, suppose one has evidence, E, for a proposition P, and that E does not rule out the possibility that one does not know P. If E does not rule out this possibility, then, after one has discovered E, further inquiry can, in principle, reveal to one that one does not know P. But if further inquiry were to reveal this, then it would surely give one reason to stop believing P (since one should not believe things that one does not know). So it is plausible that, if E does not rule out the possibility that one does not know P, then it is not conclusive in the sense just defined, and hence plausible that, if knowledge requires evidence that is conclusive in this sense, the KK principle holds.

(cf. Hintikka *"Knowing that One Knows" reviewed*. Synthese1970: 145-6)

1.Background —— KK principle

strong concept of knowledge

objection1: One such objection says that, when the claim is made that someone knows that p, it cannot usually be claimed that they know that they know that p, that they know that they know that they know that p, and so on (cf. Rynin 1967: 29).

objection2:knowledge from being ascribed to **animals** and **young children** (who lack the concept of knowledge and so cannot know that they know) is not problematic for Hintikka.

The fact that one is not prepared to claim these things or when knowledge ascribed to those subjects, may show that the KK principle fails for our ordinary concept of knowledge, but it does not show that the principle fails for the strong concept that Hintikka has in mind.

1.Background —— KK principle

If the KK principle only holds for a concept of knowledge that is very different from our everyday concept, then why should one be interested in it?

According to Hintikka, its interest derives from the fact that (in spite of the differences between our everyday concept and the strong concept) there are "many philosophers, traditional as well as contemporary" who use the strong concept of knowledge for which the principle holds (1970: 148).

What kind of knowledge concept do we need?

What is the nature of evidence E?

1.Background——Internalism, Externalism and the KK principle

$Kp = Bp \wedge p \wedge Jp$

Internalism theory of knowledge

knowledge is true belief that is based on adequate evidence or reasons, where the adequacy of our evidence or reasons is something that one can determine by introspection and reflection.

Externalism theory of knowledge

true belief that is produced by a reliable process. The reliability of the processes that produce our beliefs is not something that one can determine by introspection and reflection; it is a matter for empirical investigation.

1.Background——Internalism, Externalism and the KK principle

In general, internalist theories of knowledge say that the property which distinguishes knowledge from mere true belief is internal to our cognitive perspective.

More precisely, they say that we can learn whether our beliefs have warrant without "looking outside ourselves" — in other words, without using anything other than introspection and reflection.

Externalist theories say that warrant may be external to our cognitive perspective, and that empirical investigation may be needed to ascertain which of our beliefs have it.

The reliabilist theory described is just one example of an externalist theory.

1.Background——Internalism, Externalism and the KK principle

It is natural for internalists to endorse something like the KK principle:

FOR knowing that one knows that p = knowing that one's belief that p is warranted.

It is natural for internalists to say that one is always in a position to know whether one's beliefs are warranted,because it can always be realized by introspection and reflection.

(1)	K*p*	(assumption)
(2)	K*p* ≡ [B*p* & *p* is true & good reasons for B*p*]	(internalism)
(3)	B*p*	(from (1), (2))
(4)	K[B*p*]	(from (3), assuming one knows what one's beliefs are)
(5)	K[good reasons for B*p*]	(epistemic access to reasons)
(6)	K[B*p* & *p* is true & good reasons for B*p*]	(from (1), (4), (5), assuming knowledge is closed under conjunction)
(7)	KK*p*	(from (2), (6))

1.Background——Demonstration of the Externalists

It is also natural for externalists to reject this principle:

FOR, if warrant may be external to our cognitive perspective, then there is no special reason to expect those who know that p to be in a position to know that their belief that p is warranted.

(1) K*p*	(assumption)
(2) ~Reason to believe [B*p* is reliable]	(assumption)
(3) ~K[B*p* is reliable]	(from (2))
(4) K*p* ≡ [B*p* & *p* is true & B*p* is reliable]	(externalism)
(5) K*p* ⇒ KK*p*	(KK principle, assumed for *reductio*)
(6) KK*p*	(from (1), (5))
(7) K[B*p* & *p* is true & B*p* is reliable]	(from (4), (6))
(8) K[B*p* is reliable]	(from (7), assuming knowledge distributes across conjunction)
(9) ~[K*p* ⇒ KK*p*]	(from (1), (3), (4), (5), (8), by *RAA*)

1.Background——Demonstration of the Externalists

TWO PROBLEMS:

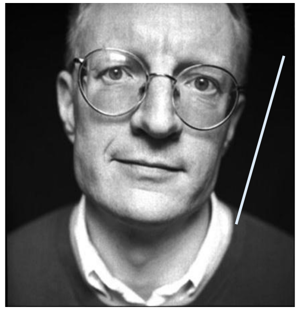
1.to argue from someone not having any reason to believe x to their not knowing x is to assume **internalism.** A thoroughgoing externalist must allow that someone can know that they know that p whether or not they have reason to think that their belief that p is reliably formed.

2.But the argument is fallacious:
to derive (7) from (4) and (6) involves substitution within an intensional context. Grant the externalist the equivalence of Kp and [Bp & p is true &Bp is reliable] as in (4); it does not follow that someone who knows that they know that p, i.e. KKp, must then know the conjunction [Bp & p is true & Bp is reliable], nor therefore that they must know the third conjunct. To think otherwise is to commit an intensional fallacy.

1.Background——Demonstration of the Externalists

This assumes that the **reliabilist's definition** of knowledge is meant as a statement of necessary co-extension, not of intensional equivalence. This is how reliabilism is standardly understood.

——its proponents and opponents; see Shapiro (2006)



The simple-minded idea that externalist accounts of knowledge have us know without knowing that we know; the latter idea is often based on an **illicitly internalist** understanding of "know" when the sentence in its scope includes epistemic vocabulary.

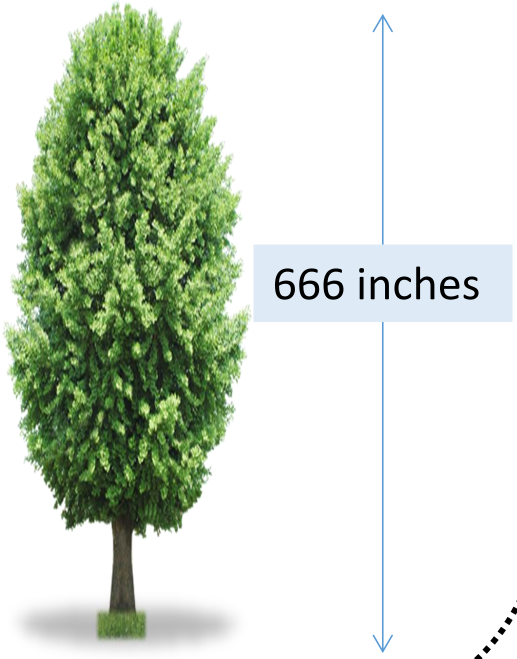
— . Contextualism, subject-sensitive invariantism and knowledge of knowledge. *Philosophical Quarterly* 55 (2005: 231 n. 14)

2.Williamson's puzzle

In chapter 5 of *Knowledge and its Limits*, Williamson formulates an argument against the principle (KK) of epistemic transparency, or luminosity of knowledge which always been called as “Williamson’s puzzle”.The argument is as follow:

I knows that if the tree is i+1 inches tall, then I don't know that the tree is not i inches tall.

I knows that this tree is not 0 inche tall.



2.Williamson's puzzle

Let q_{i+1} be the proposition that " the tree is i+1 inches tall",and K means "know". In a careful analysis of this argument, we actually have the following premises:

$(1_i) K(q_{i+1} \rightarrow \neg K \neg q_i)$ 认知局限

$(KK) Kp \rightarrow KKp$ 正内省公式

$(CLO) (p \rightarrow q) \rightarrow (Kp \rightarrow Kq)$[4] 演绎封闭性原则

$(T) Kp \rightarrow p$[5] 知蕴含真

$(F_{666}) q_{666}$ 事实条件

$(CON) \neg(p \wedge \neg p)$ 矛盾律

2.Williamson's puzzle

The complete processes from these premises to the (2_{i+1}) are as followings:

$(1_i)K(q_{i+1}\rightarrow\neg K\neg q_i)$	(Cognitive limits)
$(2_i)K\neg q_i$	(Assumption)
$(1_{i=})K(K\neg q_i\rightarrow\neg q_{i+1})$	[equivalent form of (1_i)]
$(3_i)KK\neg q_i$	[(2_i) KK principle]
$(2_{i+1})K\neg q_{i+1}$	[$(1_{i=})$,(3_i) Closure principle]

2.Williamson's puzzle

Then, by repeating the argument for values of i from 0 to 665,starting from (2_0) we reach the conclusion (2_{666}).

(2_{666})$K\neg q_{666}$

We get the contradiction by the following way:

(T_{666})$K\neg q_{666} \rightarrow \neg q_{666}$	[(2_{666}) , T]
($F_{\sim 666}$)$\neg q_{666}$	[(2_{666}),(T_{666}),MP]
(F_{666})q_{666}	(Objective fact)
(~CON)$\neg q_{666} \wedge q_{666}$	[($F_{\sim 666}$),(F_{666}),Conjuction]

2.Williamson's puzzle

Giving the premises (1_0), . . . , (1_{665}),(2_0), (CLO), (T),(KK),and some logical rules, we can deduce the false conclusion (2_{666}). Therefore, at least one of (1_0), . . . , (1_{665}), (2_0), (CLO), and(KK) is to be rejected.

Williamson has already defend the premise (1_i) for all i,and he would not give up (CLO),(T)and logical rules,(2_0) is obviously true as well. Consequently, (KK) is the premise to be rejected.

But we can’t stop thinking that weather all these conditions are right?Even if they are right to some extent,can they get on well with each other in the same context?Should we all blame,like what Williamson said ,KK principle?

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

The closure principle used in this argument is one of the premises that always be questioned. Precisely what is meant by the claim that knowledge is closed under entailment? One response is that the following straight principle of closure of knowledge under entailment is true:

(S-CLO)If person S knows p, and p entails q, then S knows q.

Here we call this straight principle as "strong verson of closure principle" which contained in Williamson's puzzle.But this principle is too controversial, since we can know one thing, *p*, but fail to see that *p* entails *q*, or for some other reason fail to believe *q*. Since knowledge entails belief , we fail to know *q*.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Hawthorne (2005) used to defend this principle on the assumption that “p is equivalent to q in all the possible worlds”.

Williamson(2000:117) naturally support this principle,thinking that we can extend our knowledge by applying deduction to what we know supports this strong verson of closure principle.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Regardless of whether their defences are valid or not, the following principles seem to be more acceptable than the one above:

(W-CLO)If person S knows p, and knows that p entails q, then S knows q.

We called this principle as "weak verson of closure principle". When we look back to the Williamson's puzzle,if the (W-CLO)was used in the process of the argument instead of (S-CLO),it seems that we can dispel the doubts about the closure.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

So the key is whether this transformation can be realized without changing Williamson's intention:

（T） K(p→q)→(p→q) （Truth condition）

（S-CLO）(p→q)→(Kp→Kq) （strong verson of closure principle ）

（W-CLO） K(p→q)→(Kp→Kq) [(T) (S-CLO)Syllogism rules]

It means that if we admit the truth of “strong verson of closure principle”and “truth condition”,“weak verson of closure principle” is just their logical result.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

We shouldn't doubt Williamson's support for the first two rules.Therefor, if we use"weak verson of closure principle" to modify the original argument like the following form,it will not against Williamson's intention:

(1_i) $K(q_{i+1}\rightarrow\neg K\neg q_i)$	(Cognitive limits)
($1_{i=}$) $K(K\neg q_i\rightarrow\neg q_{i+1})$	[equivalent form of (1_i)]
(W-CLO) $K(K\neg q_i\rightarrow\neg q_{i+1})\rightarrow(KK\neg q_i\rightarrow K\neg q_{i+1})$	(weak verson of closure principle)
(1_{i-}) $KK\neg q_i\rightarrow K\neg q_{i+1}$	[($1_{i=}$) (W-CLO) MP]
(2_i) $K\neg q_i$	(Assumption)
(3_i) $KK\neg q_i$	[(2_i) KK principle]
(2_{i+1}) $K\neg q_{i+1}$	[(1_{i-}) (3_i) MP]

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Dretskein(2003,2005) is the cricual one who against this principle.In his analysis of knowledge, tracking condition is a necessay condition which leads to（W-CLO）’s failure

We still don’t discuss the reliability of his argument.The point we want to fource here is if Williamsom’s anti-KK argument can completely get rid of the use of any verson of closure principle.

For Magoo’s problem,it seems that we can’t restructure the puzzle without closure principle.But there is another argument,given by Williamson, which against KK principle indirectly,similar in the structure to the Willamson’s puzzle, has nothing to do with any verson of closure principle.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Anti-luminosity argument

In Williamson’s opinion ,a condition C is defined to be luminous if and only if (L) holds:

(L) For every case α, if in α condition C obtains, then in α, one is in a position to know that C obtains.

We can see that if KK principle is right ,“knowing ” is a such condition C.So the KK principle is an special example of luminosity.Anti-luminosity is a way to against KK principle.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Anti-luminosity argument

Let $t_0, t_1, \ldots, t_n$ be a series of times at one millisecond intervals from dawn to noon. Let α_i be the case at t_i ($0 \le i \le n$).Suppose that S feels cold in in the dawn,and very slowly warms up, S feels hot by noon.Let “C”means “S feel cold”, Let“KP” stand for “S knows that P”,If “feeling cold ” is luminous ,then we have :

(L) For all α_i such that $0 < i < n$, if C in α_i, then KC.

Williamson’s main assumption is :

($1_{i'}$)For all α_i such that $0 < i < n$, if KC in α_i , then C in α_{i+1}.[8]

We assume that:

($2_{i'}$) C in α_i

Then by(L),($1_{i'}$) and(3_i),we have

($2_{i+1'}$) C in α_{i+1}

The following is certainly true, for α_0 is at dawn, when one feels freezing cold:

($2_{0'}$) In α_0 one feels cold.

By repeating the argument from ($2_{i'}$) to ($2_{i+1'}$) n times, for ascending values of i from 0 to n, we reach this from ($2_{0'}$):

($2_{n'}$) In α_n one feels cold.

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

"In any case, we may conjecture that, for any condition C, if one can move gradually to cases in which C obtains from cases in which C does not obtain, while considering C throughout, then C is not luminous, Because we can get and lose our knowledge gradually,"know"is not a luminous condition in this case .In other words,KK principle should be rejected.

(Williamson.T 2000:109)

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

Willianmson's puzzle $_2$

So let $t_0, t_1, \ldots, t_n$ be a series of times at one millisecond intervals from the time S got a piece of knowledge to the time S lose that piece of knowledge,and let α_i be the case at t_i $(0 \leq i \leq n)$, “KP”stand for “S knows that P”, $(\ldots)\alpha_i$ stand for the condition in brackets obtained in α_i,we have following argument to against KK:

$(1_{i'})(KKP)\alpha_i \rightarrow (KP)\alpha_{i+1}$	$(0 \leq i \leq n)$	(reliability of knowledge)
$(KK)(KP)\alpha_i \rightarrow (KKP)\alpha_i$	$(0 \leq i \leq n)$	(KK principle)
$(2_{i'})(KP)\alpha_i$		(assumption)
$(3_{i'})(KKP)\alpha_i$		[$(2_{i'})$ KK MP]
$(2_{i'+1})(KP)\alpha_{i+1}$		[$(3_{i'})$ $(1_{i'})$ MP]

3.Analyze of Williamson's puzzle —— Avoiding the closure principles

By repeating the argument from ($2_{i'}$) to ($2_{i+1'}$) n times, for ascending values of i from 0 to n, we reach this from ($2_{0'}$):

$$(2_{n'})\ (KP)\alpha_n$$

Williamsom's puzzle$_1$ can be modified with weak verson of closure principle which is easy to accept and Williamsom's puzzle$_2$ even doesn't need closure principle.It proved that whether we weak the closure principle or just abandon it,the puzzle is still there.Therefore, doubting the validity of the closure problem is not the point to dispel Williamson's puzzles.

3.Analyze of Williamson's puzzle —— the type of knowledge

We notice that there is a premise rely on the cognitive limits of human:

(1_i) Mr Magoo knows that if the tree is i+1 inches tall, then he does not know that the tree is not i inches tall.

whether our acquisition of knowledge is always accompanied by such limitations?

It seems that we have cognitive limitations to some extent when we get the animal knowledge,but why our cognitive limits influence the knowledge we get from books,the knowledge that we reasoned ?

3.Analyze of Williamson's puzzle —— the type of knowledge

So,we distinguish the type of knowledge here and see whether his argument can be established.Let "Ks" express visual knowledge, "Kt" means knowledge that is reflected or reasoning, and "Ki" can bring into any kind of knowledge,then ,the whole process will be like this:

(1_{i*})	$K_t(q_{i+1} \rightarrow \neg K_s \neg q_i)$	(Cognitive limits)
$(1_{i*=})$	$K_t(K_s \neg q_i \rightarrow \neg q_{i+1})$	[equivalent form of (1_{i*})]
(W-CLO)	$K_i(K_s \neg q_i \rightarrow \neg q_{i+1}) \rightarrow (K_i K_s \neg q_i \rightarrow K_i \neg q_{i+1})$	(weak verson of closure principle)
(1_{i*-})	$K_i K_s \neg q_i \rightarrow K_i \neg q_{i+1}$	[$(1_{i*=})$ (W-CLO) MP]
(2_{i*})	$K_s \neg q_i$	(Assumption)
(3_{i*})	$K_t K_s \neg q_i$	[(2_{i*}) KK principle]
(2_{i*t})	$K_t \neg q_{i+1}$	[(1_{i*-}) (3_{i*}) MP]

◆ 3.Analyze of Williamson's puzzle —— the type of knowledge

In a world,if we distinguish the type of knowledge,then Williamson's puzzle$_1$ can't be established,so his opposition to the KK principle is unsuccessful.

↓

But is this method still valid for Williamson's puzzle$_2$?

$(1_{i^{*'}})(K_iK_iP)\alpha_i \rightarrow (K_iP)\alpha_{i+1}$	$(0 \leq i \leq n)$	(reliability of knowledge)
$(KK)(K_iP)\alpha_i \rightarrow (K_tK_iP)\alpha_i$	$(0 \leq i \leq n)$	(KK principle)
$(2_{i^{*'}})(K_iP)\alpha_i$		(assumption)
$(3_{i^{*'}})(K_tKiP)\alpha_i$		$[(2_{i'})$ KK MP$]$
$(2_{i^{*'}+1})(K_iP)\alpha_{i+1}$		$[(3_{i'})\ (1_{i'})$ MP$]$

3.Analyze of Williamson's puzzle —— the type of knowledge

The reliability seems like a attribute of knowledge as a whole,and we never restrict the way to access to knolwedge in α_0. So,the premise ($1_{i*'}$) can always be substituted, regardless of what the cognitive operators are. By continuing this argument, we will still get contradictory results.

But why is that so? We all know that, iterative operation is the key to the processes of Williamson's puzzles. As long as the iteration can be prevented, this problem can be solved. In Williamson's puzzle_1,distinguishing the type of knowledge can indirectly avoid iteration.But in Williamson's $\text{puzzle}_{2,}$the argumentation process has nothing to do with cognitive channel,therefore distinguishing the type of knowledge can not stop the the iteration.

3.Analyze of Williamson's puzzle —— the type of knowledge

All this reminds us that if there is a way that can prevent the iterations in both Williamson's puzzles,then all the problems can be solved.

common premises:KK principle ,truth condition, some logical rules and reasonable assumptions.

The primises which progress a condition C from one case to another are (1i) and (1i') in the two arguments.If they are all correct, then there is reason to say that other premises are invalid.But are they all correct?

◆ 3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i′)

$(1_{i'})$For all α_i such that $0 < i < n$, if KC in α_i , then C in α_{i+1}.

"Consider a time t_i between t_0 and t_n, and suppose that at t_i one knows that one feels cold. Thus one is at least reasonably confident that one feels cold, for otherwise one would not know. Moreover, this confidence must be reliably based, for otherwise one would still not know that one feels cold. Now at t_{i+1} one is almost equally confident that one feels cold, by the description of the case. So if one does not feel cold at t_{i+1}, then one's confidence at t_i that one feels cold is not reliably based, for one's almost equal confidence on a similar basis a millisecond later that one felt cold is mistaken…… One's confidence at t_i was reliably based in the way required for knowledge only if one feels cold at t_{i+1}."

(T. Williamson 2000:97)

3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i’)

From this explanation, we can see that Williamson believes reliability is a necessary attribute of knowledge. He also uses a chapter to demonstrate the reasonableness of this view,It can be summed up as the following three points :

First, the reliability of knowledge should be understood as the degree of “outright belief”, not a high probability event.

Second,If one believes p truly in a case α, one must avoid false belief in other cases sufficiently similar to α in order to count as reliable enough to know p in α.

Third,The reliability of knowledge promote knowledge to play an effective role in causal interpretation of action.

3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i’)

OBJECTIONS 1 :Brueckner and Fiocco (2002)
Neta and Rohrbaugh (2004)

They give examples to show that it is possible that the beliefs which constitute knowledge are true in one case and flase in another sufficiently similar case.

OBJECTIONS 2 :Williamson’s interpretation makes his own views incoherent.

For one thing,he claimed that knowledge is a kind of factive mental state.It means knowledge should be different with the change of facts.

For another,the reliability of knowledge which Williamson advocated required that once a condition is known in a case, it must be retained in all similar cases.

similar cases ≠ same case

How to understand the same knowledge can be produced in different facts?

3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i′)

The answer may be hidden in his interpretation of the premise (1_i) in Williamson's puzzle$_{1:}$

"He wonders how tall it is. Evidently, he cannot tell to the nearest inch just by looking. His eyesight and ability to judge heights are nothing like that good. Since he has no other source of relevant information at the time, he does not know how tall the tree is to the nearest inch……. Equally, if the tree is i-1 or i+1 inches tall, he does not know that it is not i inches tall. Anyone who can tell by looking that the tree is not i inches tall, when in fact it is i+1 inches tall, has much better eyesight and a much greater ability to judge heights than Mr Magoo has.…….In this story, Mr Magoo reflects on the limitations of his eyesight and ability to judge heights. Mr Magoo knows the facts just stated."

(Williamson.T 2000:114-115)

3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i′)

The key word of this paragraph is "Cognitive limits".So,in Williamson's view when the states of a certain condition in two situations is "fully similar", people think they are "the same" because they can't distinguish the "similarity", so that knowledge is steadily transmitted and iterated in such states.

But can such an explanation be convincing?

One the one hand ,we have talked about that even the premise (1_i) has it's scope of application.Therefore, it can't be used to explain the general principle of knowledge.

One the other hand ,such an explanation is suspected of conceptual change:

"two cases are the same case " ≠ "two similar cases are hard to distinguished"

Therefore, when a belief is maintained under the similar circumstances, the stability of knowledge formed by it should be changed with the change of cases .

3.Analyze of Williamson's puzzle —— The analyze of (1i)and (1i’)

So,Williamson’s interpretation of the premise (1_i) can’t defend the acceptability of ($1_{i'}$),his views are still confronted with the problem of incoherent.Further more ,even if people have cognitive limitations, can such limitations be accurately portrayed? And even if it can be depicted, is it really like what (1_i) said?

In this way, both (1_i) and ($1_{i'}$) face their own problems,but don’t they have a internal connection? Even if we deny the possibility of explaining ($1_{i'}$) ’s acceptability with (1_i),but the structures of these two premises are so similar that we will not only think about whether there is any other connection between them, but also whether they can be dealt with in a unified way.The answer is positive.They can be linked by a priciple called “**margin for error**”.

4.Margin for error theory —— the relastionship with (1i) and (1i')

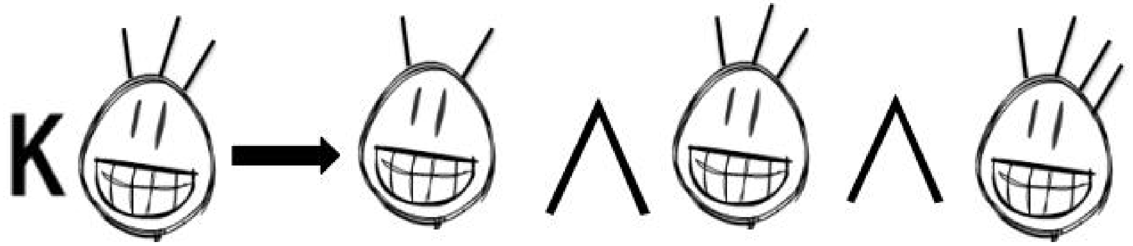
On the one hand,we human have cognitive limits,so the perception of things is not so accurate.On the other hand,we also require the reliability of the beliefs that constitute knowledge,which shouldn't be easily refuted.Therefore,Williamson thinks that what we need is to give our beliefs an "margin for error".Then he puts forward original theory:

Margin for error: For all cases α and β, if | v(α)-v β)|< c and in α one is in a position to know that C obtains then C obtains in β.

(v(α) here stant for the value the condition C in α, c stant for a very small positive real number.)

4.Margin for error theory —— the relastionship with (1i) and (1i')

The “margin for error ” is based on these two requirements,so it can explain how ($1_{i'}$)and (1_i)work in turn. Take the paradox of the valley pile as an example,let c=1,G(x)means“A valley pile is made up by x grains”,K means “know”.According to the “margin for error” theory,we have:



$K[G(m)] \rightarrow [G(m-1) \wedge G(m) \wedge G(m+1)]$

If $|n-m| \leq 1$ then, $K[G(m)] \rightarrow G(n)$ **(1i')**

If $|n-m| \leq 1$ then, $K[\neg G(m)] \rightarrow \neg G(n)$

⇕

$G(n) \rightarrow \neg K[\neg G(m)]$ **(1i)**

4.Margin for error theory —— the relastionship with (1i) and (1i')

The relationship among (1_i),$(1_{i'})$ and margin for error theory can be more clearly when we analyze the resource of the margin for error theory.

Williamson used to give an arguement in Chapter 4 of *Knowledge and it's limits* to show the origin of the this theory.The argument based on the following conditions:

(6) Suppose that for non negative real numbers v, such as the height of the tree, for every case α, whether the condition C obtains in α depends only on the value v(α) of v in α.

4.Margin for error theory —— the relastionship with (1i) and (1i')

(7) For all cases α and β, if v(α) = v(β) then C obtains in α if and only if C obtains in β.

(8) for some small positive real number c,for all cases α and non-negative real numbers u, if | u-v(α)|< c and in α one believes that C obtains then, for some case β close to α, v(β)= u and in β one believes that C obtains.

(9) For all cases α and β, if β is close to α and in α one knows that C obtains, then in β one does not falsely believe that C obtains.

(10) For all cases α, if in α one is in a position to know that C obtains then, for some case β, v(α)= v(β) and in β one knows that C obtains.

(11) For all cases α, if in α one knows that C obtains then in α one believes that C obtains.

◆ **4.Margin for error theory** —— the relastionship with (1i) and (1i')

Then suppose that one is in a position to know C obtains in α and $| v(\alpha)-v(\beta)| < c$. By (10), for some case α^*, $v(\alpha) = v(\alpha^*)$ and in α^* one knows that C obtains. Thus $|v(\alpha^*)-v(\beta)| < c$ and, by (11), in α^* one believes that C obtains. Consequently, by (8), for some case β^* close to α^*, $v(\beta^*) = v(\beta)$ and in β^* one believes that C obtains.Since β^* is close to α^* and in α^* one knows that C obtains, by (9) in β^* one does not falsely believe that C obtains.Therefore, C obtains in β^*. Since $v(\beta^*) = v(\beta)$, C obtains in β by (7).

Then we have :

For all cases α and β, if $| v(\alpha)-v\ \beta)| < c$ and in α one is in a position to know that C obtains then C obtains in β.

4.Margin for error theory —— the relastionship with (1i) and (1i')

(8) for some small positive real number c,for all cases α and non-negative real numbers u, if | u-v(α)|< c and in α one believes that C obtains then, for some case β close to α, v(β)=u and in β one believes that C obtains.

(9) For all cases α and β, if β is close to α and in α one knows that C obtains, then in β one does not falsely believe that C obtains.

(8) is just the portrayal of "cognitive limits".But this "cognitive limit" is not directly applied to the concept of knowledge, but used in the scope belief .Then by the relationship between "know" and "believe",making this principle play a role in knowledge.(9) is the reliability of knowledge.

4.Margin for error theory —— Blockade of KK Principle

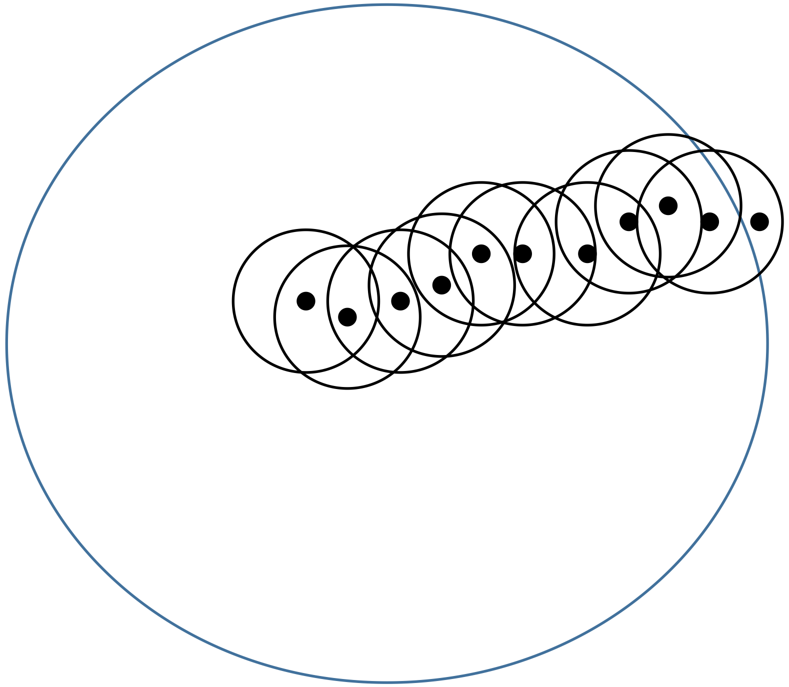
This principle plays an important role in Williamson's series of arguments,which he thought that “margin for error principle usually blocks luminosity”.He gave us an example to explain how it all happened.

He believes that knowledge has a safe boundary, like a circle drawn on a wall,and our belief is like a shot.When a bullet falls into the circle, it means that beilief is true,and thus that piece of knowledge can be formed.In other words, knowledge is a safe shot.

“Know C” means	one shot
“Know that Know C”	two shot
“Know that Know that ……Know C”	Continuous shooting

4.Margin for error theory —— Blockade of KK Principle

The repeated use of the KK principle means that the circle is drawn continuously each time with a new landing point as the center of the circle and the radius of *c*



Although ***c*** is a very small non-negative real number, after continuous accumulation, there will always be moments beyond the security boundary of C.

5.Question the margin for error theory —— Intuitive counterexamples

Brueckner and Fiocco once talked about the following example :

Consider ... the situation of a generally well-informed citizen N.N. who has not yet heard the news from the theater where Lincoln has just been assassinated. Since Lincoln is dead, he is no longer President, so N.N. no longer knows that Lincoln is President (knowing is factive). However, N.N. is in no position to know that anything is amis.(Williamson.T 2000:23）

And they think if we let t be one millisecond before Lincoln dies, and let 'L' stand for"Lincoln is President",then we have "KL" at t_i and "!L" at $t_{i+1.}$So they said that " Thus, unless Williamson can somehow distinguish the Lincoln example from that of S's feeling cold, the support for (R) is wholly undermined and the anti-luminosity argument is blocked" (Brueckner. A. and Fiocco. M.O. 2002:288)

5.Question the margin for error theory —— Intuitive counterexamples

But it is not a good challenge to the verson of the margin for error theory we give above.

For that in the two cases before and after Lincoln's murder,the condition C need to be valued is “Lincoln is President”.But in these two cases the value of this condition are great different,althouht other conditions may sufficiently similar.Since that weather Lincoln is President is a factual condition,it’s value in a case has nothing to do with people's perception.So in these two cases ,there isn’t a small non-negative real number *c* as the margin for error .It does not accord with the hypothesis which margin for error theory required.

5.Question the margin for error theory —— Intuitive counterexamples

But the example of “one’s feeling cold” is different ,the condition C need to be valued is someone’s feeling.Judging wether this condition is obtained in α_i and α_{i+1} is not because that the two cases are only one millisecond apart, but the valuation of this condition in α_i and α_{i+1} satisfy the formula $| v(\alpha_i)-v (\alpha_{i+1})|< c$.That is the distinction between Lincoln example and that of S's feeling cold which Brueckner and Fiocco wanted.

Let’s put it simply,our use of the margin for error principle does not depend on the similarity of the situations but on the approximation of the valuations of a same condintion in two cases. That’s why when we analyze the margin for error priciple above,we ignore the cases.

5.Question the margin for error theory —— Intuitive counterexamples

In fact, we don't have to work so hard to imagine two similar cases to construct a counterexample.Just look at the following instance from the Williamson's puzzle[1]:

Mr Magoo knows that the tree is not 667 inches tall,then the tree is not 666 inches tall.

In this instance,the condition C need to be valued is "is the height of that tree".Mr Magoo's eyesight is not enough to distinguish the error of 1 inche,so there is a ***c*** (margin for error) for the tree height judgment. In this way, when the theory of margin for error is established, "the tree is not 666 inches tall" should be a true proposition, which is obviously wrong.

5.Question the margin for error theory —— Intuitive counterexamples

Some may question that Mr Magoo shouldn’t know the proposition that “this tree is not 667 inches tall” in that situation , but margin for error theory does not require that in a particular situation, as long as Mr Magoo can know that “the tree is not 667 inches tall” in a certain situation,that is why Williamson use “in position to know”, then this counterexample is established.

5.Question the margin for error theory —— The scope of application

Williamson thinks that knowledge has a safe boundary and the examples he gives are cleverly applied to meet this demand .Such that people always feel cold in a certain temperature range or remember certain piece of knowledge within a certain time frame.But think about the following beliefs:

- "The earth is round."
- "My name is HU Lanshuang"
- "The skyscraper has 50 stories"

Do they have safe boundaries or aren't they qualified to form knowledge?

Obviously,for these beliefs there isn't any circle drawn on the wall for shot.There just a point there and no space for iteration. So,like the Lincoln's example,the margin for error principle apparently not applicable to these kind of knowledge.

5.Question the margin for error theory —— The scope of application

May be Williamson himself realized this problem too,and he made a clever adaptation.He knows that the following proposition is bound to arouse suspicion as a precondition:

If Mr Magoo knows that the tree is i inches tall ,then the tree is i+1 inches tall.

Because the condition "the tree is i inches tall" is a factive condition which doesn't have safe boundary.But it's negative proposition "the tree is not i inches tall" does have a certain scope of application.Therefor in Williamson's $puzzle_1$,the main part of the premise (1_i) is the negative substitution of the margin for error theory .But the negative substitution like the following from is pretty weird too:

If Mr Magoo knows that the tree is not i inches tall ,then the tree is not i+1 inches tall.

5.Question the margin for error theory —— The scope of application

This is the counterexample we have talked about.So Williamson usd it’s equivalent form to be the mean part of premise (1_i) instead of it.

If the tree is i+1 inches tall ,then Mr Magoo doesn’t know the tree is not i inches tall.

Strangely enough, although the two statements are equivalent, one is obviously unacceptable and the other is interpretable.Perhaps it's because people have different requirements for "knowing" and "not knowing",and Williamson just used it skillfully.

◆ 5.Question the margin for error theory —— The scope of application

Even if this transformation is allowed, does it mean that all conditions without security boundaries can be solved by margin for error’s negative substitution and it’s equivalence?

The answer is still no.We still don't know how to use margin for error theory to transform the following propositions:

- “The earth is round.”
- “My name is HU Lanshuang”

Even though there is a certain scope of application of the condition “the earth is not square” or “my name is not LIU Hulan”.But those shapes which are not square or those names which are not LIU hulan don’t have iterative relationship between each other,but the heights that isn’t 666 inches does.

5.Question the margin for error theory —— The scope of application

Futher more,only iterative relationship is not enough,since we always don’t agree the following proposition:

If the skyscraper has 50 stories, S doesn’t know the skyscraper has 49 stories.

Because if we stand in appropriate position ,our eyesight can still distinguish the height of one story ,but no matter where we stand, it seems that we can't distinguish the height of one story. So another prerequisite is that people have cognitive difficulties with this iterative relationship.

5.Question the margin for error theory —— The scope of application

In short, the use of the margin for error principle depends on the following three conditions:

- The knwledge which have a safe boundary.
- The beliefs in the safe boundary have iterative relationship. .
- People have cognitive difficulties with this iterative relationship.

5.Question the margin for error theory —— The scope of application

Under such restrictions, the scope of application of this principle is very limited.

Someone may say that ,even if this principle is not a general principle of knowledge, it can still be an effective rule in a certain field of knowledge.So,at least in that field ,KK principle is invalid ,then KK principle is not an effective description rule of knowledge.But don't forget the counterexample we have found,Williamson's series of practices merely hide the problem.In fact,the margin for error principle not only has a finite scope of application,but also have a source problem.

5.Question the margin for error theory —— source problem

Since this margin for error theory is derived from a series of known premises, we can know whether this principle is valid by examining the premises and the process of the inference.In the course of the inspection, two premises attracted our attention.They are the premises (6) and (9):

(6)Suppose that for non negative real numbers v, such as the height of the tree, for every case α, whether the condition C obtains in α depends only on the value v(α) of v in α.

(9) For all cases α and β, if β is close to α and in α one knows that C obtains, then in β one does not falsely believe that C obtains.

5.Question the margin for error theory —— source → problem

The premise (6) shows that whether a condition is obtained in a case is only related to the value of this condition under this situation, and has nothing to do with others. Precondition (9) shows that, as long as one condition may be known in one case, then, in a case similar to it , no matter what the value of that condition is, one won't falsely to believe the condition obtains .

If "one does not falsely believe that condition obtains" and "It actually obtains" mean different things,then the premise (6) and (9) do not have a contradiction, but Williamson just did the same explanation from the reasoning processes he gave:

" ……by (9) in β one does not falsely believe that C obtains.Therefore, C obtains in β*. ……"*

(Williamson.T 2000:128)

5.Question the margin for error theory —— source problem

Williamson should give an explanation about why "one does not falsely believe that condition obtains" means "It actually obtains", because if they mean the same thing ,then weather a condition C obtains in α will not depends only on the value v(α) of v in α,that contrary the requirement of (6).

So if we insist on the rationality of premise (6), then "In β* one does not falsely believe that C obtains.Therefore, C obtains in β*" is a wrong reasoning process and the theory of margin for error can not be reached.

◆ 5.Question the margin for error theory —— Double standards problem

Some may think that,literally speaking, “one does not falsely believe that condition obtains” and “It actually obtains”can both give the same explanation,cause that “not falsely believe” means “believe that condition obtain is true”,then “the condition obtains”.Williamson probably thought in the same way.But there is a serious problem of double standards in doing so.

There are double standards for judging whether a condition is obtained in a case. The premise (6) means that weather a condition in a case is obtained only depends on the valuation of the condition in that case, and the condition (9)claim that weather a condition is obtained in a case determined by weather it has been know by someone in another similar case. These two standards play a role in one argument at the same time, but not in a reasonable way.

5.Question the margin for error theory —— Double standards problem

Willianson's puzzle$_1$

"the tree isn't 0 inche tall"is true

"the tree is not 666"inches tall is false

"the tree is not 1-665 inches tall"

actural situation of the tree

is none of the bussiness of the situation of the tree ,but deverid from a principle of cognitive limits by human.

why can people's cognitive ability affect the actual situation of tree height?
why we still require that knowledge inferred under cognitive limitations still imply truth?

5.Question the margin for error theory —— Double standards problem

That doesn’t make sense ,because we human have cognitive limits just means that sometime we don’t have ability to figure out the truth. Imagine the following :

Mr Magoo saw a tree there one day ,a man who accurately measured the height of the tree told him that “This tree is 665 inches tall”,then he knew that the tree is 665 inches tall. At the same time the next day, he passed the tree. He looked carefully at the tree and thought it was no different from yesterday, so his knowledge of the height of the tree remained unchanged. But in fact, the tree grew 1 inch in a day, but Mr. Margu's vision was not enough to distinguish.

5.Question the margin for error theory —— Double standards problem

Represent two days of the same time with α and β respectively. Then we know that v(α)=665 ,v (β)=666 and| v(α)-v (β)|<c,this is in line with the application requirements of "margin for error" principle.So,according to the margin for error principle Mr Margoo in pisition to know that " This tree is 665 inches tall"in α,then "the tree is 665 inches tall "obtains in β.But in fact the tree is 666 inches tall in β.

So if we want to acknowledge both the influence of objective facts and the influence of cognitive limits of human on a condition,we can't avoid the conflicting result.

5.Question the margin for error theory —— Double standards problem

Furthermore, Williamson's double standard problem can be further extended to his discussion of the attribute of knowledge.

knowledge

- facitive — objective , entail truth, not depend on subjects
- mental state — intentional,inexact cognitive limitations

→ how can a belief that constitute knowledge is true and inexact at the same time?

5.Question the margin for error theory —— Double standards problem

There is a feasible way to solve this problem —weakening the truth condition.

- regarding“truth” as a cognitive concept

  the proposition of “knowing” is true only if it is relative to the cognitive subject.

- regarding “truth” as a presupposition.

  “truth”means a certain “guarantee” ,even though this guarantee doesn’t work, it doesn’t prevent it from being presupposed as a pragmatic act.

5.Question the margin for error theory —— Double standards problem

Williamson naturally disagrees with the method above.

On the one hand ,he explicitly objected the idea that "truth" is a kind of cognitive concept: "I completely reject the assertion that truth is a cognitive concept of any kind." The "truth" contained in knowledge is absolute truth.

On the other hand, he insisted that people have cognitive limitations, "knowledge is inexact, and always be known in an inexact way"(Williamson.T 1992:217).

It means that the inexact knowledge under a limit cognitive ability must implicates absolute truth.Such attitude to knowledge unavoidable lead to some contradictory results.

However, he completely attributed the contradiction to "luminosity" and "KK principle", which we think are really innocent.

6. Summary

1. Although the closure principle is a rather suspect condition, it is not the main reason for the puzzles,cause it even doesn't appare in Williamson's $puzzle_{2.}$

2. KK principle and other rules, though, appear in both puzzles,but only play a minor role in iterative operations.The primises which progress a condition C from one case to another are (1_i) and ($1_{i'}$) in two arguments.

3. When they are explained separately, both (1_i) and ($1_{i'}$) face their own problems,such as the problem of incoherence and rationality.

4. They also can be interpreted as different forms of the "margin for error theory" which usually block the KK principle. But after a careful examination of the margin for error principle ,we found that it is not only limited in it's scope of application, but also has a source problem.

6.Summary

5. Therefore, if we insist on both the margin for error principle and the truth condition in a one argument, we will have contradictory results because of the double standard problem. That is why the Williamson’s puzzles arises.Of course,we can't all blame the KK principle,which means that Williamson's opposition to the KK principle is not valid.

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