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Conditionals

Theories of Minimal Changes and Disputes

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Purposes of This Presentation

1. Share my interests
2. Have a overview about conditionals, and learn some general ideas of Minimal Theories in conditionals.
3. Feel fun in this kind of topics.
   And maybe we can cooperate together in the future.

Introduction to Conditionals

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Introduction to Conditionals

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Material Implication

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What is conditional?

Paradigmatically*, a conditional declarative* sentence in English* is one which contains ”if” and ”then”.

Examples:

1. If I were warm, (then) I would remove my jacket.
2. If it is raining, (then) we are taking a taxi.

What is conditional?

Paradigmatically*, a conditional declarative* sentence in English* is one which contains ”if” and ”then”.

Examples:

1. If I were warm, (then) I would remove my jacket.
2. Were I warm, I would remove my jacket.

Indicative VS Subjunctive/Counterfactual

1. If Oswald didn't kill Kenndy, someone else did.*
2. If Oswald hadn't kill Kenndy, someone else would have.

Well-known Oswald/Kennedy minimal pair due to [Adams, 1970]

Indicative VS Subjunctive/Counterfactual

1. If shakespeare didn’t write Hamlet, someone else did.
2. If shakespeare hadn’t write Hamlet, someone else would have.

- Intuitively, the Indicative Conditionals are true, while the Subjunctive Conditionals are false.

Special Conditionals

- Counterlegal
- If the gravitational constant were to take on a slightly higher value in the immediate vicinity of the earth, then people would suer bone fractures more frequently.

Special Conditionals

- Counteridentity
- If I were the pope, I would support the use of the pill in India.

Special Conditionals

- Backtracker conditional & Non-backtracker conditional
- If Reagan were dead, Hinckley would have been a better shot.
- If Hinckley had been a better shot, Reagan would be dead.

Our Focus in This Presentation

The conditionals we are concentrating are declarative Subjunctive Conditionals like:

- If Oswald hadn't kill Kenndy,someone else would have.
- If my bicycle were broken this afternoon, I would be late for this presentation.

And we don't handle special conditionals in this presentation.

Notation

We formalize such conditionals as "A > B"(A Corner B), following Stalnaker's notation.

Why Do Conditionals Matter?

- We use them in different situations: ordinary conversations, making decision, scientific enterprises and philosophical queries.
- What in the world makes a declarative conditional sentence true?
- Giving a proper semantics is not that easy, since conditionals have their own personality.

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Invalid Patterns

Transitivity

$$(A > B) \to [(B > C) \to (A > C)]$$

Failure of Transitivity

If Hoover had been a Communist, he would have been a traitor.
If Hoover had been born in Russia, he would have been a Communist.
$\not\Rightarrow$ If Hoover had been born in Russia, he would have been a traitor.[Stalnaker, 1968]

Invalid Patterns

Strengthening/Weakening* the Antecedent

$$(A > B) \to [(A \wedge C) > B]$$

Failure of Strengthening the Antecedent

If kangaroos had no tails,they would topple over.
$\not\Rightarrow$If kangaroos had no tails but used crutches,they would topple over.[Lewis, 1973]

Invalid Patterns

Contraposition

$$(A > B) \rightarrow (\neg B > \neg A)$$

Failure of Contraposition

If it had rained,there wouldn't have been a terrific cloudburst.
$\not\Rightarrow$If there had been a terrific cloudburst, it wouldn't have rained.[Adams, 1975]

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Material Implication

$$\boxed{A > B \leftrightarrow A \to B}$$

1. If my height were 1.7meters, then my height is over 1.5 meters.
2. If my height were 1.7meters, then my height is less than 1.6 meters.*
3. If my height were 1.7meters, then my height is over 1.6 meters.
4. If my height were 1.7meters, then my height is over 1.8 meters.*

Material Implication

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4. If my height were 1.7meters, then my height is over 1.8 meters.*

The moral of these examples is that when the antecedent of an English subjunctive conditional is false, the truth value of the conditional is not determined by the truth values of the antecedent and the consequent of the conditional alone.

Conditionals cannot be material implication or any other truth function.

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Strict Conditionals

The truth value of a conditional is not always determined by the actual truth values of its antecedent and consequent, but perhaps it is determined by the truth values which its antecedent and consequent take in some other possible worlds.

Strict Conditionals

$$A > B \leftrightarrow \Box(A \rightarrow B)$$

Understood from two perspectives

- A > B is true if B is true at every physical alternative* to the actual world at which A is true.
- A > B is true if it is physically* impossible that A be true and B false.(Necessity between A and B)

If the proposal is proper, then we should expect some minimal properties which all modal logics share in conditionals, such as:

- $\Box(A \to B) \wedge \Box(B \to C) \to \Box(A \to C)$
- $\Box(A \to B) \to \Box(A \wedge C \to B)$
- $\Box(A \to B) \to \Box(\neg A \to \neg B)$

If the proposal is proper, then we should expect some minimal properties which all modal logics share in conditionals, such as:

- Translating to Conditionals:
- $(A > B) \to [(B > C) \to (A > C)]$
- $(A > B) \to [(A \wedge C) > B]$
- $(A > B) \to (\neg B > \neg A)$
- Which are not satisfied in Conditionals as said before.

- Giving above theories, you may start to wonder what are the PROPER ways to deal with conditionals.
- Before come back to our main interests in minimal change theories, let's review a brief history of conditionals.

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Although conditional logic has been studied rather intensively during the last 50 years or so, the topic has both ancient and medieval roots (starting in the Stoic school), as the monograph [Sanford, 1989] explains in details.

Before 1968−Cotenability Theory of Conditionals

A first contemporary wave of work, such as[Chisholm, 1946], [Goodman, 1955] and [Rescher, 1964].

The basic idea which these proposals share is that the conditional A>B is true in case A , together with some set of laws and true statements, entails B.

But those works did not result in an attempt to provide formal models for the semantical structure of conditionals.

The Logic of Ontic* Conditionals

Classical paper: [Stalnaker, 1968]

which deploys a possible worlds semantics for conditionals and offers an axiom system as well.

The Logic of Probabilistic Conditionals

Classical Work: [Adams, 1975]

Which develops the idea that the probability of (non-nested) conditionals is given by the corresponding conditional probability.

The Logic of Epistemic Conditionals

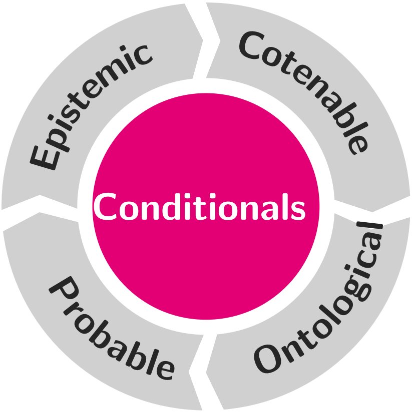
Classical Work: [Gärdenfors, 1978]

Which deals with conditionals in terms of (non-probabilistic) belief revision policies.

The basic idea is that:

A $<$ B is accepted with respect to K iff B $\in$ K*A.

Scope of Theories



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Minimal Changes and Stalnaker’s Proposal

Disputes on UA, LA and CEM

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Summary

- Strict Conditional is not the proper proposal for conditionals.
- Possible worlds semantics associated with modal logic is very attractive.
- A conditional is TRUE just in case its consequent is true at every member of some set of worlds at which its antecedent is true.

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Basic Idea of Minimal Theories

In order to make the antecedent of conditional true, we make minimal changes to our actual* world.

As a result, we obtain worlds* which are similar to actual world as much as possible.Then we consider whether the consequence is true or not in those worlds.

Basic Idea of Minimal Theories

In order to know the truth of a given conditional,

We consider whether B is true in the most similar worlds* to world i which A is true .

- The antecedent is already true in actual world,
- The antecedent is FALSE in actual world. Interesting Case

Stalnaker's Proposal

Stalnaker [1968] proposes that the conditional A > B is true just in case B is true at the world most like the actual world at which A is true.

Possible worlds correspond to these epistemically ideal situations.

Stalnaker's Restrictions on Similarity

1. Any world is more similar to itself than is any other world;
2. The A-world closest to world i is always at least as close as the A&B world closest to i;
3. If the A- world closest to i is a B-world and the B-world closest to i is a A-world, then the A -world closest to i and the B -world closest to i are the same world.

- Those restrictions can turn into a set of restrictions on the items of the models.

Stalnaker's Restrictions on Similarity

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- The class of simplied* Stalnaker models determines or characterizes the conditional logic C2.

Stalnaker's Restrictions on Similarity

1. Any world is more similar to itself than is any other world;
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3. If the A- world closest to i is a B-world and the B-world closest to i is a A-world, then the A -world closest to i and the B -world closest to i are the same world.

- None of Transitivity, Contraposition, and Strengthening Antecedents is contained in C2.

simplied Stalnaker Models

Simplified Stalnaker Models:

- Ordered quadruple $\langle I, R, s, [\,]\rangle$
- *I* is a set of possible worlds
- s is a partial world selection function which assigns to sentence A and a world i in *I* a world s(A, i) (the A-world closest to i)
- $[\,]$ is a function which assigns to each sentence A a subset [A] of *I* (all those worlds in *I* at which A is true)

Restrictions on Models:

S1 s(A,i)$\in$[A];

S2 $\langle i, s(A, i) \rangle \in R$;

S3 if s(A, i) is not defined then for all j$\in$*I* such that $\langle i, j \rangle \in R$, $j \notin R$;

S4 if i$\in$ [A], then s(A, i)=i;

S5 if s(A, i)$\in$[B] and s(B, i)$\in$[A], then s(A, i)= s(B, i)

S6 i$\in$[A $<$ B] iff s(A, i)$\in$[B] or s(A, i) is undefined.

The **conditional logic** determined by Stalnaker's model theory **C2**, which is the smallest conditional logic which is closed under the two inference rules

RCEC $$\frac{b \leftrightarrow c}{(a > b) \leftrightarrow (a > c)}$$

RCK $$\frac{(b_1 \wedge \ldots b_n) \rightarrow b}{(a > b_1) \ldots \wedge (a > b_n) \wedge (a > b)} \text{(n} \geq \text{0)}$$

and which contains all substitution instances of the theses

PC	Any axiomatization of propositional calculus
ID	$A > A$
MP	$(B > C) \rightarrow (B \rightarrow C)$
MOD	$(\neg A > A) \rightarrow (B > A)$
CSO	$[(A > B) \wedge (B > A)] \rightarrow ((A > C) \leftrightarrow (B > C))$
CV	$[(A > C) \wedge \neg(A > \neg B)] \rightarrow ((A \wedge B) > C)$

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Stalnaker's Uniqueness Assumption

Stalnaker thinks that:

At least when the antecedent of a conditional is logically possible, there is always a unique possible world which satisfies the above condition.

For those antecedents which are not logically possible, Stalnaker conceives an Absurd World in which all propositions are true.

Lewis's Rejection to Uniqueness Assumption

Lewis argues that there may be no unique A- world which is closer to i than is any other A-world. Consider a straight line printed in a book and suppose that :

- A line were longer than it is.

CEM−Conditional Excluded Middle

A virtue of the Uniqueness Assumption is that it validates the principle of Conditional Excluded Middle.

CEM: $(A > B) \vee (A > \neg B)$

if A is possible, then B must be either true or false at the closest -world.

If we follow Lewis's advice and drop the Uniqueness Assumption, we must give up CEM.

Counterexample to CEM

1. If Bizet and Verdi had been compatriots, Bizet would have been Italian.
2. If Bizet and Verdi had been compatriots, Verdi would have been French.[Quine, 1950]

Stalnaker's Way to Defend CEM

Stalnaker suggests that(1) and (2) are semantically indeterminate.

- Either if Bizet and Verdi had been compatriots, Bizet would have been Italian, or (Bizet and Verdi had been compatriots) Verdi would have been French.

It is TRUE nevertheless because it would be true under any reasonable resolution of the indeterminate.

Fraassen's Way to Defense CEM

If we accept Stalnaker's semantics together with a multiplicity* of world selection functions,it turns out that every instance of CEM is supertrue* even though it may be the case that neither disjunct of some instance of CEM is supertrue.

In fact, all the members of C2 are supertrue when we apply Van Fraassen's method of supervaluation.

Lewis's Proposal

Lewis formulate a logic and a semantics for conditionals which resembles Stalnaker's but which does not include CEM.

- Replace the US with the weaker Limit Assumption, thus replace world selection with class selection.(CS model)
- Replace CEM with CS.(Logic VC,which is weaker than C2)

Class Selection Models:

- Ordered triple $\langle I, f, [\,]\rangle$
- *I* is a set of possible worlds
- f is a world selection function which assigns to sentence A and a world i a **subset** of*I* (all the A- worlds closest to i)
- $[\,]$is a function which assigns to each sentence A a subset [A] of *I* (all those worlds in *I* at which A is true)

Restrictions on Models:

CS1 if $j \in f(A,i)$, then $j \in [A]$;

CS2 if $i \in [A]$, then $f(A, i)=i$;

CS3 if $f(A, i)$ is empty, then $f(B, i) \cap [A]$ is also empty;

CS4 if $f(A, i) \subseteq [B]$ and $f(B, i) \subseteq [A]$, then $f(A, i)=f(B, i)$;

CS5 if $f(A, i) \cap [B] \neq \emptyset$, then $f(A \wedge B, i) \subseteq f(A, i)$

CS6 $i \in [A < B]$ iff $f(A, i) \in [B]$.

The **conditional logic** determined by Class Selection Model theory **VW**, which is the smallest conditional logic which is closed under the same rules as those listed for **C2** and which contains all those theses used in defining **C2** except that we replace CEM with

CS $(A \wedge B) \to (A > B)$

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Pollock's Rejection to Lewis's Proposal

Pollock rejects Lewis's semantics and the conditional logic VC because of the following reasons:

1. Reject the thesis CV:
   $[(A > C) \wedge \neg(A > \neg B)] \rightarrow ((A \wedge B) > C)$
2. Counterexample: Conceive a circuit.
3. Embrace the Generalized Consequence Principle (GCP):
   If Γ is a set of sentences, and for each B $\in \Gamma$, and $\Gamma \vdash$ C, then A > C is true.

Pollock's Proposal

- Replace the condition (CS5) with (CS5')
  $f(A \vee B, i) \subseteq f(A, i) \cup f(B, i)$
- Replace CV with CA:
  $[(A > C) \wedge (B > C)] \rightarrow ((B \vee A) > C)$(Logic SS)

Pollock's Minimal Theory

Pollock's Minimal Theory is different from Stalnaker -Lewis's theoriesPollock conceives of what would count as a minimal change quite differently from the way Stalnaker& Lewis do.

- Set of true basic strong subjunctive generalizations*
- Set of true simple states of affairs*
- The justification conditions for a subjunctive conditional A > B are stated in terms of making minimal changes in these two sets in order to accommodate .

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Summary of Minimal Theories

UA/LA /CEM

- If Lewis's counterexamples to UA/LA/CEM are conclusive,
- Then the only adequate theory is Lewis' SOS semantics.

Summary of Minimal Theories

GCP

- If the GCP is a principle which we wish to preserve,
- Then Lewis' SOS semantics is also inadequate.

Summary of Minimal Theories

Common

Thesis CS: $(A \wedge B) \rightarrow (A > B)$

- When the antecedent of the conditional is true, the actual world is the unique closest antecedent world and hence the only world to be considered in evaluating the conditional.
- But it may contradict to our intuition.

Summary of Minimal Theories

Common

Thesis CS: $(A \wedge B) \to (A > B)$

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Thank You