Categorical Quantum Mechanics and Its Applications

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Outline

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- A change is also called a process.

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We have to use brackets here to avoid confusion as follows:

$$A \xrightarrow{g \circ f} C = A \xrightarrow{f} B \xrightarrow{g} C$$

Associativity

- Finite sequential composition makes sense without brackets:

$$A_1 \xrightarrow{f_1} A_2 \xrightarrow{f_2} \cdots \xrightarrow{f_{n-1}} A_n$$

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- This important property is called associativity.

Identity

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- As a consequence, composite process with identity process involved has the following property:

$$(A \xrightarrow{1_A} A \xrightarrow{f} B) = (A \xrightarrow{f} B) = (A \xrightarrow{f} B \xrightarrow{1_B} B)$$

$$i.e., \qquad f \circ 1_A = f = 1_B \circ f$$

Categories

If we summarise the above properties of processes, then we have the definition of a category:

A category $\mathfrak{C}$ consists of:

- a class of objects $ob(\mathfrak{C})$;
- for each pair of objects A, B, a set $\mathfrak{C}(A, B)$ of morphisms from A to B;
- for each triple of objects A, B, C, a composition map

$$\begin{array}{ccc} \mathfrak{C}(B, C) \times \mathfrak{C}(A, B) & \longrightarrow & \mathfrak{C}(A, C) \\ (g, f) & \mapsto & g \circ f; \end{array}$$

- for each object A, an identity morphism $1_A \in \mathfrak{C}(A, A)$,

satisfying the following axioms:

- associativity: for any $f \in \mathfrak{C}(A, B), g \in \mathfrak{C}(B, C), h \in \mathfrak{C}(C, D)$, there holds $(h \circ g) \circ f = h \circ (g \circ f)$;
- identity law: for any $f \in \mathfrak{C}(A, B), 1_B \circ f = f = f \circ 1_A$.

Slogan

A category can be seen as a closed system of processes!

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$$(A \xrightarrow{g \circ f} C) = (A \xrightarrow{f} B \xrightarrow{g} C)$$

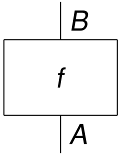
- The simple property of an identity arrow is not shown as simple as a tautology:

$$(A \xrightarrow{1_A} A \xrightarrow{f} B) = (A \xrightarrow{f} B) = (A \xrightarrow{f} B \xrightarrow{1_B} B)$$

Processes as diagrams

We introduce boxes and wires to denote processes:

- A general process $A \xrightarrow{f} B$ can be represented by a box:



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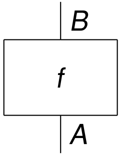
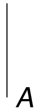


- The identity process $A \xrightarrow{1_A} A$ can be represented as a wire:



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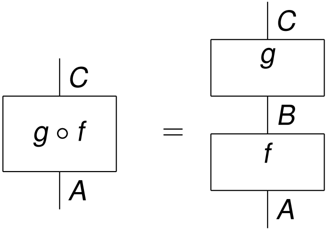
Deficiency is now turned into efficiency:

- Equality of processes without brackets:



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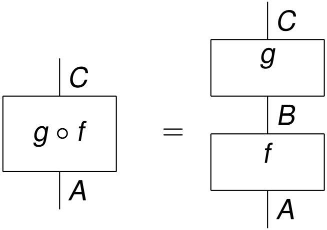
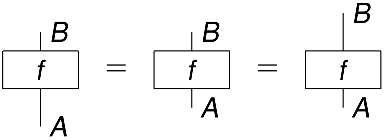
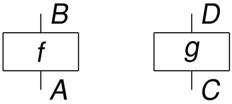


- Property of an identity is shown as a tautology:



Spatial composition of processes

Processes not only happen sequentially in time, but also happen simultaneously in space. Two arbitrary simultaneous processes f, g can be represented by diagrams as follows:



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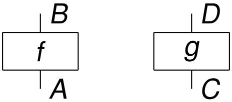

Placing two diagrams in parallel can also be seen as a (spatial) composition denoted by $\otimes$. The resulted composite process from the above composition is just $f \otimes g$.

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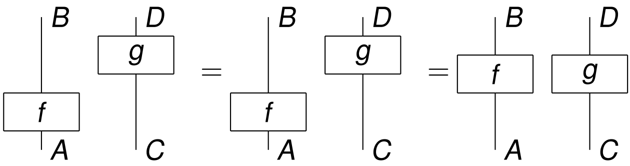
- An process can also be empty, which means nothing happened. Empty process is denoted by an empty diagram as well.
- Apparently, a box slides freely along a wire still represent the same process. Therefore, the following equalities are just a tautology:



Strict Monoidal Category

To sum up, we now arrive at the definition of Strict Monoidal Category which you definitely do not want to memorize.

A strict monoidal category consists of:

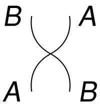
- a category $\mathfrak{C}$;
- a unit object $I \in ob(\mathfrak{C})$;
- a bifunctor $- \otimes - : \mathfrak{C} \times \mathfrak{C} \longrightarrow \mathfrak{C}$,

satisfying

- associativity: for each triple of objects A, B, C of $\mathfrak{C}$, $A \otimes (B \otimes C) = (A \otimes B) \otimes C$; for each triple of morphisms f, g, h of $\mathfrak{C}$, $f \otimes (g \otimes h) = (f \otimes g) \otimes h$;
- unit law: for each object A of $\mathfrak{C}$, $A \otimes I = A = I \otimes A$; for each morphism f of $\mathfrak{C}$, $f \otimes 1_I = f = 1_I \otimes f$.

Introducing the swap

- Two objects (systems) can swap their positions, this can be represented by the following diagram:



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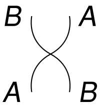
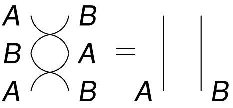
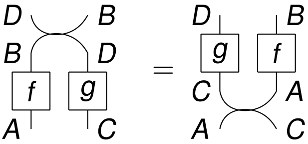


- Swapping a swap will undo a swap:



Naturality of swaps and SMC

- Boxes can move freely through a swap:



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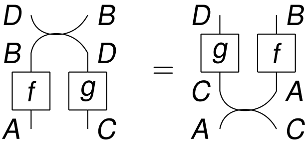


- Now we obtain a strict symmetric monoidal category :
  A strict monoidal category $\mathfrak{C}$ is symmetric if it is equipped with a natural isomorphism

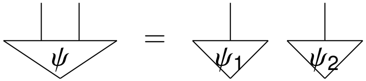
$$\sigma_{A,B} : A \otimes B \to B \otimes A$$

for all objects A, B, C of $\mathfrak{C}$ satisfying:

$$\sigma_{B,A} \circ \sigma_{A,B} = 1_{A \otimes B}, \;\; \sigma_{A,I} = 1_A, \;\; (1_B \otimes \sigma_{A,C}) \circ (\sigma_{A,B} \otimes 1_C) = \sigma_{A,B \otimes C}.$$

Introducing Cap and Cup

- All states in the classical world separate:



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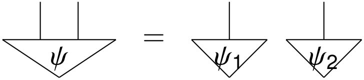

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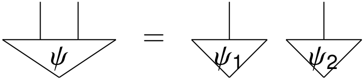

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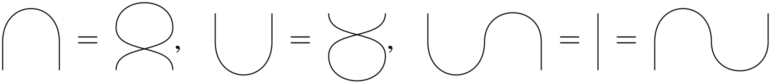
- Cap and cup satisfy the following rules:



Self-dual strict compact closed category

With the cap and cup, we have a self-dual strict compact closed category:
A self-dual strict compact closed category is a strict symmetric monoidal category $\mathfrak{C}$ such that for each object A of $\mathfrak{C}$, there exists two morphisms

$$\epsilon_A : A \otimes A \to I, \quad \eta_A : I \to A \otimes A$$

satisfying:

$$(\epsilon_A \otimes 1_A) \circ (1_A \otimes \eta_A) = 1_A, \quad (1_A \otimes \epsilon_A) \circ (\eta_A \otimes 1_A) = 1_A.$$

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- A cap is interpreted as the linear map $\sum_{j=0}^{d-1} \langle jj|$.

ONB measurements in CQM

- Suppose we wish to measure a quantum state ρ in an ONB (orthonormal basis) $\{|x_i\rangle\}$, where ρ is a positive operator. The probability of getting the i–th measurement outcome is computed using the Born rule:
  $Prob(i, \rho) = Tr(|x_i\rangle\langle x_i|\,\rho) = \langle x_i|\rho|x_i\rangle$

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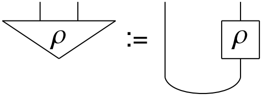
- The operator ρ can be represented as 

, which corresponds to a state vector 

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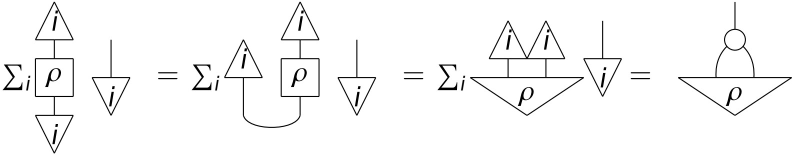
- Then the diagram for the probability distribution vector can be derived as:



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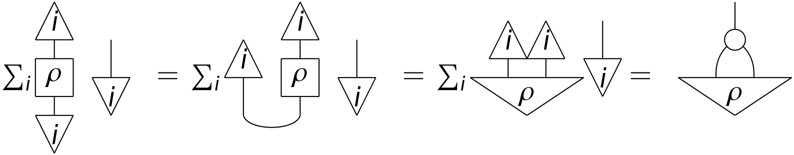
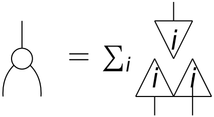


- Operate on a state then obtain a probability distribution, that is exactly a measurement:



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- The Key idea of Categorical Quantum Mechanics (CQM) is to represent quantum processes by string diagrams and then reason with diagrams by graphical rewriting, in an intuitive way, while the underlying mathematics is hidden.

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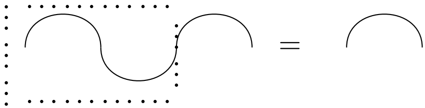
- The Key idea of Categorical Quantum Mechanics (CQM) is to represent quantum processes by string diagrams and then reason with diagrams by graphical rewriting, in an intuitive way, while the underlying mathematics is hidden.
- By rewriting we mean replace a sub-diagram with another diagram according to a graphical rule. Here is an example:



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- Qubit QM means complex vector spaces of dimensions 2^n and linear maps between them. Therefore, everything is based on the computational basis $\{|0\rangle, |1\rangle\}$.
- The way for evolving into ZX-calculus is to fill in the boxes with spiders.

Generators of the ZX-calculus

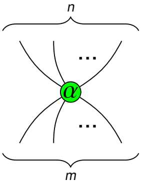
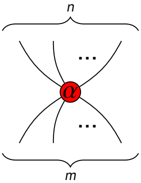
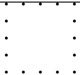
$R_{Z,\alpha}^{(n,m)} : n \to m$		$R_{X,\alpha}^{(n,m)} : n \to m$	
$H : 1 \to 1$		$\sigma : 2 \to 2$	
$\mathbb{I} : 1 \to 1$		$e : 0 \to 0$	
$C_a : 0 \to 2$		$C_u : 2 \to 0$	

Table: Generators of qubit ZX-calculus

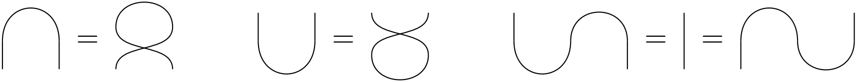
where $m, n \in \mathbb{N}$, $\alpha \in [0, 2\pi)$, and e represents an empty diagram.

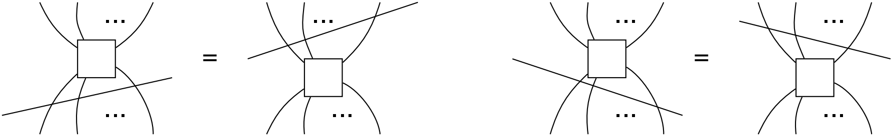
Generators of the ZX-calculus

$L : 1 \to 1$ 	$T : 1 \to 1$ 

Table: New generators with $\lambda \geq 0$.

Structural rules of the ZX-calculus





Non-structural rules of the ZX-calculus

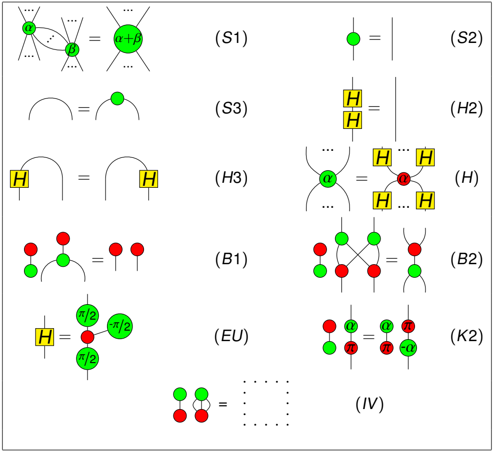


Figure: Non-structural ZX-calculus rules, where $\alpha, \beta \in [0,\ 2\pi)$.

Note that all the rules enumerated in Figures 1 still hold when they are flipped upside-down. Due to the rule (H) and (H2), the rules in Figure 1 have a property that they still hold when the colours green and red swapped.

Non-structural rules of the ZX-calculus

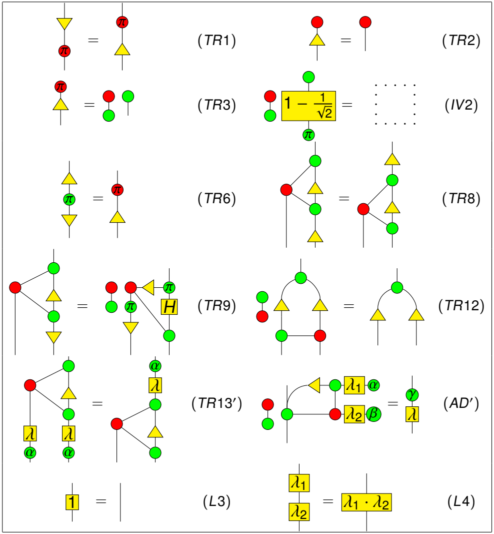
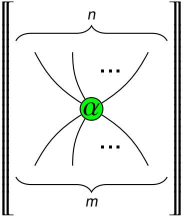
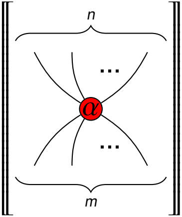


Figure: Extended ZX-calculus rules, where $\lambda, \lambda_1, \lambda_2 \geq 0, \alpha, \beta, \gamma \in [0,\ 2\pi)$; in (AD′), $\lambda e^{i\gamma} = \lambda_1 e^{i\beta} + \lambda_2 e^{i\alpha}$.The upside-down version of these rules still hold.

Standard interpretation for the ZX-calculus

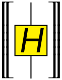
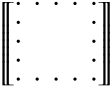


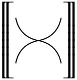
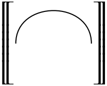
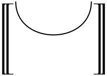
$$= |0\rangle^{\otimes m}\langle 0|^{\otimes n} + e^{i\alpha}\,|1\rangle^{\otimes m}\langle 1|^{\otimes n},$$

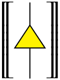
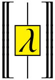


$$= |+\rangle^{\otimes m}\langle +|^{\otimes n} + e^{i\alpha}\,|-\rangle^{\otimes m}\langle -|^{\otimes n},$$

Standard interpretation for the ZX-calculus

 $= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$,  $= 1$,  $= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$,

 $= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$,  $= \begin{pmatrix}1\\ 0\\ 0\\ 1\end{pmatrix}$,  $= \begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}$,

 $= \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$,  $= \begin{pmatrix}1 & 0\\ 0 & \lambda\end{pmatrix}$.

$$[\![D_1 \otimes D_2]\!] = [\![D_1]\!] \otimes [\![D_2]\!], \quad [\![D_1 \circ D_2]\!] = [\![D_1]\!] \circ [\![D_2]\!],$$

Three properties of the ZX-calculus

- Now we are ready to define three important properties of the ZX-calculus: soundness, universality and completeness. Note that if a diagram D_1 in the ZX-calculus can be rewritten into another diagram D_2 using the ZX rules, then we denote this as $ZX \vdash D_1 = D_2$.

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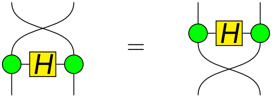
  The ZX-calculus is called universal if for any linear map L, there must exist a diagram D in the ZX-calculus such that $[\![D]\!] = L$.

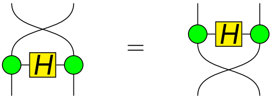
Three properties of the ZX-calculus

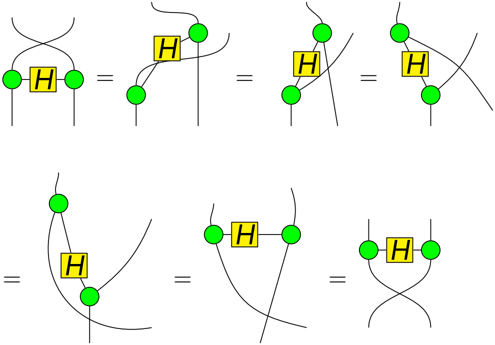
- Definition
  The ZX-calculus is called complete if for any two diagrams D_1 and D_2, $[\![D_1]\!] = [\![D_2]\!]$ must imply that $ZX \vdash D_1 = D_2$.

Examples of quantum diagram reasoning

- 

Examples of quantum diagram reasoning

- 

- 

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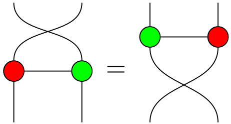
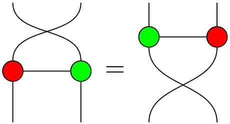
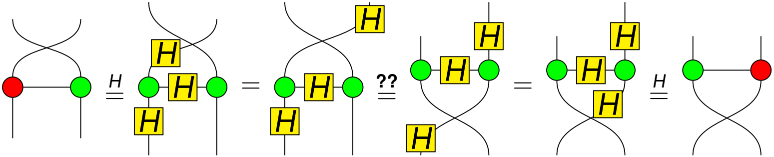
- 

Examples of quantum diagram reasoning

- 

- 

Basic quantum gates in ZX

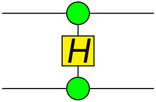
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mapsto$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto$$

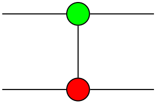
$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \mapsto$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \mapsto$$

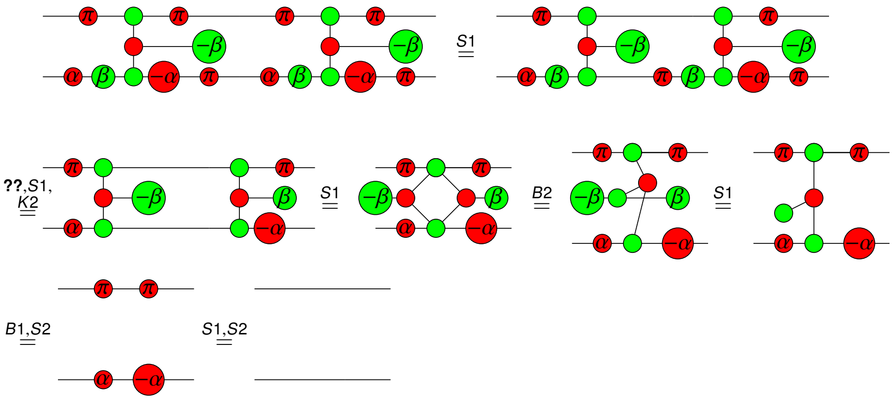
$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \mapsto$$



$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \mapsto$$

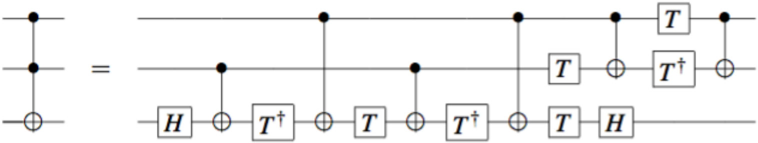


Simplify quantum circuits in ZX-calculus

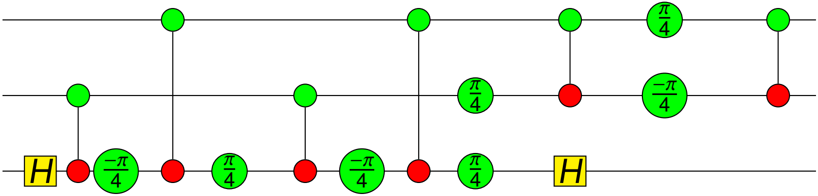


Toffoli gate in standard form

The Toffoli gate is known as the “controlled-controlled-not” gate.
The standard circuit form of Toffoli gate is given as follows:

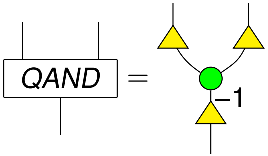


We express the circuit form of Toffoli gate in ZX-calculus as follows:



Toffoli gate in ZX form with triangles

- The quantum AND gate has the following form in ZX:



where the triangle with a −1 on the top-left corner is the inverse of the normal triangle.

Toffoli gate in ZX form with triangles

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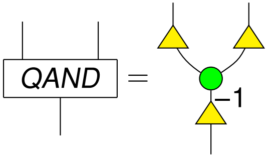
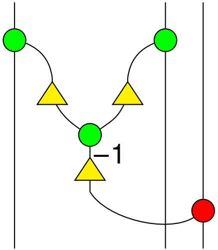


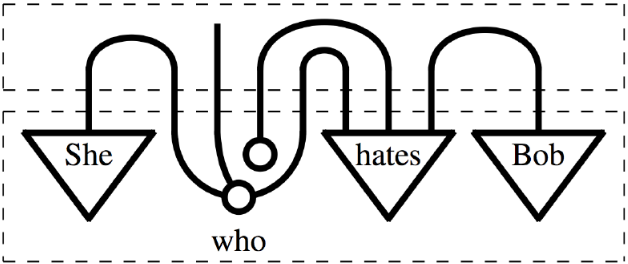
where the triangle with a −1 on the top-left corner is the inverse of the normal triangle.

- Then we can have a simple form of Toffoli gate in ZX-calculus with trianlges as follows:



Graphical calculus for Linguistics

Relative pronouns:



M. Sadrzadeh, B. Coecke & S. Clark (2013–2014) *The Frobenius anatomy of word meaning I & II*. Journal of Logic and Computation. arXiv:1404.5278

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NewScientist

WEEKLY 11 December 2010

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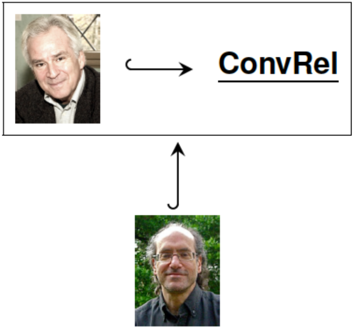
Compositional cognition



J. Bolt, B. Coecke, F. Genovese, M. Lewis, D. Marsden & R. Piedeleu (2017) *Interacting Conceptual Spaces I : Grammatical Composition of Concepts*. arXiv:1703.08314

Y. Al-Mehairi, B. Coecke & M. Lewis (2016) *Compositional Distributional Cognition*. QI'16.

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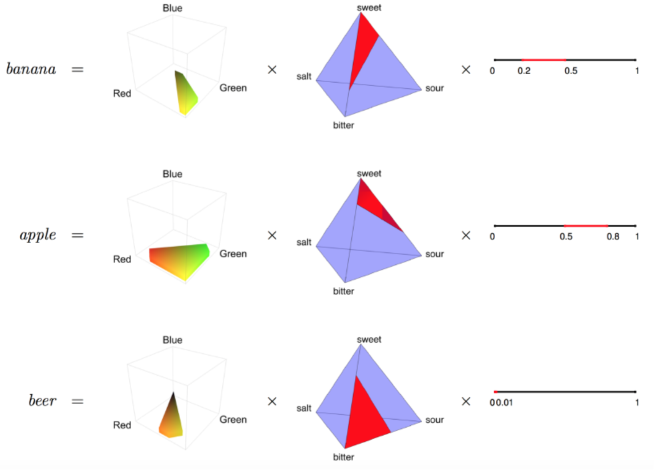
Cognitive concepts in conceptual space



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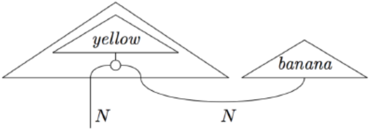
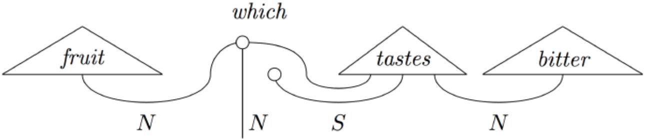
Composing concepts in string diagrams





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Further work

- Generalise the completeness result of the ZX-calculus from qubit to qudit for arbitrary dimension d.

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Further work

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- Achieve a complete axiomatization of the ZX-calculus with mixed dimensions.
- Formalise the Causality theory in the ZX-calculus.
- Efficient algorithm for Toffoli+H quantum circuits.