Global Property versus Local Update Transformer
– A Comparison of DEL and ETL

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Outline

1. Introduction

- A variety of logical systems today describe intelligent interacting agents over time. DEL and ETL among them are two well-known frameworks.
- Given an initial epistemic model and sequences of DEL event models one can generate an ETL model. This provides a concrete way of relating DEL and ETL.
- In [1], it has been proved that an ETL model is generatable by some event model iff it satisfies certain properties. In [2], it has been discussed that an ETL-like model is generatable by a fixed $\mathcal{U}$ iff it satisfies certain properties.

- There doesn’t exist an update transformer corresponding to the **Pr** property of ETL models.
- We can find a minimum set of **determinate ETL properties** (property). And try to characterize definite ETL properties.
- Given a set of determinate ETL properties (property), we can find a corresponding update transformer. And try to find an uniform transforming method.

2. Preliminaries

We start by only single agent (the situation of multi-agents is similar) and a (possibly infinite) set of events Σ. A **history** is a finite sequence of events from Σ. We write Σ^* for the set of histories built from elements of Σ. For a history h, we write he for the history h followed by the event e.

Definition (ETL Frames)

Let Σ be a set of events. A **protocol** is a set $H \subseteq \Sigma^*$ closed under prefixes. An **ETL frame** is a tuple $(\Sigma, H, \to)$ with H a protocol and a binary relation $\to$ on H.

Definition (ETL Model)

An **ETL model** is a tuple $(\Sigma, H, \rightarrow, V)$ with $(\Sigma, H, \rightarrow)$ an ETL frame and V a valuation function $V : At \rightarrow 2^H$.

An example from [3].

Definition (the Property Pr of ETL Models)

If $he \to h'$ then there is an event f with $h' = h''f$ and $h \to h''$. It corresponds to the property of synchronous perfect recall:

- **Synchronicity** iff for all $h, h' \in H$, if $h \to h'$ then $len(h) = len(h')$.
- **Perfect Recall** iff for all $h, h' \in H$, $e, e' \in \Sigma$ with $he, h'e' \in H$, if $he \to h'e'$, then $h \to h'$

Definition (the Property Nm of ETL Models)

Iff for all $h, h' \in H$, $e, e' \in \Sigma$ with $he, h'e' \in H$, if there are $h'', h''' \in H$ with $h''e, h'''e' \in H$ such that $h''e \to h'''e'$ and $h \to h'$, then $he \to h'e'$.

Definition (Epistemic Model)

Let At be a set of atomic propositions. An **epistemic model** is a tuple $(S, \rightarrow, V)$ where S is a non-empty set, $\rightarrow$ is a binary relation on $S(\rightarrow \subseteq S \times S)$ and V a valuation function ($V : At \rightarrow 2^S$).

Definition (Event Model)

An **event model** $\mathcal{U}$ is a tuple $(\Sigma, \rightarrowtail, Pre)$, where Σ is a nonempty set, $\rightarrowtail \subseteq \Sigma \times \Sigma$ and $Pre : \Sigma \rightarrow \mathcal{L}_{EL}$ is the pre-condition function.

Definition (Update Product $\otimes$)

The **product update** $\mathcal{M} \otimes \mathcal{U}$ of an epistemic model $\mathcal{M} = (S, \rightarrow, V)$ and event model $\mathcal{U} = (\Sigma, \rightarrowtail, Pre)$ is the epistemic model $(S', \rightarrow', V')$ with

1. $S' = \{(s, e) \mid s \in S, e \in \Sigma \text{ and } \mathcal{M}, s \Vdash Pre(e)\}$
2. $(s, e) \rightarrow (s', e')$ iff $s \rightarrow s'$ in $\mathcal{M}$ and $e \rightarrowtail e'$ in $\mathcal{U}$
3. $(s, e) \in V'(P)$ iff $s \in V(p)$

Definition (Update Transformer)

Given an epistemic model $\mathcal{M} = (S, \rightarrow, V)$ and an event model $\mathcal{U} = (\Sigma, \rightarrow, Pre)$, a **update transformer** $\circledast$ is a function:$\mathbb{M} \times \mathbb{B} \rightarrow \mathbb{M}(\mathbb{B} \text{ is the class of all event models })$ such that $S_{\mathcal{M} \circledast \mathcal{U}} \subseteq S_{\mathcal{M}} \times \Sigma_{\mathcal{U}}$. Note that $\circledast$ represents a class of model transformers, but $\circledast$ itself is not a concrete model transformer.

Now let $\mathbb{E} = \{(\mathcal{U}, e) \mid \mathcal{U} \text{ is an event model and } e \in dom(\mathcal{U})\}$ be the class of all event models. A **DEL protocol** is a set $P \subseteq \mathbb{E}^*$ closed under the initial segments. Given a DEL protocol P, let σ denote an element of P. We write σ_n for the initial segment of σ of length $n(n \leqslant len(\sigma))$ and write $\sigma_{(n)}$ for the nth component of σ. For example, if
$\sigma = (\mathcal{U}_1, e_1)(\mathcal{U}_2, e_2)\ldots(\mathcal{U}_n, e_n)$, then
$\sigma_2 = (\mathcal{U}_1, e_1)(\mathcal{U}_2, e_2), \sigma_{(2)} = (\mathcal{U}_2, e_2)$. Finally, let $Ptcl(\mathbb{E})$ be the class of all protocols, i.e.,
$Ptcl(\mathbb{E}) = \{P \mid P \subseteq \mathbb{E}^* \text{ and } P \text{ is closed under initial segments. } \}$

state-dependent DEL protocol
unifrom DEL protocol

σ-generated epistemic model
ETL Model Generated from a Uniform DEL Protocol.
$Forest(\mathcal{M}, P) = (\Sigma, H, \rightarrow, V)$, where $(\Sigma, H, \rightarrow, V)$ is the union of all models of the form $\mathcal{M}^{\sigma}$ with $\sigma \in P$.

Definition (p-Generated Model)

Let $\mathcal{M} = (W, \rightarrow, V)$ be an epistemic model and p, a state-dependent DEL-protocol on $\mathcal{M}$. The **p-generated model at level** n, $\mathcal{M}^{n,p} = (W^{n,p}, \rightarrow^{n,p}, V^{n,p})$, is defined by induction on n:

1. $W^{0,p} = W, \rightarrow^{0,p} = \rightarrow$ and $V^{0,p} = V$.
2. $w\sigma \in W^{n+1,p}$ iff (1)$w \in D(\mathcal{M})$, (2)$len(\sigma) = n + 1$, (3)$w\sigma_n \in W^{n,p}$, (4)$\sigma \in p(w)$, and (5)$\mathcal{M}^{n,p}, w\sigma_n \models pre(\sigma_{(n)})$.
3. For $w\sigma, v\sigma' \in W^{n+1,p}$, $w\sigma \rightarrow^{n+1,p} v\sigma'$ iff $w\sigma_n \rightarrow^{n,p} v\sigma'_n$ and $\sigma_{(n+1)} \rightarrowtail \sigma'_{(n+1)}$.
4. For each $P \in AT$, $V^{n+1,p}(P) = \{w\sigma \in W^{n+1,p} \mid w \in V(P)\}$.

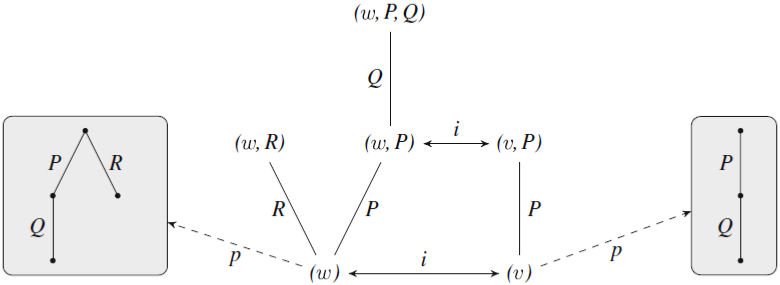
Definition (Generated ETL Model)

Let $\mathcal{M} = (W, \to, V)$ be an epistemic model and p a state-dependent DEL protocol on $\mathcal{M}$. An ETL model $Forest(\mathcal{M}, p) = (H, \to, V')$ is defined as follows:

1. $H = \{h \mid \text{there is a } w \in W, \sigma \in \bigcup_{w \in W} p(w) \text{ with } h = w\sigma \in W^{len(\sigma),p}\}$.
2. For all $h, h' \in H$ with $h = w\sigma$ and $h' = v\sigma'$, $h \to h'$ iff $len(\sigma) = len(\sigma')$ and $w\sigma \to^{len(\sigma),p} v\sigma'$.
3. For each $P \in AT$ and $h = w\Sigma \in H$, $h \in V'(P)$ iff $h \in V^{len(\sigma),p}(P)$.

Since each DEL protocol P is closed under prefixes, the definition above indeed describes an ETL model.

An example from [1]:



$$\mathbb{F}(\mathbf{X}) = \{\mathit{Forest}(\mathcal{M}, p) \mid \mathcal{M} \text{ an epistemic model and } p \in \mathbf{X}\}$$

3. A Comparision I

Theorem (Main Representation Theorem)

Let $\boldsymbol{X}^{uni}_{DEL}$ be the class of uniform DEL protocols. If an ETL model is in $\mathbb{F}(\boldsymbol{X}^{uni}_{DEL})$ then it satisfies propositional stability, synchronicity, perfect recall local no miracles, as well as local bisimulation invariance.

If an ETL model satisfies the finiteness assumption, propostiional stability, synchronicity, perfect recall, local no miracles, and local bisimulation invariance, then it is in $\mathbb{F}(\boldsymbol{X}^{uni}_{DEL})$.

Proof.

1. Suppose that $\mathcal{N} = (\Sigma, H, \rightarrow, V) \in \mathbb{F}(\mathbf{X}_{DEL}^{uni})$. We just show that $\mathcal{N}$ satisfies local bisimulation invariance. Suppsoe that $h, h' \in H$ with $h \sim^* h'$, h and h' are epistemically bisimilar, and $he \in H$ for some event $e \in \Sigma(= D(p))$. We must show $h'e \in H$. It's enough to show $h'e \in D(\mathcal{M} \otimes \mathcal{U}_1 \cdots \otimes \mathcal{U}_n \otimes \mathcal{U})$. This follows from two facts: (1) $h' \in D(\mathcal{M} \otimes \mathcal{U}_1 \cdots \otimes \mathcal{U}_n)$ and (2) $h' \vDash pre(e)$.

2. Suppose $\mathcal{N} = (\Sigma, H, \rightarrow, V)$ is an ETL model satisfying the above properties. We must show there is an epistemic model $\mathcal{M}_{\mathcal{N}}$ and a DEL protocol $P_{\mathcal{N}}$ such that $\mathcal{N} = Forest(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$. For the initial epistemic model, let $\mathcal{M}_{\mathcal{N}} = (W, \rightarrow, V')$ with $W = \{h \mid len(h) = 1\}$. □

Proof.

Call a history $h \in H$ **maximal** if there is no $h' \in H$ such that $h \prec h'$. For each maximal history $h \in H$, define the closure of h, denoted $C(h)$, to be the smallest set that contains all finite prefixes of h, and if $h' \in C(h)$ and $h' \sim^* h''$, then also $h'' \in C(h)$.
$H = \bigcup\{C(h) \mid h \text{ is a maximal history}\}$.

We define, for each maximal history $h \in H$ and $j = 1, \ldots, len(h) - 1$, an event model $U_j^h = (S_j^h, \rightarrowtail, pre)$ as follows:

1. $S_j^h = \{e \in \Sigma \mid \text{ there is a history } h \text{ of length } j + 1 \text{ in } H \text{ with } h = h'e\}$.
2. For each $e, e' \in S_j^h$, define $e \rightarrowtail e'$ provided there are histories h and h' of length $j + 1$ ending in e and e' respectively, such that $h \rightarrow h'$.
3. for each $e \in S_j^h$, let $pre(e)$ be the formula that characterizes the set $\{h \mid he \in H \text{ and } len(h) = j\}$. Such a formula does exist, due to local bisimulation invariance and the finiteness assumption.

□

Proof.

Finally, let $P_{\mathcal{N}} = \{(U)_j^h \mid h \text{ is a maxiaml history in } H \text{ and } j \leqslant len(h)\}$. It's easy to see that $Forest(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$ and $\mathcal{N}$ have the same set of histories. All that remains is to prove that the epistemic relations are the same in $Forest(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$ and $\mathcal{N}$:

The proof is by induction on the length of h and h'. For the induction step:

$(\longrightarrow)$ let $h_1 = he$ and $h_2 = h'e'$. Suppose $h_1 \sim h_2$ in $\mathcal{N}$. Then by perfect recall, $h \sim h'$ in $\mathcal{N}$. So by IH, $h \sim h'$ in $Forest(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$ as well. By the definition given above, $e \rightarrowtail e'$ in the appropriate event model $U_j^{h_m}$ for a maximal history h_m and $j = len(h_1)$. It follows by the definition of product update that $h_1 \sim h_2$ in $Forest(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$. □

Proof.

($\longleftarrow$) Assume $h_1 \sim h_2$ in $\mathit{Forest}(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$. Then, by definition of product update, $h \sim h'$ in $\mathit{Forest}(\mathcal{M}_{\mathcal{N}}, P_{\mathcal{N}})$ and $e \rightarrowtail e'$ in the appropriate event model. By the way the event model is defined, there must be some x and x' with $xe \sim x'e'$ in $\mathcal{N}$, and therefore, by local no miracles, also $he \sim h'e'$ in $\mathcal{N}$. □

Theorem

Let $\boldsymbol{X}_{DEL}$ be the class of all state-dependent DEL-protocols. Then, an ETL model is in $\mathbb{F}(\boldsymbol{X}_{DEL})$ iff it satisfies propositional stability, synchronicity, perfect recall, and local no miracles.

The above theorems identify the minimal properties that any DEL generated model must satisfy, and thus describe exactly what type of agent is presupposed in the DEL framework.

In [2], we have some relative results:

- **Pre, Inv, Nm** and **Pr** characterize the $\otimes$-generatable extended models under universal protocols.
- **PPre, Det, Inv, Nm** and **Pr** characterizes the $\otimes$-generatable extended models under arbitrary protocols.

While there are some differences:

- extended models w.r.t. some fixed $\mathcal{U}$, generatable model modulo Σ-bisimulation.
- proof strategy:

$$\mathcal{N}, t \equiv_{LDEL} \mathcal{N}^t_C, |t| \equiv_{LDEL} (N^t_C)^-, |t| \equiv_{LDEL} F^{\otimes}((N^t_C)^-), |t|$$

A Comparision II

- There doesn't exist an update transformer corresponding to the **Pr** property of ETL models.
- We can find a minimum set of **determinate ETL properties** (property). And try to characterize definite ETL properties.
- Given a set of determinate ETL properties (property), we can find a corresponding update transformer. And try to find an uniform transforming method.
- Reflection. Why is product update so important?
  Intuitively. Technically.

References

1. J. van Benthem, J. Gerbrandy, T. Hoshi, T., and E. Pacuit. Merging frameworks for interaction. *Journal of Philosophical Logic*, October 2009.
2. Wang and G. Aucher. An Alternative Axiomatization of DEL and its Applications. *Proceedings of IJCAI 2013*, May 2013.
3. A. Baltag, S. Smets. (Eds.) *Johan van Benthem on Logic and Information Dynamics*. Springer, 2014.

Thanks for your attention!!