Frege's use of function–argument analysis

Frege’s use of function–argument analysis (FAA) is fundamental to Frege’s new logic: many of his characteristic doctrines flow from this.

Frege’s logic is fundamental to analytic philosophy: e.g. Frege the key influence on Wittgenstein.

An important case study for methodology.
- Paying close attention to the genesis of Frege’s doctrines is methodologically and philosophically revealing.
- Can be used to test not only Frege’s own methodological account but also that of others.

Frege as hedgehog?

Hedgehog

- Small spiny nocturnal pig-snouted insectivorous mammal of genus Erinaceus, rolling itself up into a ball for defence.
- Porcupine, sea-urchin, or other animal similarly armed with spines.
- (Mil.) small self-contained defensive position bristling with fortifications on all sides.
- Prickly seed-vessel of some plants, e.g. corn crowfoot.
- Person hard to get on with, whence ~gy.

(Concise Oxford Dictionary, 6th ed. 1976)

Romany belief: sharp inner/outer distinction, symbolized in hedgehog; also animal who lives in margins; at death, Roma turn clothes inside out.

The hedgehog and the fox

“The hedgehog knows one big thing, the fox knows many things.”

- Isaiah Berlin’s essay about Tolstoy (a fox who wants to be a hedgehog), compared with Dostoyevsky.
- Gel’man talks of the “famous foxiness” of linguistic philosophy, and of its “hedgehog-like conviction that foxiness is the answer to everything” (1959, 195-6).

Plan

1) Methodological motivations: DJ model, elucidation, explication
2) Frege’s first step
3) Early development of Frege’s conception of a concept
4) Middle development of Frege’s conception of a concept
5) Later development of Frege’s conception of a concept
6) Conclusions
Methodological motivations: DJ model

(K) Concepts are functions (whose values are truth-values).
- Frege’s ‘discovery’ of (K) fundamental to his new logic (first expounded in 1879); yet, surprisingly, this is only made explicit in 1891
- raises problems for discovery-justification (DJ) model
  - discovery of a truth or creation of a picture (invention of a new way of seeing things)?
  - are concepts functions, or is just that they (or at least some of them) can be represented as functions?
  - if latter, then genesis is philosophically important
  - cannot be stated, let alone justified, within Frege’s ‘Begriffsschrift’ (BS)
  - so how is it ‘justified’?
    - if ‘justified’ by success of functional framework in practice, then such a practice must be established and its advantages seen
    - historical understanding needed in both cases

Frege on elucidation

- elucidation = fixing the basic concepts (specifying the Bedeutungen of basic concept words) (1)
  - requires a ‘meeting of minds’, ‘guessing what we have in mind’
- elucidatory logical analysis = articulating clearly the sense of the simple signs
  - preparing the building stones, taking place before construction of the system
- but gap in Frege’s account: very little said about this process, which seems to involve replacement of our vague basic concepts by exact ones (explication)
  - more is done in Frege’s actual practice than he seems to admit: elucidation with an historical dimension
    - e.g. Parts I-III of Grundlagen, and essays FC, SB, CO

Carnap on explication

- explication (core sense) = the replacement of a vague concept (explicandum) by a more exact concept (explicatum) (2)
  - explication (stricter sense) = replacement by defining exact concept
- clarification = making explicandum practically clear by informal explanations and examples (3)
- but gap in Carnap’s account: nothing said about how exact basic concepts are arrived at
  - call this ‘flexplication’ = non-definitional explication, adopting flexible means to get basic concepts understood
  - might also be considered as elucidation, though historical dimension must also be appreciated

Methodological map

Introduction of function-argument analysis

- Habilitationsschrift (1874): the seed from which arithmetic grows is addition
  - associated with iteration of an operation, represented by an appropriate function
  - the concept of the quantity of a function allows us to connect different areas
- 1874-79: nothing published to indicate how he got from this idea of the unifying potential of this concept to the application of FAA to logic – but e.g. 1879 paper talks of generalization
- BS (1879): first talk of FAA in §9 (7)
  - one-place functions, i.e. one-place functional expressions, stand for concepts
  - but he doesn’t yet say that concepts are functions

2 Frege’s first step

- cf. §308 of Wittgenstein’s PW (4)
  - philosophical problems typically arise from moves we make at the very beginning of our thinking
    - Frege’s use of FAA commits us to a particular way of looking at things
    - W concerned to expose the ‘proto-philosophical level’ (Goldfarb)
    - extraordinary how much of Frege’s philosophy comes from his use of FAA, which smuggles in presuppositions
- BS, preface: “replacement of the concepts subject and predicate by argument and function will prove itself in the long run” (6)
  - leads to reinterpretations such as of ‘All humans are mortal’
  - but he only drew attention to the significance of his notation for generality, and the superior power of his logic, after BS
  - implications only gradually dawned on Frege
**Function-argument analysis as a form of representation**
- Frege’s use of FAA introduced without fanfare
  - not justified, just used
  - comparison with Russell (had to fight his way to new logic)
- Frege’s use of FAA his first step, the movement he thought innocent – and never questioned
  - philosophical rationale offered later
- FAA as a form of representation
  - what counts as a function part of the symbolism, not of what is symbolized
- Frege’s own philosophy (esp. his logic) a first step in analytic philosophy, only later appreciated as such

**Taylor on uncovering origins**
- most effective way to uncover ‘first steps’ is through a genetic account (5)
  - frees one from grip of picture
- Frege’s logico-linguistic turn as revolutionary as Descartes’ epistemological turn
  - proper understanding requires undoing the forgetting
  - philosophy is essentially historical
  - historicizing Wittgenstein’s own approach

**3 Early development of Frege’s conception of a concept**
- reviews of BS led Frege to compare his system with Boole’s (8)
  1) judgements (judgeable contents) prior to concepts
  2) → ‘unsaturatedness’ of concepts
  3) concept/object distinction
  4) subsumption/subordination distinction
- all fundamental to Frege’s philosophy, arising from thinking through his use of FAA

**Frege’s theses concerning concepts (rooted in FAA)**
- Letter to Marty, 29.08.1882 (9)
  - A distinction must be drawn between concept and object.
  - A distinction must be drawn between subsumption and subordination.
  - Judgements (judgeable contents) are prior to concepts.
  - Concepts are unsaturated.
  - Judgable contents have no unique analysis.
  - Concepts must be sharply defined.
  - The realm of the conceptual is the realm of the enumerable.
  - Existential judgements are assertions about concepts.
  - all, except (E), only explicitly formulated after BS
  - with qualifications to (C) and (E), content later split into Sinn and Bedeutung, all are endorsed for the rest of Frege’s life and are fundamental to his philosophy

**4 Middle development of Frege’s conception of a concept**
- Frege’s conclusion to letter to Marty: refers to ‘vicious circle’ – accepting BS requires seeing what it can do, and vice versa (10)
  - Stumpf responds, and advises Frege to explain his ideas “first in ordinary language”
  - Grundlagen (1884):
    - Parts I-III: critique of existing conceptions of number, clarification of his key ideas and claims
  - Introduction: 3 fundamental principles (11)
    1) anti-psychologistic principle
    2) context principle
    3) concept/object distinction

**Grundlagen: two further theses**
- (I) Number statements are assertions about concepts.
- (J) Every concept has an extension, which is an object.
  - (I) just a generalization of (III)
  - (J) never explicitly stated, but implicit in §§ 104, 107 (12)
- explicit definition of number just sprang upon us.
- (Na) The Number that belongs to the concept F is the extension of the concept ‘equinumerous to the concept F’.
  - only motivation the equivalence between:
- (Nd) The extension of the concept ‘equinumerous to the concept F’ = the extension of the concept ‘equinumerous to the concept G’.
  - very little talk of functions (just §11) or FAA (just implicit in §§ 70-83)
  - Frege’s ‘innocent’ first step apparently forgotten!
Theses (A) – (H) as found in *Grundlagen*

(A) endorsed as third principle; §§ 27 (fn.), 38, 51, 68 (fn.), and 97.
(B) §§ 47, 53.
(C) reflected in context principle; but not explicitly formulated.
(D) not stated.
(E) not stated, but implicit especially in §§ 62-9.
(F) §§ 54, 74.
(G) §§ 14, 24, 40, 48, 87.
(H) generalized as (I).

mention of (D) and (E) would have required talk of FAA, omitted in *Grundlagen*; may also be true of (C)

lack of talk of FAA encourages us to overlook Frege’s first step or treat it as innocent, like Frege himself

perhaps Frege saw his form of representation as transparent

5 Later development of Frege’s conception of a concept

- key issue in this development: problems with the notion of content that leads to split into the notions of Sinn and Bedeutung
- early notion of content offered as a justification of use of FAA, which involves idea that the same value can be yielded by different functions for appropriate arguments
- conceptual content = value of prop. functions
- (Lhc) Hydrogen is lighter than carbon dioxide.
- (Hch) Carbon dioxide is heavier than hydrogen.
- have same content, but reflecting different FAAs
- various problems
- mismatch opens up between ‘intuitions’ about sameness of content and equivalences within Frege’s logical system, e.g. concerning identity statements and Cantor-Hume Principle

‘Function and concept’ (1891)

- with SB and CO, FC can be seen as offering Frege’s definitive elucidation of his basic concepts
- “My starting-point is what is called a function in mathematics” (p.1)
- clarification of conception of function, introduction of truth-values, value-ranges and Axiom V
- crucial question of the paper: “how has the Bedeutung of the word ‘function’ been extended by the progress of science?” (p.12)
- two directions in which this has happened:
  1) extending the field of mathematical operations
  2) extending the field of possible arguments and values for functions
- Frege goes further still: notion of function generalised to allow any object whatsoever as either argument or value of function

Truth-values as the values of some functions (concepts)

- key passage: FC, pp.12-13
- Frege applies FAA to identity statements and finds truth-values to be the best (only?) candidates for the values of the relevant functions
- generalized to all statements
- “We thus see how closely that which is called a concept in logic is connected with what we call a function. Indeed, we may say at once: a concept is a function whose value is always a truth-value.” {14}
- extending (philosophically elaborating) functional form of representation, proposing that (Pa) be read as (Pb):
  (Pa) ‘p’ is true.
  (Pb) ‘p’ stands for (bedeutet) the True.

Truth-values as objects

- distinction between (unsaturated) concept and (saturated) object absolute
- notion of function generalised to allow any object whatsoever as either argument or value of function, understood as Bedeutungen
- truth-values must themselves be objects {15}
- but no definition of object possible: can only be elucidated
  - so (K) itself is not a proper (constructive) definition
- Frege later talks of having ‘discovered’ the two truth-values (as objects); in reality, he simply pushed his functional framework as far as it could go, ‘projecting’ appropriate entities to match his symbolic forms

Conclusions: the elucidatory story

- focus: development of Frege’s concept of a concept, which lies at the basis of his new logic (and logicist project)
  - involves replacement of our ordinary concept of a concept by a more exact concept (a concept = a function whose value is a truth-value)
  - historically, this took over a decade to be elucidated
- the ‘insight’ that concepts are functions may have been implicit from the start, but needed much philosophical elaboration to be made explicit (tidying after the event, projecting backwards)
- between BS and GG, there was a great deal of elucidation: explanation, historical elucidation, clarification, introduction of further concepts, etc: all this much richer than Frege – and Carnap – officially allow
- but all rooted in FAA, seeking to establish the functional framework
- notions of elucidation, etc., are themselves part of the elucidation (extra-systematic or meta-theoretical ‘justification’)
- can take different forms, and hence be involved in processes of ‘forgetting’
Frege’s elucidation of concept of a concept

Ordinary concepts of a function, value, content, sense, reference, etc.

Clarification

Conceptual Innovation

Definition

Basic vague concepts of concept, truth, meaning, etc.

Replication

Basic exact concepts of function, truth-value, Sinn, Bedeutung, etc.

Elucidation

Concept = function whose value is a truth-value

Conclusions: the historical aspects of philosophizing

- Frege did not ‘discover’ that concepts are functions; he invented a new form of representation, the elaboration of which led him to ‘project’ new entities, introduce new ‘justificatory’ (philosophical) concepts, and lay down new definitions.

- Its advocacy/defence requires historical elucidation.
  - Coming to see how it offers a better way of looking at things (formalizing inferences, etc.)
  - And in learning the system, we take the commitments on board (like accepting the existence of numbers), have our ‘intuitions’ reshaped, etc.; normative dimension here

- Its critique requires genetic understanding.
  - Diagnosing philosophical problems and counterintuitive implications requires uncovering first steps, underlying analogies, metaphors and pictures, exposing presuppositions, recognizing rhetorical talk (e.g. ‘discovery’) and reshaping of intuitions, etc.

Final thought: defining as semiotically telescoping

- Concepts are functions (whose values are truth-values).
  - Not statement of fact (picture of a state of affairs) but encapsulation of a process (recommendation of a way of looking at things).
  - Definitions may form part of the framework of a conceptual system but they are also the end-product of an intellectual process, so have a problematic status.
  - All concepts in which an entire process is semiotically telescoped elude definition. (Nietzsche, On the Genealogy of Morals) (16)
  - All definitions in which an entire process is semiotically telescoped elude conceptualization. (Nietzsche)
  - All definitions in which an entire process is semiotically telescoped require genealogical elucidation. (Fregeaney)

Telescoping Frege in Jena

Telescoping Frege in Jena: in good company

Telescoping Frege in Jena: vague object