

# Frege's use of function–argument analysis

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## 1 Introduction

- function-argument analysis (FAA) fundamental to Frege's new logic: many of his characteristic doctrines flow from this
- Frege's logic fundamental to analytic philosophy: e.g. Frege the key influence on Wittgenstein
  - an important case study for methodology
    - paying close attention to genesis of Frege's doctrines is methodologically and philosophically revealing
  - can be used to test not only Frege's own methodological account but also that of others

## Frege as hedgehog?



## Hedgehog

- Small spiny nocturnal pig-snouted insectivorous mammal of genus *Erinaceus*, rolling itself up into ball for defence
- porcupine, sea-urchin, or other animal similarly armed with spines
- (Mil.) small self-contained defensive position bristling with fortifications on all sides
- prickly seed-vessel of some plants, e.g. corn crowfoot
- person hard to get on with, whence ~gy  
(*Concise Oxford Dictionary*, 6<sup>th</sup> ed. 1976)

Romany belief: sharp inner/outer distinction, symbolized in hedgehog; also animal who lives in margins; at death, Roma turn clothes inside out

## The hedgehog and the fox

"The hedgehog knows one big thing, the fox knows many things."

- Isaiah Berlin's essay about Tolstoy (a fox who wants to be a hedgehog), compared with Dostoyevsky
- Gellner talks of the "famous foxiness" of linguistic philosophy, and of its "hedgehog-like conviction that foxiness is the answer to everything" (1959, 195-6)



## Plan

- 1) Methodological motivations: DJ model, elucidation, explication
- 2) Frege's first step
- 3) Early development of Frege's conception of a concept
- 4) Middle development of Frege's conception of a concept
- 5) Later development of Frege's conception of a concept
- 6) Conclusions

## Methodological motivations: DJ model

- (K) Concepts are functions (whose values are truth-values).
- Frege's 'discovery' of (K) fundamental to his new logic (first expounded in 1879); yet, surprisingly, this is only made explicit in 1891
  - raises problems for discovery-justification (DJ) model
    - discovery of a truth or creation of a picture (invention of a new way of seeing things)?
      - are concepts functions, or is just that they (or at least some of them) *can be represented as functions*?
      - if latter, then genesis is philosophically important
    - cannot be stated, let alone justified, within Frege's 'Begriffsschrift' (BS)
      - so how is it 'justified'?
      - if 'justified' by success of functional framework in practice, then such a practice must be established and its advantages seen
  - historical understanding needed in both cases

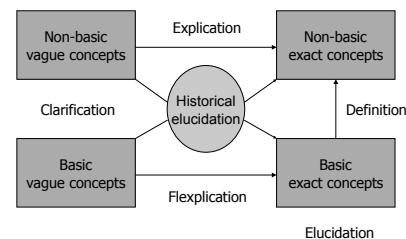
## Frege on elucidation

- elucidation = fixing the basic concepts (specifying the *Bedeutungen* of basic concept words) {1}
  - requires a 'meeting of minds', 'guessing what we have in mind'
- elucidatory logical analysis = articulating clearly the sense of the simple signs
  - preparing the building stones, taking place *before* construction of the system
- but gap in Frege's account: very little said about this process, which seems to involve replacement of our vague basic concepts by exact ones (explication)
- more is done in Frege's actual practice than he seems to admit: elucidation with an historical dimension
  - e.g. Parts I-III of *Grundlagen*, and essays *FC*, *SB*, *CO*

## Carnap on explication

- explication (core sense) = the replacement of a vague concept (*explicandum*) by a more exact concept (*explicatum*) {2}
  - explication (stricter sense) = replacement by defining exact concept
- clarification = making *explicandum* practically clear by informal explanations and examples {3}
- but gap in Carnap's account: nothing said about how exact basic concepts are arrived at
  - call this 'flexpication' = non-definitional explication, adopting flexible means to get basic concepts understood
  - might also be considered as elucidation, though historical dimension must also be appreciated

## Methodological map



## 2 Frege's first step

- cf. §308 of Wittgenstein's *PI* {4}
- philosophical problems typically arise from moves we make at the very beginning of our thinking
    - Frege's use of FAA commits us to a particular way of looking at things
    - W concerned to expose the 'proto-philosophical level' (Goldfarb)
    - extraordinary how much of Frege's philosophy comes from his use of FAA, which smuggles in presuppositions
  - *BS*, preface: "replacement of the concepts *subject* and *predicate* by *argument* and *function* will prove itself in the long run" {6}
    - leads to reinterpretations such as of 'All humans are mortal'
    - but he only drew attention to the significance of his notation for generality, and the superior power of his logic, *after BS*
    - implications only gradually dawned on Frege

## Introduction of function-argument analysis

- *Habilitationsschrift* (1874): the seed from which arithmetic grows is addition
  - associated with iteration of an operation, represented by an appropriate function
  - the concept of the quantity of a function allows us to connect different areas
- 1874-79: nothing published to indicate how he got from this idea of the unifying potential of this concept to the application of FAA to logic – but e.g. 1879 paper talks of generalization
- *BS* (1879): first talk of FAA in §9 {7}
  - one-place functions, i.e. one-place functional expressions, stand for concepts
  - but he doesn't yet say that concepts *are* functions

## Function-argument analysis as a form of representation

- Frege's use of FAA introduced without fanfare
  - not justified, just used
  - comparison with Russell (had to fight his way to new logic)
- Frege's use of FAA his first step, the movement he thought innocent – and never questioned
  - philosophical rationale offered later
- FAA as a form of representation
  - what counts as a function part of the symbolism, not of what is symbolized
- Frege's own philosophy (esp. his logic) a first step in analytic philosophy, only later appreciated as such

## Taylor on uncovering origins

- most effective way to uncover 'first steps' is through a genetic account {5}
  - frees one from grip of picture
- Frege's logico-linguistic turn as revolutionary as Descartes' epistemological turn
  - proper understanding requires undoing the forgetting
- ➔ philosophy is essentially historical
  - historicizing Wittgenstein's own approach

## 3 Early development of Frege's conception of a concept

- reviews of BS led Frege to compare his system with Boole's {8}
  - 1) judgements (judgeable contents) prior to concepts
  - 2) → 'unsaturatedness' of concepts
  - 3) concept/object distinction
  - 4) subsumption/subordination distinction
- all fundamental to Frege's philosophy, arising from thinking through his use of FAA

## Frege's theses concerning concepts (rooted in FAA)

- Letter to Marty, 29.08.1882 {9}
- (A) A distinction must be drawn between concept and object.
  - (B) A distinction must be drawn between subsumption and subordination.
  - (C) Judgements (judgeable contents) are prior to concepts.
  - (D) Concepts are unsaturated.
  - (E) Judgeable contents have no unique analysis.
  - (F) Concepts must be sharply defined.
  - (G) The realm of the conceptual is the realm of the enumerable.
  - (H) Existential judgements are assertions about concepts.
- all, except (E), only explicitly formulated *after* BS
- with qualifications to (C) and (E), content later split into *Sinn* and *Bedeutung*, all are endorsed for the rest of Frege's life and are fundamental to his philosophy

## 4 Middle development of Frege's conception of a concept

- Frege's conclusion to letter to Marty: refers to 'vicious circle' – accepting BS requires seeing what it can do, and vice versa {10}
  - Stumpf responds, and advises Frege to explain his ideas "first in ordinary language"

→ *Grundlagen* (1884):

Parts I-III: critique of existing conceptions of number, clarification of his key ideas and claims

Introduction: 3 fundamental principles {11}

- 1) anti-psychologistic principle
- 2) context principle
- 3) concept/object distinction

## *Grundlagen*: two further theses

- (I) Number statements are assertions about concepts.
- (J) Every concept has an extension, which is an object.
  - (I) just a generalization of (H)
  - (J) never explicitly stated, but implicit in §68 (fn.)
- explicit definition of number just sprung upon us:
  - (Ne) The Number that belongs to the concept *F* is the extension of the concept 'equinumerous to the concept *F*'.
  - only motivation the equivalence between:
  - (Na) The concept *F* is equinumerous to the concept *G*.
  - (Nd) The extension of the concept 'equinumerous to the concept *F*' = the extension of the concept 'equinumerous to the concept *G*'.
  - only other mentions of extensions: §§ 69, 104, 107 {12}
- very little talk of functions (just §1) or FAA (just implicit in §§ 70-83)
  - Frege's ('innocent') first step apparently forgotten!

## Theses (A) – (H) as found in *Grundlagen*

- (A) endorsed as third principle; §§ 27 (fn.), 38, 51, 68 (fn.), and 97.
- (B) §§ 47, 53.
- (C) reflected in context principle; but not explicitly formulated.
- (D) not stated.
- (E) not stated, but implicit especially in §§ 62-9.
- (F) §§ 54, 74.
- (G) §§ 14, 24, 40, 48, 87.
- (H) generalized as (I).
  - mention of (D) and (E) would have required talk of FAA, omitted in *Grundlagen*; may also be true of (C)
- lack of talk of FAA encourages us to overlook Frege's first step or treat it as innocent, like Frege himself
  - perhaps Frege saw his form of representation as transparent

## 5 Later development of Frege's conception of a concept

- key issue in this development: problems with the notion of content that leads to split into the notions of *Sinn* and *Bedeutung*
- early notion of content offered as a 'justification' of use of FAA, which involves idea that the same value can be yielded by different functions for appropriate arguments
  - conceptual content = value of prop. functions
- (Lhc) Hydrogen is lighter than carbon dioxide.
- (Hch) Carbon dioxide is heavier than hydrogen.
- have same content, but reflecting different FAAs
- ‡ various problems
  - mismatch opens up between 'intuitions' about sameness of content and equivalences within Frege's logical system, e.g. concerning identity statements and Cantor-Hume Principle

## 'Function and concept' (1891)

- with *SB* and *CO*, *FC* can be seen as offering Frege's definitive elucidation of his basic concepts
- "My starting-point is what is called a function in mathematics" (p.1)
- clarification of conception of function, introduction of truth-values, value-ranges and Axiom V
- crucial question of the paper: "how has the *Bedeutung* of the word 'function' been extended by the progress of science?" (p.12)
- two directions in which this has happened:
  - 1) extending the field of mathematical operations
  - 2) extending the field of possible arguments and values for functions
- Frege goes further still: notion of function generalised to allow any object whatsoever as either argument or value of function

## Truth-values as the values of some functions (concepts)

- key passage: *FC*, pp.12-13
- Frege applies FAA to identity statements and finds truth-values to be the best (only?) candidates for the values of the relevant functions
- generalized to all statements
  - "We thus see how closely that which is called a concept in logic is connected with what we call a function. Indeed, we may say at once: a concept is a function whose value is always a truth-value." {14}
- extending (philosophically elaborating) functional form of representation, proposing that (Pa) be read as (Pb):
  - (Pa) 'p' is true.
  - (Pb) 'p' stands for (*bedeutet*) the True.

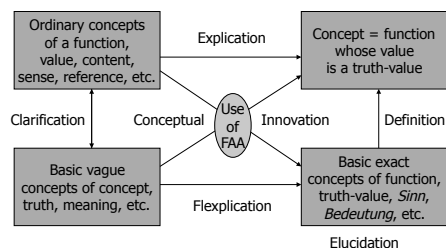
## Truth-values as objects

- distinction between (unsaturated) concept and (saturated) object absolute
- notion of function generalised to allow any object whatsoever as either argument or value of function, understood as *Bedeutungen*
- ➔ truth-values must themselves be objects {15}
- but no definition of object possible: can only be elucidated
  - so (K) itself not a proper (constructive) definition
- Frege later talks of having 'discovered' the two truth-values (as objects); in reality, he simply pushed his functional framework as far as it could go, 'projecting' appropriate entities to match his symbolic forms

## Conclusions: the elucidatory story

- focus: development of Frege's concept of a concept, which lies at the basis of his new logic (and logicist project)
  - involves replacement of our ordinary concept of a concept by a more exact concept (a concept = a function whose value is a truth-value)
- historically, this took over a decade to be elucidated
  - the 'insight' that concepts are functions may have been implicit from the start, but needed much philosophical elaboration to be made explicit (tidying after the event, projecting backwards)
- between *BS* and *GG*, there was a great deal of elucidation: explication, historical elucidation, clarification, introduction of further concepts, etc: all this much richer than Frege – and Carnap – officially allow
  - but all rooted in FAA, seeking to establish the functional framework
- notions of elucidation, etc., are themselves part of the elucidation (extra-systematic or meta-theoretical 'justification')
  - can take different forms, and hence be involved in processes of 'forgetting'

## Frege's elucidation of concept of a concept



## Conclusions: the historical aspects of philosophizing

- Frege did not 'discover' that concepts are functions; he invented a new form of representation, the elaboration of which led him to 'project' new entities, introduce new 'justificatory' (philosophical) concepts, and lay down new definitions
- its advocacy/defence requires historical elucidation
  - coming to see how it offers a better way of looking at things (formalizing inferences, etc.)
  - and in learning the system, we take the commitments on board (like accepting the existence of numbers), have our 'intuitions' reshaped, etc.: normative dimension here
- its critique requires genetic understanding
  - diagnosing philosophical problems and counterintuitive implications requires uncovering first steps, underlying analogies, metaphors and pictures, exposing presuppositions, recognizing rhetorical talk (e.g. of 'discovery') and reshaping of intuitions, etc.

## Final thought: defining as semiotically telescoping

- (K) Concepts are functions (whose values are truth-values).
- not statement of fact (picture of a state of affairs) but encapsulation of a process (recommendation of a way of looking at things)
    - definitions may form part of the framework of a conceptual system but they are also the end-product of an intellectual process, so have a problematic status
  - ❖ All concepts in which an entire process is semiotically telescoped elude definition. (Nietzsche, *On the Genealogy of Morals*) {16}
  - ❖ All definitions in which an entire process is semiotically telescoped elude conceptualization. (Nietzsche)
  - ❖ All definitions in which an entire process is semiotically telescoped require genealogical elucidation. (Fregeanay)

## Telescoping Frege in Jena



## Telescoping Frege in Jena: in good company



## Telescoping Frege in Jena: vague object

