

Leibniz, Digital Philosophy and UAI

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- 1 An Overview of Leibniz's Philosophy
- 2 Digital Philosophy—Lovely and Trustworthy :-)
- 3 Leibniz's Universe and How to Live in Leibniz's Universe

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- 1 An Overview of Leibniz's Philosophy
 - Ontology
 - Epistemology

Two Premises

- The Law of Contradiction
- The Law of Sufficient Reason \implies 'Indistinguishable \rightarrow Identical'

God's Perfection

- Omnipotence (power)
- Omniscience (knowledge)
- Sovereign goodness (will)

Mechanical Universe

- As God calculates, so the world is made.
- Everything is Mechanical.
- or else, God cannot reveal and explain nature to us. (sufficient reason)

Monadology—Let there be Monads!

- God can bring a real world into existence merely by decreeing it.
- All simple substances or **Monads** are **incorporeal automata**.
- Corporeal mass is not a substance but a phenomenon resulting from Monads.
- Extension and motion, are not substances, but true phenomena.
- Time and space are not things, but orders of things.

Supreme Perfection

“It follows from the **supreme perfection** of God, that in creating the universe He has chosen the best possible plan, in which there is

- the greatest variety together with the greatest order;
- the best arranged time and space;
- the maximum effect produced by the simplest means;
- the highest levels of power, knowledge, happiness and goodness in the creatures that the universe could allow.

For since all the possibles in the understanding of God laid claim to existence in proportion to their perfections, the actual world, as the resultant of all these claims, must be the most perfect possible. And without this it would not be possible to give a reason why things have turned out so rather than otherwise.”

Supreme Perfection → *The Best Possible World*

- the **simplicity** of means should be **balanced** against the **richness** of effects/ends.
- *minimize* decrees/order/hypotheses/mathematical laws \implies
maximize richness/diversity/variety/fruitfulness
- *maximize* power/knowledge/happiness/goodness

Monad → Soul → Human → God

- Monad + distinct perception + memory = Soul
- Soul + awareness/consciousness/reflection of abstract/eternal truth = Human
- Human aware of 'I' (Universe = 'I' + Universe \ { 'I' })
- Human + Supreme Perfection = God?

Monadology—Leibniz's Universe of Monads

- Each monad is a living substance.
- Every living substance is made up of still smaller ones, and so on down to infinity.
- There are infinite levels of life among monads.
- Some of which are more or less dominant over others.
- God is the greatest Monad.
- There is interconnection/communication of monads between each other.
- Each monad is a perpetual living mirror of the universe.

Pre-established Harmony

- **Pre-established Harmony** between final causes and efficient causes.
- The Universe is collection of **Compossibles**.
- The actual Universe = The richest compound.

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What is a Law?

- finite points vs. infinite lines
- same effect vs. several causes
- choose the simplest hypothesis at hand that explains more phenomena
- find better ones

'One suffices to derive all out of nothing.'

"Our characteristic will reduce the whole to numbers, so that reasons can also be weighed, as if by a kind of statics. For probabilities, too, will be treated in this calculation and demonstration, since one can always estimate which of the given circumstances will more probably occur."

— Leibniz

Universal Calculus

- universal characteristic
- reduce knowledge to numbers (0s and 1s)
- systematically generate all possible knowledge
- decidability—given any problem, just calculate!
- incomplete information?—calculate the probability!

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 - Feng Ye's Naturalism
 - Digital Philosophy

Methodological Naturalism

- no first philosophy
- inductive method of natural science



$$PS_1 \longrightarrow TT_1 \longrightarrow EE_1 \longrightarrow PS_2 \longrightarrow \dots$$

- well order *TT*s (aesthetic/utilitarian/general/simple)

Ontological Naturalism

- Finite discrete physical reality.
- There is nothing over and above the physical reality.
- No non-physical 'subject' facing 'external world'.

Applicability Puzzle

Puzzle (**Applicability Puzzle**)

Why is the applicability of classical mathematics possible?

Mathematical Physics

- (i) a mathematical structure $[m]$;
- (ii) an empirical domain $[r]$;
- (iii) a set of rules which bridge mathematical structure with the physical $[b]$.

Problem

$$\Gamma_r \cup \Gamma_m \cup \Gamma_b \vdash \phi$$

The Logical Problem of Applicability

Problem

$$\underbrace{\Gamma_r \cup \Gamma_m \cup \Gamma_b \vdash \phi \quad \mathfrak{A}_r \models \Gamma_r}_{\Downarrow}$$

$$\mathfrak{A}_r \models \phi$$

Naturalize the Applicability of Mathematics—Feng Ye

Problem

$$\underbrace{\Gamma_r \cup \Gamma'_r \vdash \phi \quad \mathfrak{A}_r \models \Gamma_r \cup \Gamma'_r}_{\Downarrow}$$

$$\mathfrak{A}_r \models \phi$$

where $\Gamma'_r \subset \Gamma_m \cup \Gamma_b$.

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What is a (Physical) Model?

- model vs. theory
- tradeoff between tractability with accuracy (Pareto)
- Copenhagen—Probabilistic
- Hidden Variables—Deterministic
- Many worlds—measurements force a possibility into an actuality, choosing a path in the tree of all possible universes.
- model asymptotically approximates physical chunk
- $\lim_{t \rightarrow \infty} \text{model} = \text{physical chunk}$
- $\lim_{t \rightarrow \infty} \text{ultimate model} = \text{the world}$
- Leibniz/Wittgenstein—Isomorphism: language $\xrightarrow{1-1}$ reality

Tegmark's 'Mathematical Universe Hypothesis' (MUH)

Hypothesis

Physical reality = Mathematical Structure.

- Mathematics approximates Mathematics—Not Physics!
- Note that not all of the recursively axiomatizable formal systems characterize universes—Compactness theorem and Löwenheim-Skolem theorem.

ToE

- $ToE = Th(\mathcal{A}_r)$
- Is it knowable/understandable/comprehensible? How?
- What is the meaning of 'knowable'? (simulate to arbitrary precision?)

Knowability

"There is no ignorabimus in mathematics."

"We must know; We will know."

— Hilbert

Hypothesis

Knowability = Decidability/Computability

Knowable/Unknowable

- Gödel— $Th(\mathfrak{N})$ is unknowable: $Th(\mathfrak{N})$ is complete but not r.e.; $Th(PA)$ is r.e. but not complete.
- Tarski— $Th(\mathfrak{A})$ is knowable.
- For a finite structure \mathfrak{A} in a finite language, $Th(\mathfrak{A})$ is decidable.
- If our universe is finite and discrete, then it is knowable.

Knowability Puzzle

Puzzle (Knowability Puzzle)

Why is natural science possible?

"God does not play dice."

"God always takes the simplest way."

"Subtle is the Lord, however he is not malicious."

"The most incomprehensible thing about the world is that it is comprehensible."

— Einstein

Leibniz—God created the best possible world balancing the simplicity of means against the richness of ends.

Knowable subset of MUH



$$\mathcal{M}_T := \{\mathfrak{A} : Th(\mathfrak{A}) \text{ is r.e.}\}$$



$$\mathcal{M}_T \subset MUH$$

Leibniz's Universe

- $Th(\mathfrak{A})$ can be coded to be a computable real number.
- Why not generalize \mathcal{M}_T a little bit?

$$\mathcal{M}_D := \{p : \exists x (U(p) = x)\}$$

where U is the universal monotone Turing machine.

Multiverses

Loosely (very lazily :-),

Theorem

$$\mathcal{M}_T \subset \mathcal{M}_D \in \mathcal{M}_D \setminus \mathcal{M}_T$$

- Schmidhuber's Dovetailer (Lenvin Search).
- Leibniz's **infinite level of living substances**.
- Some computing process can be characterized by some $\mathfrak{A} \in \mathcal{M}_T$, but, is the **time** of computation = the time of universe \mathfrak{A} ??? Can a low level Monad predict a high level Monad by simulating it??? (Knowability? Wolfram: Principle of Computational Irreducibility?)
- Gödel's Incompleteness Theorem.

Why should we prefer abstract digital universe to tangible physical reality?

- Applicability Puzzle (Digitalism = Naturalism)
- Knowability Puzzle (Digitalism > Naturalism)
- Learning = Recollection (Digitalism = Platonism)
- Church-Turing Thesis

Church-Turing Thesis

Thesis (Church-Turing Thesis)

effective calculable = recursive = Turing Computable

=

finite definable = Herbrand-Gödel computable

=

representable in any consistent formal system extending \mathcal{R}

=

λ -definable = flowchart (or 'while') computable

=

neural network with a tape computable = Conway's 'game of life'

=

Post/Markov/McCarthy/Kolmogorov and Uspensky ...

Task

Task

Explain the emergence of physical phenomena rather than the applicability of mathematics.

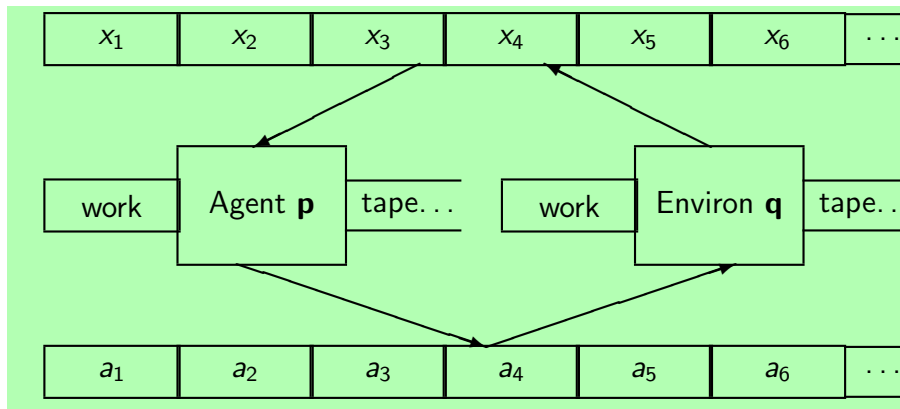
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Leibniz's Universe of Monads

- A pool of programs computing/simulating each other or themselves.
- Best possible? 'Simplicity = Kolmogorov Complexity'; but 'Richness = ?'
- Leibniz's compossible?

Chronological Turing Machine



ToE

The Monad p aware of 'I' tries to discover ToE.

$$\langle p^*, q^* \rangle [h] := \arg \min_{p, q} \{ |\langle p, q \rangle| : U(p, U(\langle p, q \rangle, \epsilon)) = h^* \}$$

Is there any better way to predict the future?

- Calculate the probability to estimate which of the given circumstances will more probably occur.



$$\text{probability} = \frac{\text{consistent universes}}{\text{all of the universes}}$$

Universal Probability

Definition (Universal Probability)

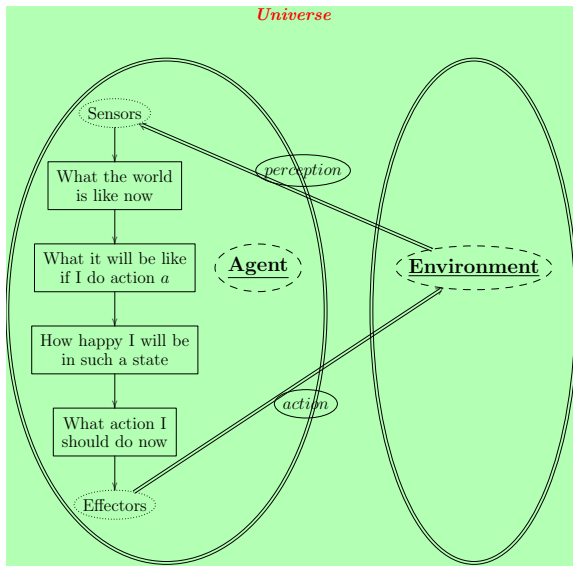
$$M(x) := \sum_{p:U(p)=x^*} 2^{-|p|}$$

where U is a universal monotone Turing machine.

$$\begin{aligned} M(x) &= \sum_p 2^{-|p|} \llbracket U(p) = x^* \rrbracket \\ &= \lim_{n \rightarrow \infty} \frac{\sum_{p:|p| \leq n} 2^{n-|p|} \llbracket U(p) = x^* \rrbracket}{2^n} \\ &\approx \lim_{n \rightarrow \infty} \frac{|\{p \in \chi^n : U(p) = x^*\}|}{2^n} \end{aligned}$$

Subjective Belief-Objective Environment

$$M(x(h)|a(h)) = \sum_{p \rightsquigarrow h} 2^{-|p|}$$

$$\text{Universe} = 'I' + \text{Universe} \setminus \{ 'I' \}$$


What if we live in a Stochastic Universe?



$\mathcal{M}_U := \{\rho: \mathcal{X}^* \cup \mathcal{X}^\infty \rightarrow [0, 1] : \rho \text{ is an enumerable semimeasure}\}$

- A theory that predicts universe ω with probability $\mu(\omega)$ and experience h in universe ω with probability $\pi(h|\omega)$, induces a probability distribution $P(h) := \sum_{\omega} \pi(h|\omega)\mu(\omega)$. The observed noise can then be coded in $-\log_2 P(h)$ bits.

Probabilistic ToE



$$|p_\pi| + |p_\mu| - \log_2 P(h)$$

- Anthropic Principle requires $\pi(h|\omega) = 1$, which leads to MDL if we ignore $|p_\pi|$.

$$|p_\mu| - \log_2 \mu(h)$$

Universe = 'I' + Universe \ {'I'}

$$\arg \min_{\frac{\pi}{\mu}} \{ |p_{\mu}^{\pi}| - \log_2 \frac{\pi}{\mu}(h) \} = \arg \max_{\frac{\pi}{\mu}} \{ 2^{-|p_{\mu}^{\pi}|} \cdot \frac{\pi}{\mu}(h) \} = \arg \max_{\frac{\pi}{\mu}} \{ P \left[\frac{\pi}{\mu} | h \right] \}$$

How to live in \mathcal{M}_U ?

$$M(x) = \sum_p 2^{-|p|} \llbracket U(p) = x^* \rrbracket$$

Definition (Universal Bayes Mixture)

$$\xi_U(x) := \sum_{\nu \in \mathcal{M}_U} w_\nu^U \nu(x)$$

where $w_\nu^U := 2^{-K(\nu)}$ is the universal prior.

$$\xi_U \in \mathcal{M}_U$$

Deterministic vs. Probabilistic

Lemma

For every $\nu \in \mathcal{M}_U$ there exists some monotone Turing machine T such that

$$\nu(x) = \sum_{p: T(p)=x^*} 2^{-|p|} \quad \text{and} \quad K(\nu) \stackrel{\pm}{=} |\langle T \rangle|$$

where $T(p) = U(\langle T \rangle p)$.

Theorem (Deterministic Representation)

$$M(x) \stackrel{\times}{=} \xi_U(x)$$

What is (Supreme) Intelligence?

Intelligence is an Equilibrium,
We just have to Identify the Game.

	Game in \mathcal{M}_D	Game in \mathcal{M}_U
Ex Post Equilibrium	Deterministic	$AI\mu$ (recursive/iterative)
Ex Ante Equilibrium	$AI\mu$ (functional)	$AI\xi$ (functional)

What is (Supreme) Intelligence?

- Intelligence of an 'agent' is an **Perfect Bayesian Equilibrium** of the Incomplete Information game with observable actions played against imaginary possible worlds (\mathcal{M}_U) only if our subjective belief system is the universal prior;
- Intelligence can be seen as an **Ex Post Equilibrium** of the Incomplete Information game with observable actions played against imaginary possible worlds (\mathcal{M}_U) only if we pretend that the true environment is the universal mixture environment;
- Intelligence can also be regarded as an **Perfect Bayesian Equilibrium** of the Incomplete Information game with observable actions played against Leibniz's possible worlds (\mathcal{M}_D) only if our subjective belief system is the Solomonoff prior.

Monad → Soul → Human → God

Welcome to the
Desert/Paradise of the Real!
Thank you!