

Listen to me!

Public announcements to agents that pay attention — or not

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1 Introduction

In public announcement logic it is assumed that announcements are perceived by all agents. In this work we take one step back from that point of view.

When an announcement is made, it may well be that some agents were not paying attention and therefore did not hear it. Also, there may be uncertainty among the agents about who is paying attention and who not, and therefore, who heard the message and who not.

Additional to the usual set of propositional variables we add designated variables for each agent, that express that the agent is paying attention.

A given state of a Kripke model therefore contains information about which agents are paying attention and which agents are not paying attention. This determines the meaning of what we call *attention-based announcements*.

A special case is that of introspective agents that know whether they are paying attention. We axiomatize our attention-based announcement logic ABAL, including a version with introspection for beliefs and attention.

In the ABAL we can formalize a concept that has been widely discussed in the philosophical and in the cognitive science literature, namely *joint attention*. This concept has been shown to be crucial for explaining the genesis of common belief in a group of agents.

Finally, we add other dynamics to our logic, namely change of attention. This is an elementary further addition to the logical framework and this logic also has a complete axiomatization.

2 Attention-Based Announcement Logic ABAL

- AGT : a finite set of agents
- ATM : a (disjoint) countable set of propositional variables
- $H = \{h_a \mid a \in AGT\}$: a disjoint set of propositional variables.

Definition (Language)

The language \mathcal{L} of attention-based announcement logic ABAL is defined as follows, where $p \in ATM$ and $a \in AGT$.

$$\mathcal{L} \ni :: p \mid h_a \mid \neg\varphi \mid (\varphi \wedge \psi) \mid B_a\varphi \mid [\varphi]\psi$$

- q : a variable that is either $p \in ATM$ or $h_a \in H$
- $B_a\varphi$: agent a believes that φ is true
- $[\varphi]\psi$: after the public announcement of φ , ψ holds
- $h_A: \bigwedge_{a \in A} h_a$

Definition (Epistemic attention model)

An epistemic attention model is a triple $M = (S, R, V)$.

- S : a non-empty set
- R : a function assigning to each agent an accessibility relation R_a
- V : a function assigning to each propositional variable q the subset $V(q) \subseteq S$ where the variable is true.

Definition (Attention introspection)

Given an epistemic attention model $M = (S, R, V)$, the model satisfies the property of attention introspection if for all $s, t \in S$, if $(s, t) \in R_a$, then $s \in V(h_a)$ iff $t \in V(h_a)$.

When attention introspection holds, an agent knows whether she is paying attention.

Definition (Semantics of attention-based announcements)

$$M, s \models [\varphi]\psi \text{ iff } M^\varphi, (s, 0) \models \psi$$

where $M^\varphi = \{S', R', V'\}$ is defined as follows.

- $S' = S \times \{0, 1\}$
- for each agents a , $((s, i), (t, j)) \in R'_a$ iff $(s, t) \in R_a$ and:
 1. $i=0, j=0, (M, s) \models h_a$ and $(M, t) \models \varphi$, or
 2. $i=0, j=1$, and $(M, s) \not\models h_a$, or
 3. $i=1, j=1$.
- for each $p \in \text{ATM}$, $(s, 0) \in V'(p)$ iff $s \in V(p)$ and $(s, 1) \in V'(p)$ iff $s \in V(p)$

The model M^φ is the extended disjoint union of the φ restriction of M , called $M|_\varphi$, and M itself, plus — that is the extension — a number of additional accessibility pairs between states for those agents that are not attentive. Roughly speaking, $M^\varphi = M|_\varphi \oplus M$ plus some edges.

After the announcement of φ , the agents that are attentive only consider possible the 0-copies of the states of the original model M in which φ is true. In contrast, the agents that are not attentive only consider possible 1-copies of the states of the original model M .

This construction of the updated model M^φ ensures that attentive agents learn φ while inattentive agents don't learn anything.

Example

- Agents: Ann(a), Bill (b), Cath(c)
- Scene: Both a and b consider the possibility of snowfall this afternoon (p), and c comes along and says she just read the weather report: it will snow.
- Annex: Bill has seen the weather report and knows whether it will snow, while Ann does not. However, Bill never knows whether Ann is paying attention. Ann knows that Bill is attentive. Both agents know whether they are attentive.

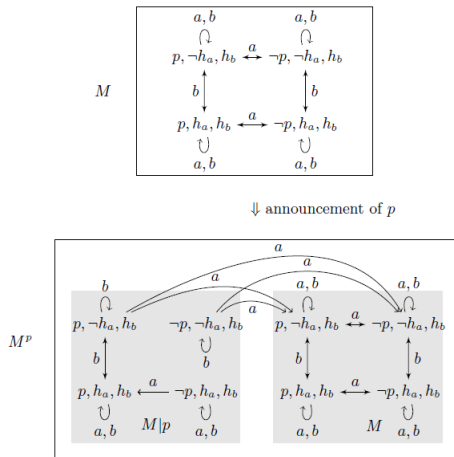


Fig. 1. Example of an attention-based announcement

Proposition (Preservation of attention introspection)

If M satisfies attention introspection then M^φ satisfies attention introspection.

Although we consider the Prop a valuable result, it takes somewhat away from the glamour when we realize that models with empty accessibility relations also satisfy attention introspection.

For example, suppose that agent a is paying attention in the actual state s (h_a is true) and also in the (uniquely) accessible state t , and where s is also considered possible. Attention introspection is satisfied. After the announcement with attentive agents $\neg h_a$, the agent a no longer considers state t possible but also no longer considers the actual state possible. Because the agent was paying attention, she has come to believe that she is not paying attention; but at the price of still also believing that she is paying attention, where the latter remains in fact the truth.

In this paper we focus on two classes of models:

- K_n : *multiple agents and no special properties of the accessibility relations.* ($n=|AGT|$)
- $K45_n^h$: *multiple agents with transitive and Euclidean accessibility relations, and with attention introspection as well.*

The set of valid \mathcal{L} formulas on the class of models K_n is called ABAL, and the set of valid \mathcal{L} formulas on the class of models $K45_n^h$ is called $ABAL^{intro}$.

3 Relation with action models

Every attention-based announcement is definable as an action model. Whether an announcement φ is heard in a given state depends on the value of h_a for every agent a in that state. The agents who hear the announcement retain all arrows pointing to states where φ holds and delete all arrows pointing to states where φ does not hold, and that is independent of the truth of φ in the actual state; whereas the agents who do not hear the announcement think that nothing has happened, i.e., also independent of the truth of φ they think that the trivial action with precondition \top happened.

Definition (Action model for attention-based announcements)

Given a formula φ , the action model for the attention-based announcement of φ is the multi-pointed action model

$\mathcal{A} = (A, R, Pre, P)$ where:

- $A = \{(i, J) \mid i \in \{0, 1\} \text{ and } J \subseteq AGT\} \cup \{w_T\}$
- R maps each agent $a \in AGT$ to

$$R_a = \{((i, J), (1, K)) \mid i \in \{0, 1\} \text{ and } a \in J\} \cup \{((i, J), w_T) \mid a \notin J\} \cup \{(w_T, w_T)\}$$
- $Pre: A \rightarrow \mathcal{L}$ is defined as follows:

$$Pre((i, J)) = \bar{\varphi} \wedge \bigwedge_{a \in AGT} \bar{h}_a$$

$$Pre(w_T) = \top$$
- $P = \{(i, J) \mid i \in \{0, 1\} \text{ and } J \subseteq AGT\}$ is the set of points.

Informally, the action model for the attention-based announcement of φ consists of $2^{n+1} + 1$ actions and has 2^{n+1} initial points (alias actual actions). Moreover, there is a ‘nothing happens’ alternative with precondition \top , that is not an initial point.

An attentive agent believes that any action point with precondition entailing φ may be the actual action. An inattentive agent believes that the action with precondition \top is the actual action.

The action model for attention-based announcements is depicted in Figure 2 for the example of two agents a and b and the announcement φ .

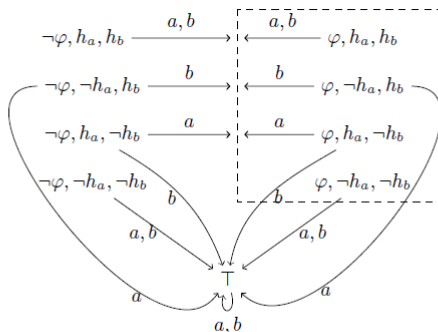


Fig. 2. The action model \mathcal{A} corresponding to an attention-based announcements φ to two agents. An arrow pointing to a box points to all actions in the box.

4 Axiomatization

Table 1 shows the axiomatization.

all propositional tautologies	$\mathbf{B}_a\varphi \rightarrow \mathbf{B}_a\mathbf{B}_a\varphi$	*
$\mathbf{B}_a(\varphi \rightarrow \psi) \rightarrow (\mathbf{B}_a\varphi \rightarrow \mathbf{B}_a\psi)$	$\neg\mathbf{B}_a\varphi \rightarrow \mathbf{B}_a\neg\mathbf{B}_a\varphi$	*
$[\varphi]\mathbf{B}_a\psi \leftrightarrow ((h_a \rightarrow \mathbf{B}_a(\varphi \rightarrow [\varphi]\psi)) \wedge (\neg h_a \rightarrow \mathbf{B}_a\psi))$	$h_a \rightarrow \mathbf{B}_ah_a$	*
$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$	$\neg h_a \rightarrow \mathbf{B}_a\neg h_a$	*
$[\varphi]\neg\psi \leftrightarrow \neg[\varphi]\psi$	From φ infer $\mathbf{B}_a\varphi$	
$[\varphi]q \leftrightarrow q$	From φ infer $[\psi]\varphi$	

Table 1. The axiomatizations for ABAL and ABAL^{intro}

- The axioms * formalize that agents have introspective beliefs and are not uncertain about what they hear.
- The axiomatization ABAL consists of all the derivation rules and axioms of Table 1.
- The axiomatization *ABAL^{intro}* consists of ABAL plus the *-ed axioms and rules.

Proposition

The axiomatization of ABAL is sound and complete for the class of K_n models. The axiomatization of $ABAL^{intro}$ is sound and complete for the class of $K45_n^h$ models.

5 Joint attention

The attention introspection axiom $h_a \rightarrow B_a h_a$ of $ABAL^{intro}$ only guarantees attention introspection for individuals, **not for groups**: it may happen that h_A is true while some $a \in A$ does not believe that h_A .

We now investigate a condition under which attention introspection obtains in terms of common belief: *joint attention or joint attentional state*.

We assume a common belief operator C_A for a subgroup A of the set of all agents, so that $C_A \varphi$ stands for ‘the agents in group A commonly believe φ ’, and which is interpreted in the usual way by the transitive closure of the union of all accessibility relations R_a for the agents in A .

Let $A \subseteq AGT$. The idea is that the agents in A have a joint attention (or are in a joint attentional state) iff each of them is looking at the source of information and focusing his attention on it and they have common belief that each of them is looking at the source of information and focusing his attention on it. Formally:

$$JointAtt_A \equiv_{def} h_A \wedge C_A h_A$$

Note that when joint attention of all agents is satisfied then attention-based announcements are the same as public announcements.

In a normal situation (what is announced is true, there is no noise in the communication channel, etc.) looking at the source of information and having a common belief that everyone is looking at the source of information (i.e., being in a joint attentional state) provide a sufficient condition for the formation of a common belief. This is captured by a validity of ABAL:

$$\models \text{JointAtt}_A \rightarrow [p]C_A p$$

We can actually characterize the formation of common belief of an atomic fact p as follows:

$$\models [p]C_A p \leftrightarrow C_A \bigwedge_{a \in A} (h_a \vee B_a p)$$

Note that the equivalence is not valid if we replace p by h_a .

6 Attention change

- Clap the hands.
- Tap on the shoulded.

We model such fine-grained attention change by an assignment. Given a set of agents $A \subseteq AGT$, we distinguish the assignment $+A$ (merely shorthand for a simultaneous assignment $a_1 := \top, \dots, a_n := \top$) that makes all agents $a \in A$ pay attention and hear subsequent announcements, from an assignment $-A$ that makes all h_a false.

To the inductive definition of the language \mathcal{L} , we add clauses for the modal operators $[+A]$ and $[-A]$, for $A \subseteq AGT$. We write $[+a_1, \dots, a_n]$ instead of $[+\{a_1, \dots, a_n\}]$. The semantics of attention-based assignment is then:

$$M, s \models [+A]\psi \text{ iff } M^{+A}, (s, 0) \models \psi$$

$$M, s \models [-A]\psi \text{ iff } M^{-A}, (s, 0) \models \psi$$

Where $M^{+A} = (S', R', V')$ is defined as follows (the definition of M^{-A} is similar).

- $S' = S \times \{0, 1\}$;
- if $a \in A$ and $s, t \in S$ then $((s, i), (t, j)) \in R'_a$ iff $(s, t) \in R_a$
and
 1. $i=0$ and $j=0$; or
 2. $i=1$ and $j=1$;
- if $a \notin A$ and $s, t \in S$ then $((s, j), (t, j)) \in R'_a$ iff $(s, t) \in R_a$
and
 1. $i=0, j=0$, and $(M, s) \models h_a$; or
 2. $i=0, j=1$, and $(M, s) \not\models h_a$; or
 3. $i=1$ and $j=1$.

- $(s, 0) \in V'(p)$ iff $s \in V(p)$, and $(s, 1) \in V'(p)$ iff $s \in V(p)$;
- if $a \in A$ then
 1. $(s, 0) \in V'(h_a)$ iff $s \in V(h_a)$, and
 2. $(s, 1) \in V'(h_a)$ iff $s \in V(h_a)$;
- if $a \notin A$ then
 1. $(s, 0) \in V'(h_a)$, and
 2. $(s, 1) \in V'(h_a)$ iff $s \in V(h_a)$.

Attention assignment preserves attention introspection. The order in successive change of attention does not matter, but it does not achieve joint attention:

- $\models [+a][+b]\varphi \leftrightarrow [+b][+a]\varphi$;
- $\not\models [+a, b]\varphi \leftrightarrow [+b][+a]\varphi$.

Thank you for your attention!