Mental Logic

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LAWRENCE ERLBAUM ASSOCIATES, PUBLISHERS
1998 Mahwah, New Jersey London

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*Some Empirical Justification of the Mental-Predicate-Logic Model**

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This chapter reports an initial test of the mental predicate logic developed in chapter 11. We examined (a) whether it can predict the relative difficulties of problems in a set of problems employing monadic predicates (i.e., predicates that take a single argument) and in a parallel set of problems employing dyadic predicates (i.e., predicates that take two arguments), and (b) whether erroneous evaluations of conclusions that are derivable on the direct-reasoning routine are relatively rare.

The general methodology is similar to that of chapter 7. Subjects were presented with a set of problems in which the schemas and number of reasoning steps varied. In each problem, one or more facts were given, followed by a conclusion. Subjects had to decide whether this conclusion was true or false, given those facts, and then to rate its subjective difficulty right after they marked a truth value.[1] All the monadic-predicate problems are shown in Appendix 1, and the parallel dyadic-predicate problems are shown in Appendix 2 at the end of this chapter. The problems are all

*The empirical work reported here is based in part on a doctoral dissertation by the first author, begun under the supervision of the second author, and completed under the supervision of the third author.

[1]Perceived difficulty was rated on a 7-point scale, in which 1 was "easier to figure out than most of the other problems," 7 was "harder to figure out than most of the problems," and 3 to 5 were "about average difficulty."

soluble by the Direct Reasoning Routine (DRR) of Table 11.4, but they vary considerably in the number of inferential steps required.

In chapter 7 we noted three possible sources of errors in problems of this sort. We called them *comprehension errors, heuristic inadequacy errors,* and *processing errors.* A comprehension error occurs when a subject construes a premise or the conclusion in an unintended manner. The starting information used by the subject is not that intended by the problem setter. Heuristic inadequacy errors occur when the subject's reasoning program fails to find a line of reasoning that solves a problem, that is, the problem is too difficult for the subject. Processing errors are due to lapses of attention, errors of execution in the application of schemas, failure to keep track of information in working memory, and the like. We assume that the likelihood of a processing error increases with problem complexity, but overall tends to be low and largely vanishes in the simplest problems where processing load is minimal.

In the work reported here we expected most errors to be processing errors. Because all problems are soluble by the DRR, we predicted that there should be no problems too difficult for subjects; hence, there should be no errors due to heuristic inadequacies. We sought to eliminate comprehension errors by careful attention to the wording of problems. We tried to avoid problems in which the premises might have "conversational implicatures" (Grice, 1975) that could lead some subjects to interpret premises in ways we did not intend. Also, where the relative scopes of quantifiers and negations had to be understood, we did preliminary work to discover a wording that subjects construed in the intended manner.

Our first prediction followed immediately from this discussion of error. Problems that can be solved immediately by the application of a single schema should be solved essentially without errors; moreover, because all problems are soluble by the DRR, the overall error rate should be low. The second, and main, prediction is that the difficulty of a problem will be a function of the number and nature of the inferential steps required by the DRR to solve it. In particular, the difficulty of a problem is predicted from the sum of the difficulties of the individual reasoning steps required to solve it. As in the work of chapter 7, we used two prediction schemes. In the main scheme we estimated difficulty weights for each primary schema from the data, the difficulty weight of a schema being an index of the difficulty of adopting and using that schema in solving a problem. We then investigated how far we could systematically predict the empirically measured difficulty of a problem from the sum of the difficulty weights of the component inferences used by the DRR to solve it. In the second prediction scheme, we assigned equal difficulty weights to all the primary schemas, taking the difficulty of a problem as proportional only to the number of component inferences, not to their nature. Thus, this scheme examines how well problem difficulty can be predicted just from the number of primary schemas used to solve a problem, that is, from the number of reasoning steps.

We used two measures of problem difficulty—the mean difficulty rating assigned to the problem by subjects who solved it correctly, and the number of errors on the problem. We used the difficulty ratings in estimating difficulty weights for schemas. In the case of the difficulty ratings, we assumed, as in chapter 7, that in doing a problem a subject forms a subjective impression of the amount of processing the problem demanded, and that this processing is, in turn, a function of the number and kind of the mental steps required to solve the problem; the difficulty rating assigned reflects this subjective impression. We used the ratings to estimate difficulty weights because they had better psychometric properties than the errors; they were less skewed, subjects showed more consensus in their ratings than in errors, and problems were differentiated better.

The following primary schemas were used in the solution of problems: Schemas 1, 2, 3i, 3ii, 4i, 4ii, 6, 7, 8i, 8ii, 11, 13i, 13ii, 14i, and 14ii of Table 11.3. (Schemas 5 and 12 were not included because they exist only in propositional form in Table 11.3; Schemas 9 and 10 were excluded because they primarily concern indirect reasoning not covered by the DRR.) In addition, several paraphrase schemas and truisms were involved. It seemed to us that to assign each of these schemas a difficulty weight would result in so many parameters (difficulty weights) being estimated from the data as to make the results uninteresting. We therefore sought to cut down on the number of parameters to be estimated from the data. A combination of common-sense intuition and preliminary work suggested the following. First, we assigned all paraphrase schemas and truisms a weight of zero.[2] Second, we assigned 8i the same weight as 8ii, 13i as 13ii, and 14i as 14ii: Both intuition and our preliminary work indicated that this was reasonable. Third, the preliminary work indicated that 3i was more difficult than 3ii, and 4i than 4ii, the differences being about equal, and similar to the difficulty of Schema 8; because, as noted in chapter 11, Schema 3i can be generated by Schema 3ii feeding Schema 8, and 4i by 4ii feeding 8, it seemed reasonable to fix the weight of Schema 3i as the weight of 3ii plus the weight of 8, and similarly, the weight of 4i as the weight of 4ii plus the weight of 8. With these decisions made, the number of schema weights to be estimated from the data is 10 (Schemas 1, 2, 3ii, 4ii, 6, 7, 8, 11, 13, and 14).

Five types of problems were used:

[2]It is consistent with the work done that the cognitive cost of the application of paraphrase rules and truisms is small compared with that of the primary inference rules. Also, in pilot work where we have had subjects write down what they infer, they often write down not the precise output of the inference rule, but what in our system would be a paraphrase of it, again suggesting little cognitive cost to paraphrasing.

1. *Control-true problems.* In these the premise and conclusion were identical and merely had to be matched. They served to anchor the low end of the rating scale.
2. *Control-false problems.* In these, either the conclusion directly negated the premise, or vice versa. Formally, these are one-step problems involving Schema 13 or Schema 14.
3. *One-step problems.* These were problems in which the conclusion could be reached from the premises in one step using one of the primary schemas.
4. *One-step plus contradiction.* These were problems in which a contradiction between the premises and the conclusion could be reached in one step using one of the primary schemas.
5. *Multistep problems.* These all involved a chain of two or more inferences; those for which the expected response was *False* also involved finding an incompatibility between the conclusion and the premises taken with the inferences made from them.

We conducted three experiments. Experiment 1 investigated the problems of Appendix 1, that is, those with monadic predicates. Experiment 2 investigated the problems of Appendix 2, that is, those with dyadic predicates. Experiment 3 investigated both the monadic and dyadic problems jointly. In Experiment 1, four data sets were obtained, each from 20 native-English-speaking undergraduates drawn from the introductory psychology course at New York University.[3] (All participants in subsequent studies were drawn from the same population.) A preliminary data set was obtained with a set of 46 of the 64 problems in Appendix 1, and three additional data sets were obtained with the complete set of 64 problems. The reason for obtaining the preliminary data set was to provide estimates of schema difficulty from one set of problems and subjects that could be used to predict problem difficulties for a different set of problems with different subjects. We obtained the three data sets using the same set of 64 problems with different subjects for two reasons. The first was for cross-validation: We wished to see how similar the weights would be across data sets, and particularly, whether weights estimated from one data set would predict problem difficulty in other data sets as well as weights estimated from those data sets. The second reason was to obtain a reasonably stable error measure by using error data summed across the three sets.

[3]In all experiments the data for a small number of subjects are not included. In Experiment 1, for example, two additional subjects are not included because they failed to follow instructions, and one subject was not included because this subject made errors on most practice problems.

All the monadic-predicate test problems in Appendix 1 concerned the following situation: The subject was asked to imagine that there are many beads and that a manufacturer put them in bags. Each problem concerned the content of a different bag. The same set of 16 practice problems was used in collecting all four data sets. The practice problems sampled the range of difficulty of the main problem set (without duplicating any). They served to familiarize subjects with the nature of the problems they would be doing, and made it possible for subjects to form a rough calibration of their internal difficulty rating scale before beginning the main set of problems.

Experiment 2 used the dyadic-predicate problems of Appendix 2. These 64 problems were constructed in one-to-one correspondence with those of Appendix 1, so that, for example, Problem 36 of the dyadic set required the same reasoning steps as Problem 36 of the monadic set. Three data sets were obtained for all 64 problems ($N = 20$ for each set), again allowing for crossvalidations of predictions, and allowing comparison between the predictions for the monadic and dyadic sets. Sixteen practice problems parallel to those in Experiment 1 were used.

For the dyadic-predicate problems of Appendix 2, participants were told to imagine that there are many beads and a manufacturer has put them in bags. The beads vary in color (e.g., blue, red, or green), shape (e.g., square, round, or triangular), and material (e.g., wooden, plastic, or metal). The problems are presented by kinds of facts such as the following: Children played with each other (who played with whom), children like or dislike (also, find or did not find, got or did not get) some particular kinds of beads, or simply what kinds of beads are in bags. A fact can also be a possible combination of these.

Experiment 3 presented each subject all of the 128 monadic- and dyadic-predicate problems from from Appendixes 1 and 2. We generated two data sets ($N = 20$ for each set), again allowing a crossvalidation of the predictions. The motivation for this was to assess whether the difficulty ratings would differ when made against the background of the two sorts of problems rather than against the background of only monadic or only dyadic problems.

In all three experiments, the task was administered individually. In Experiments 1 and 2, each subject received two booklets sequentially. The first booklet provided instructions, followed by the set of practice problems. The second booklet provided the set of test problems. The experimenter read the instructions aloud while the subjects followed them in the booklet. Throughout the task, groups of four test problems were presented on a separate sheet of paper. Subjects were instructed to specify on each problem whether the proposed conclusion was true or false, given the premises, and then to rate the problem's perceived difficulty on a line provided beneath each problem.

Within each booklet, problem order was partially counterbalanced by using two random orders and their reverse orders, with an equal number of subjects receiving each of the resulting four orders. The number of problems with true conclusions equaled the number of problems with false conclusions, both in the set of practice problems and in the sets of test problems.

The procedure used in presenting Experiment 3 was identical to that in Experiments 1 and 2, with the following exceptions. In presenting Experiment 3, three booklets were presented sequentially. The first booklet included all 32 practice problems used in Experiments 1 and 2, presented in random order. A second booklet included 40 test problems and a third included 88 test problems. After finishing the first two booklets, each subject was given $2 to buy a refreshment outside the building before returning to start working on the third booklet. Problem order was also partially counterbalanced by using one random order across all 128 problems and its reverse order.

RESULTS: THE MONADIC-PREDICATE PROBLEMS OF EXPERIMENT 1

Level of Accuracy

For the preliminary Data Set 1, there were 44 problems (omitting control-true problems) and 20 subjects, that is, 880 responses, of which 20 were errors (2.3%). Considering all problems in Data Sets 2, 3, and 4 (omitting the control-true problems), there were 61 problems and 60 subjects, that is, 3,660 responses, of which 84 were errors (2.3%). Thus, subjects were 97.7% accurate over the entire set, indicating that the problems were well within subjects' abilities, as predicted, and demonstrating once again a class of logical reasoning problems on which people do very well.

There were 17 problems common to Data Sets 2, 3, and 4, and 13 problems to Data Set 1, which could be solved in one step using one of the primary schemas. (These comprise nine one-step and eight control-false problems.) With 60 subjects and 17 problems in Data Sets 1, 2, and 3, and with 20 subjects and 13 problems in Data Set 1, we have 1,280 responses. Of these, 29 were errors (2.3%); thus, the accuracy rate was 97.7% correct. In fact, 13 of the errors were on a single problem. This presented the single premise *All the beads are round and wooden*, and the subject had to evaluate the conclusion *All the beads are wooden?* (Problem 15 of Appendix 1). We suspect that the 13 subjects who marked this as *False* did so not because they thought the beads were not wooden, but because they thought the conclusion insufficiently perspicuous—it did not give a sufficiently complete account of the beads (i.e., it offended Grice's, 1975, maxim of quantity). If this problem is omitted, accuracy is 98% on one-step problems.

Predicting Problem Difficulty

The first analyses were computed without considering any weights for individual schemas and concerned only the number of reasoning steps involved in solving a problem, as predicted by the DRR. For Data Sets 1, 2, 3, and 4, respectively, the correlations between the number of reasoning steps and the perceived mean difficulty ratings were .80, .80, .79, and .80. This shows that the number of reasoning steps alone provides substantial predictability to the perceived difficulty ratings, accounting for approximately 64% of the variance. The number of reasoning steps is, of course, correlated with overall problem length—it takes more words to convey the problems that require more reasoning steps. It is possible, therefore, that a substantial portion of the correlations between the number of reasoning steps and obtained mean difficulties could be due to problem length alone. To provide a conservative estimate of the predictability of reasoning steps alone, the correlations were computed with problem length partialed out; the resulting correlations were .68, .68, .66, and .68, for Data Sets 1, 2, 3, and 4, respectively, accounting for approximately 46% of the variance.

A second set of analyses were computed considering weights for each individual schema. Difficulty weights for the schemas were estimated as follows: For any problem, predicted difficulty = 1 + Σ(difficulty weights of schemas involved in solving it). (The constant, 1, reflects the fact that the lowest possible difficulty rating is 1.) Because there are 44 problems in Study 1 and 61 problems in the other studies (excluding control-true problems), we have 44 and 61 such equations, respectively.

Difficulty weights for the reasoning steps were estimated using the program PRAXIS (Brent, 1973; Gegenfurtner, 1992) to obtain the best least-squares fit of the predicted problem difficulties to the obtained mean difficulty ratings. (It will be recalled that 10 weights were estimated—for Schemas [from Table 11.3] 1, 2, 3ii [3i = 3ii + 8], 4ii [4i = 4ii + 8], 6, 7, 8 [8i = 8ii], 11, 13 [13i = 13ii], and 14 [14i = 14ii].) Separate estimates were made for each study.

Table 12.1 shows the correlations between predicted difficulties and obtained mean difficulty ratings for the four data sets. The correlations on the diagonal (top left to bottom right) show the quality of prediction when the predictions are based on difficulty weights estimated from the same study from which the obtained ratings come (the 10 parameters were estimated from the same data). In the other correlations, the predictions based on difficulty weights estimated from one data set are used to predict

TABLE 12.1
Correlations Between Mean Difficulty Ratings and Predicted Difficulties for the Monadic-Predicate Problems of Experiment 1

Mean Difficulty Ratings	*Predicted Difficulties*			
	Data Set 1	*Data Set 2*	*Data Set 3*	*Data Set 4*
Data Set 1	.93	.92	.89	.91
Data Set 2	.91	.92	.90	.90
Data Set 3	.89	.91	.93	.88
Data Set 4	.90	.90	.89	.89

obtained mean difficulty ratings from a different data set (no parameters estimated from the data predicted). There are no significant differences among the correlations in Table 12.1, indicating that it makes little or no difference whether estimates and obtained difficulty ratings come from the same data set or a different one. The correlations between the mean ratings across studies ranged from .92 to .95. Thus, the quality of predictions shown in Table 12.1 is hardly below the reliability of the mean ratings themselves.

There were 17 problems included in Data Sets 2 to 4 that were not included in Data Set 1 (four control-false problems, three one-step–plus–contradiction problems, and 10 multistep problems). One can therefore inquire how well the estimates from Data Set 1 predict the difficulties of these new problems that were not used in estimating the difficulty weights of Data Set 1. The correlations were .92, .88, and .89 for predictions from Data Set 1 to Sets 2, 3, and 4, respectively, indicating excellent predictability to new problems done by new subjects.

The rated difficulty of a problem correlates with problem length—the longer the problem in number of words, the more likely subjects are to rate it as more difficult. The correlations between problem length and mean rated difficulty ranged from .66 to .79 across the four data sets. Of course, because the longer problems are the ones that require more inferential steps to solve, the correlation with problem length could well be largely or entirely due to the number and kinds of inferences demanded by a problem. However, problem length itself could well make an independent contribution, and a proper conservatism requires that one investigate whether the relations between predicted and obtained difficulties remain substantial when length is partialed out.[4] These partial

[4]We treated problem length differently here from in the work reported in chapter 7. There we attempted to measure the effects of length on subjective difficulty directly. However, our method of doing this actually measured the effects of substitution complexity rather than just length (see the discussion of problem length in chap. 7). It could well be that the methods used in chapter 7 overestimated the effects of problem length by equating length with substitution complexity.

TABLE 12.2
Correlations Between Mean Difficulty Ratings and Predicted Difficulties With Problem Length Partialed Out for the Monadic-Predicate Problems of Experiment 1

Mean Difficulty Ratings	*Predicted Difficulties*			
	Data Set 1	*Data Set 2*	*Data Set 3*	*Data Set 4*
Data Set 1	.88	.86	.83	.83
Data Set 2	.84	.84	.79	.79
Data Set 3	.80	.80	.81	.72
Data Set 4	.83	.79	.76	.77

TABLE 12.3
Weights for the 10 Schemas From the Monadic-Predicate Problems in Data Sets 2, 3, and 4 of Experiment 1

Schemas	1	2	3ii	4ii	6	7	8	11	13	14
Weights	.46	.68	.73	1.55	1.28	1.59	.71	.67	.48	.95

Note. The weights were obtained from the combined 60 subjects in Data Sets 2 to 4, that is, where subjects received all 64 problems.

correlations are shown in Table 12.2. It can be seen that the correlations are indeed somewhat lower than in Table 12.1, but are nevertheless still very substantial.

The high correlations between the observed mean difficulties and the predictions remain quite high when schema weights were generated from the same or from different sets of subjects, which indicates that the weights across studies were quite similar. The best estimate of the weights for each schema, therefore, would be those obtained from the combined data sets. Table 12.3 shows the weights for each of the 10 schemas obtained from all 60 subjects in Data Sets 2, 3, and 4, that is, for those subjects who received the entire set of 64 problems.

RESULTS: THE DYADIC-PREDICATE PROBLEMS OF EXPERIMENT 2

Level of Accuracy

Considering all the problems in all three data sets (again, omitting the control-true problems), there were 61 problems and 60 subjects, that is, 3,660 responses. Of these, 128 (3.5%) were errors. Thus, subjects were 96.5% accurate over the entire set, indicating that these problems of dyadic-predicate sort were also well within subjects' capacity, as predicted, and demonstrating, once again, another class of logical reasoning problems on which people do very well.

For 17 one-step problems (including 8 control-false problems), the error rate was 2.6% (there were 1,020 responses; of these, 27 were errors). Unlike the data for Experiment 1, for which a large number of errors were made on Problem 15, none of the subjects made an error on Problem 15 on the dyadic-predicate problems. Inspection of the two parallel problems in Appendixes 1 and 2 reveals why the dyadic-predicate version did not elicit the same sort of response as did the monadic-predicate version: Unlike the problem presented with a monadic predicate, which presented the same topic (the subject noun phrase of the sentence) in both the premise and the conclusion (i.e., "beads"), the topic of the premise in the dyadic version (i.e., "girls") differs from the topic in the conclusion (i.e., "boys"). This switch of topics in the dyadic version mitigates the grounds for the Gricean violation that is likely the reason for the *False* response to the monadic version.

Analyses Based on the Number of Reasoning Steps

For Data Sets 1, 2, and 3, the correlations between the number of steps and three sets of perceived mean ratings are .74, .74, and .79, respectively, which, although about 5% lower than those for Experiment 1, again are substantial. Overall they account for approximately 57% of the variance. When the problem length is partialed out, the three partial correlations are .67, .66, and .74, respectively, accounting for approximately 45%, 43%, and 55% of the variance, respectively. Consideration of the number of reasoning steps alone thus provides a reasonable prediction of perceived problem difficulty.

Analyses Based on Schema Weights

Cross-sample validation analyses for the dyadic-predicate problems are presented in Tables 12.4 and 12.5. Table 12.4 shows the correlations between predicted difficulties and obtained mean difficulty ratings for the three data sets. Table 12.5 shows the corresponding correlations when the length of problems is partialed out. In comparison with the values reported for the monadic-predicate problems in Experiment 1, the correlations in Tables 12.4 and 12.5 are consistently lower, but are still quite substantial, indicating high predictability of the model for the dyadic-predicate problems. Taken collectively, the correlations represented in Tables 12.4 and 12.5 reveal that the sum of schema weights accounts for approximately 71% of the variance, and the amount of variance accounted for when problem length is partialed out is approximately 55%. As was

TABLE 12.4
Correlations Between Predicted Problem Difficulties and Obtained Mean Ratings From the Dyadic-Predicate Problems in Data Sets 1, 2, and 3 in Experiment 2

Obtained Difficulties	Predicted Difficulties		
	Data Set 1	*Data Set 2*	*Data Set 3*
Data Set 1	.83	.82	.83
Data Set 2	.85	.83	.84
Data Set 3	.86	.85	.87

TABLE 12.5
Correlations Between Predicted Problem Difficulty and Obtained Difficulty When Problem Length is Partialed Out for the Dyadic-Predicate Problems in Data Sets 1 to 3 of Experiment 2

Obtained Difficulties	Predicted Difficulties		
	Data Set 1	*Data Set 2*	*Data Set 3*
Data Set 1	.72	.79	.78
Data Set 2	.70	.72	.73
Data Set 3	.73	.73	.80

the case for the monadic-predicate problems, the high correlations among the three data sets for dyadic-predicate problems show that the best estimates would be derived from the combined data for all 60 subjects in Data Sets 1, 2, and 3. These weights are shown in Table 12.6.

Comparison of the weights in Table 12.6 with those in Table 12.4 reveals an overall similarity, but not a perfect correspondence. (The rank-order correlation between the two sets = .76.) The overall similarity between the weights in Table 12.6 and those in Table 12.4 suggests that the relative amount of cognitive effort required to apply each schema remains stable across predicates that take a single argument and those that take two arguments. This overall similarity, however, does not hold for Schema 3ii, which when presented with a monadic predicate, and rated against other schemas presented in monadic-predicate problems, is rated as relatively easy, but when presented with a dyadic predicate, and rated against

TABLE 12.6
Schema Weights Obtained for the Dyadic-Predicate Problems From the Combination of Data Sets 1, 2, and 3 in Experiment 2

Schemas	1	2	3ii	4ii	6	7	8	11	13	14
Weights	.58	.51	1.18	1.40	1.54	1.39	.58	.64	.60	1.08

other schemas presented in dyadic-predicate problems, is rated as relatively difficult. It is possible, of course, that the difference for Schema 3ii between Tables 12.4 and 12.6 relies on the contexts in which the ratings were made, that is, against a background of 64 monadic-predicate problems in the one case, and against a background of 64 dyadic-predicate problems in the other case. Alternatively, it may be that Schema 3ii takes relatively more effort when applied to a dyadic predicate. The data reported in the next section were obtained from subjects presented all 128 problems from the two parallel sets of monadic- and dyadic-predicate problems, allowing assessment of whether the differential ratings for monadic- and dyadic-predicate problems are robust against a background of both problem types.

RESULTS: THE MONADIC- AND DYADIC-PREDICATE PROBLEMS OF EXPERIMENT 3

Level of Accuracy

Considering all problems in two data sets (omitting the control-true problems), there were 122 problems and 40 subjects, that is, 4,880 responses. Of these, 106 were errors (2.2%). The 97.8% accuracy rate replicates the findings for the monadic- and dyadic-predicate problems reported in Experiments 1 and 2, and again demonstrates that these are deductive reasoning problems on which people do very well. In Data Sets 1 and 2, there were 34 one-step problems and 40 subjects, that is, 1,360 responses. Of these, 19 were errors (1.4%), showing that subjects did extremely well on solving one-step problems.

Analyses Based on the Number of Reasoning Steps

For Data Sets 1 and 2, for the combined monadic and dyadic problems, the correlations between the number of reasoning steps and two sets of obtained mean difficulty ratings are .82 and .81, respectively, accounting for approximately 66% of the variance. This shows that the number of reasoning steps alone also provides substantial predictability. When problem length is partialed out, the two correlations are .74 and .72, respectively, accounting for approximately 53% of the variance.

We now consider the obtained mean difficulty ratings on monadic predicate problems and on dyadic predicate problems separately. The correlation between the number of reasoning steps and obtained mean difficulty ratings on only monadic predicate problems is .81 (.69 when the problem length is partialed out), for both data sets. The correlations between the number of reasoning steps and obtained mean difficulty ratings on only dyadic-predicate problems are .76 and .75 (.71 on both when problem length is partialed out), respectively, for Sets 1 and 2. Note that the correlations for the monadic-predicate problems are equivalent to those in Experiment 1, and those for the dyadic-predicate problems are equivalent to those in Experiment 2.

Analyses Based on Schema Weights

We turn first to the monadic-predicate problems. The four correlations between the predictions from the two sets of schema weights and the two sets of obtained mean difficulty ratings ranged from .91 to .93, demonstrating once again that observed mean difficulty is predicted equally well by a set of weights obtained from the same data set or from a different data set. Again, the best estimate of schema weights is given by the combined 40 subjects of Data Sets 1 and 2, and these weights are presented in the first row of Table 12.7. The predicted difficulties for the monadic predicate problems provided by the weights in Row 1 of Table 12.7 have a correlation = .91 with the obtained mean difficulties of the combined data sets of Experiment 1.

We turn now to the dyadic predicate problems of Data Sets 1 and 2. The four correlations between the problem difficulties predicted from these two sets of weights and their observed mean difficulties range from .83 to .86. The schema weights for dyadic-predicate problems obtained from the 40 subjects of Data Sets 1 and 2 together are shown in Row 2 of Table 12.7. The predicted difficulties for the dyadic problems generated from these weights have a correlation of .86 with the obtained mean difficulty ratings for the combined dyadic-predicate data sets in Experiment 2.

The similarity of the weights for monadic and dyadic problems in Rows 1 and 2 of Table 12.7 to those in Tables 12.3 and 12.6, respectively, is illustrated by the correlations between the problem difficulties predicted by the models obtained from Experiment 3 and those obtained from Experiments 1 and 2, $r = .91$ for the monadic predicate problems and $r =$

TABLE 12.7
Schema Weights Obtained From the Monadic- and Dyadic-Predicate Problems, and From all Problems of Experiment 3

Types of Problems	*Schema Numbers and Weights*									
	1	*2*	*3ii*	*4ii*	*6*	*7*	*8*	*11*	*13*	*14*
Monadic part	.50	.61	.55	1.38	1.43	1.53	.80	.49	.48	.95
Dyadic part	.64	.62	1.13	1.52	1.72	1.47	.64	.63	.62	1.10
Combination	.57	.62	.83	1.45	1.57	1.50	.72	.56	.55	1.02

.86 for dyadic predicate problems. This indicates that the weights obtained from the separate presentation of monadic- and dyadic-predicate problems in Experiments 1 and 2 are sufficiently robust that they survive to presentation of the two types of problems together in Experiment 3.

The presentation of both types of problems to the same subjects allows construction of a single parametric model, that is, with weights estimated for each schema from all 128 problems including both the monadic- and dyadic-predicate problems (i.e., ignoring whether a problem's predicates are monadic or dyadic). The four correlations between the problem difficulties predicted by the weights from Data Sets 1 and 2, respectively, and the observed mean difficulties for the two sets range from .91 to .92, revealing that the two data sets and their weights are extremely similar. The weights for two problem types taken together obtained from the 40 subjects in the combined data set are presented in Row 3 of Table 12.7.

The correlations between problem difficulties predicted by the overall weights in Row 3 of Table 12.7 for the monadic-predicate problems of Experiments 1 and 3 are .91 and .92, respectively, and for the dyadic-predicate problems of Experiments 2 and 3 are .86 each. Thus, the weights of the combined monadic and dyadic problem set provide excellent predictions for both the monadic and dyadic problems considered separately. This suggests that the predictions obtained from the weights reported in Row 3 of Table 12.7 (the combined monadic and dyadic set) are highly correlated with the predictions obtained from the monadic and dyadic sets separately, and indeed in both cases the $r = .99$.

The obtained mean difficulties for any data set (monadic, dyadic, or combined), therefore, can be predicted quite accurately from the weights obtained from any data set (monadic, dyadic, or combined). Ignoring whether problems present monadic or dyadic predicates and treating the problems as a single set when generating schema weights has no apparent predictive cost.

GENERAL RESULTS

Let us first consider overall performance on evaluating the proposed conclusions. In all data sets from all three experiments where subjects received all 64 problems, a total 180 subjects made 13,080 responses, and of these 354 were errors (2.71%). (Of these, 20 errors, 17 monadic and 3 dyadic, were on Problem 15.) Thus, the overall accuracy was 97.29%. This result shows two things: First, it demonstrates a class of predicate-logic reasoning problems that are well within subjects' reasoning abilities. Second, it shows that the parametric models are based on 97.29% of the overall responses (given that perceived difficulty ratings were counted only where evaluations were correct).

The fact that subjects almost always made the logically appropriate judgment, however, does not, by itself, show that subjects arrived at these judgments in the way described by the mental-logic theory. The analyses of perceived problem difficulty, however, provide evidence indicating that subjects were reasoning on these problems in the way described by the theory. The theory claims that the subjects in this study evaluated the conclusions to these problems after going through a line of reasoning in which schemas are applied sequentially. Each application of a schema is assumed to require some cognitive effort, and the total effort expended on a single problem soluble directly on the DRR is equal to the sum of the efforts for each schema application. This led to the prediction that perceived problem difficulty would correlate with the number of reasoning steps required to solve a problem. The overall results show that for the 180 subjects presented all 64 problems, the correlation between the number of reasoning steps and the obtained mean ratings is .83, accounting for approximately 69% of the variance, which substantially supports the prediction. Indeed, even with problem length partialed out the correlation is .75, accounting for 56% of the variance.

Furthermore, the theory predicted that problem difficulty would be even more closely correlated with the summed difficulties estimated for each schema used in each reasoning step proposed by the model. Combining all nine data sets into one (i.e., including together all 180 subjects who received all 64 problems) generates the schema weights shown in Row 1 of Table 12.8. (Rows 2 and 3 of Table 12.8 present the weights generated separately for monadic-predicate problems from the 100 subjects who received all 64 monadic-predicate problems and the 100 subjects who received all 64 dyadic-predicate problems, respectively.) Table 12.9 presents the difficulties for each problem predicted from the weights in Table 12.7, together with the obtained mean difficulties for these problems calculated from the data from all three experiments. The correlation between the predicted difficulties from Row 1 of Table 12.8 and the obtained mean difficulty ratings of all data sets combined is .93, accounting for approximately 86% of the variance; even with problem length partialed out, the correlation is

TABLE 12.8
Schema Weights Obtained From the Combination of Data Sets From Experiments 1, 2, and 3 Overall and its Monadic and Dyadic Parts

	Schema									
Data Set	*1*	*2*	*3ii*	*4ii*	*6*	*7*	*8*	*11*	*13*	*14*
Overall	0.54	0.61	0.90	1.46	1.47	1.50	0.67	0.62	0.54	1.02
Monadic	0.47	0.65	0.66	1.48	1.34	1.56	0.75	0.60	0.49	0.95
Dyadic	0.60	0.56	1.16	1.44	1.61	1.42	0.60	0.64	0.61	1.09

TABLE 12.9
Predicted Problem Difficulties and Obtained Mean Difficulty Ratings from Experiments 1, 2, and 3

	Predicted Difficulties								
	10 Parameters			*3 Parameters*			*Obtained Means*		
Problems	*Monadic*	*Dyadic*	*Overall*	*Monadic*	*Dyadic*	*Overall*	*Monadic*	*Dyadic*	*Overall*
4.	1.49	1.61	1.54	1.57	1.60	1.59	1.50	2.19	1.85
5.	1.49	1.61	1.54	1.57	1.60	1.59	1.45	1.46	1.45
6.	1.95	2.09	2.02	1.89	2.10	1.99	1.95	2.09	2.02
7.	1.95	2.09	2.02	1.89	2.10	1.99	2.26	2.19	2.23
8.	1.49	1.61	1.54	1.57	1.60	1.59	1.81	2.00	1.91
9.	1.49	1.61	1.54	1.57	1.60	1.59	1.41	1.65	1.53
10.	1.95	2.09	2.02	1.89	2.10	1.99	2.46	3.24	2.84
11.	1.95	2.09	2.02	1.89	2.10	1.99	2.12	2.42	2.27
12.	1.66	2.16	1.90	1.89	2.10	1.99	2.13	2.62	2.37
13.	2.48	2.44	2.46	2.52	2.48	2.50	2.80	3.07	2.93
14.	1.47	1.60	1.54	1.57	1.60	1.59	1.66	1.81	1.73
15.	1.65	1.56	1.61	1.57	1.60	1.59	1.70	2.80	2.28
16.	1.75	1.60	1.67	1.57	1.60	1.59	2.63	2.77	2.69
17.	2.34	2.61	2.47	2.52	2.48	2.50	2.42	2.63	2.53
18.	2.56	2.42	2.50	2.52	2.48	2.50	2.77	3.07	2.92
19.	1.60	1.64	1.62	1.57	1.60	1.59	2.18	3.38	2.74
20.	1.75	1.60	1.67	1.57	1.60	1.59	2.42	2.03	2.22
21.	1.96	2.21	2.08	2.14	2.20	2.18	2.14	3.28	2.70
22.	2.14	2.17	2.15	2.14	2.20	2.18	2.13	2.06	2.10
23.	2.83	3.22	3.01	3.09	3.08	3.09	2.69	3.00	2.84
24.	3.51	3.51	3.52	3.41	3.58	3.49	3.07	3.75	3.41
25.	2.42	2.69	2.56	2.46	2.70	2.58	2.11	2.14	2.13
26.	2.14	2.17	2.15	2.14	2.20	2.18	1.98	1.88	1.93
27.	2.70	2.69	2.69	2.46	2.70	2.58	2.33	2.47	2.40
28.	2.15	2.77	2.44	2.46	2.70	2.58	2.85	3.00	2.92
29.	2.09	2.25	2.15	2.14	2.20	2.18	1.97	1.58	1.78
30.	2.09	2.25	2.16	2.14	2.20	2.18	1.73	2.02	1.88
31.	2.90	3.37	3.11	3.03	3.30	3.17	3.42	3.45	3.44
32.	3.72	3.65	3.67	3.66	3.68	3.68	3.36	3.85	3.60
33.	4.64	4.80	4.70	4.55	4.78	4.67	4.30	4.73	4.50
34.	2.41	2.76	2.57	2.46	2.70	2.58	2.45	2.80	2.63
35.	3.32	3.04	3.13	3.09	3.08	3.09	3.08	2.86	2.97
36.	5.24	5.44	5.32	5.12	5.38	5.26	5.15	6.47	5.82
37.	4.18	4.48	4.33	4.17	4.50	4.35	3.47	4.21	3.83
38.	3.70	3.64	3.67	3.66	3.68	3.68	3.17	3.12	3.15
39.	3.77	3.93	3.85	3.60	3.90	3.76	3.53	3.46	3.50
40.	3.76	4.00	3.86	3.60	4.90	3.76	3.72	3.62	3.67
41.	3.16	3.36	3.24	2.46	2.70	2.58	2.93	4.38	3.58
42.	3.13	3.00	3.07	3.09	3.08	3.09	4.24	2.78	3.48
43.	3.90	4.03	3.97	3.41	3.58	2.49	3.21	3.61	3.41
44.	3.83	3.68	3.75	3.66	3.68	3.68	4.09	3.79	3.94

(Continued)

TABLE 12.9
(Continued)

	Predicted Difficulties								
	10 parameters			3 parameters			Obtained means		
Problems	*Monadic*	*Dyadic*	*Overall*	*Monadic*	*Dyadic*	*Overall*	*Monadic*	*Dyadic*	*Overall*
45.	3.09	3.21	3.14	3.09	3.08	3.09	3.19	3.47	3.33
46.	3.58	3.82	3.68	3.66	3.68	3.68	3.53	4.12	3.82
47.	4.38	4.81	4.57	4.55	4.78	4.67	3.94	4.67	4.25
48.	5.13	5.41	5.24	5.12	5.38	5.26	5.04	4.91	4.97
49.	2.71	2.81	2.75	2.71	2.80	2.77	2.71	3.08	2.90
50.	4.26	4.11	4.19	3.98	4.18	4.08	4.32	3.63	3.97
51.	3.17	3.29	3.23	3.03	3.30	3.17	3.67	3.35	3.52
52.	4.16	4.07	4.13	3.98	4.18	4.08	3.86	3.82	3.84
53.	3.48	3.78	3.62	3.66	3.68	3.68	3.46	3.40	3.42
54.	3.02	3.33	3.18	3.03	3.30	3.17	3.15	3.41	3.28
55.	2.95	3.04	3.00	3.09	3.08	3.09	2.95	3.59	3.27
56.	2.22	2.20	2.21	2.14	2.20	2.18	2.96	2.20	2.45
57.	3.97	4.18	4.07	3.98	4.18	4.08	4.47	4.00	4.23
58.	2.71	2.81	2.75	2.71	2.80	2.77	2.98	2.71	2.84
59.	2.82	2.84	2.83	2.71	2.80	2.77	3.33	2.75	3.04
60.	3.88	3.60	3.74	3.66	3.68	3.68	4.66	4.14	4.39
61.	2.82	2.84	2.83	2.71	2.80	2.77	2.40	2.64	2.52
62.	5.24	5.44	5.32	5.12	5.38	5.26	5.01	4.53	4.76
63.	5.13	5.41	5.24	5.12	5.38	5.26	4.86	4.99	4.93
64.	2.41	2.76	2.57	2.46	2.70	2.58	2.53	2.77	2.65

Note. Problems 1 to 3 were control-true problems and are excluded from this table. Sixty subjects received only monadic-predicate problems, 60 subjects received only dyadic-predicate problems, and 40 subjects received both monadic and dyadic problems. Thus, there were 100 monadic cases, 100 dyadic cases, and 200 cases overall. The predicted difficulties and observed means were obtained from all these subjects.

.89, accounting for 79% of variance. The overall weights, even when differences between monadic and dyadic predicate are ignored, provide an extremely good prediction of perceived problem difficulty.

Construction of a Parsimonious Model

In the preceding analyses, a different weight was assigned to each of the 10 core schemas, and a 10-parameter model was used to predict problem difficulties. Given that the 10-parameter model worked extremely well, it seemed useful to consider whether a more parsimonious model with a smaller number of parameters could be constructed without significant cost to predictability. Figure 12.1 plots the weights for each schema in Row 2 of Table 12.8 (the monadic part) against the weight in Row 3 of

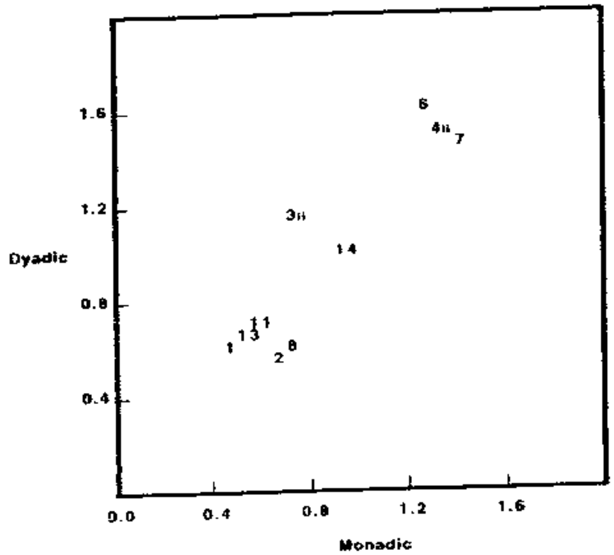

FIG. 12.1. Cross tabulation of schema weights based on perceived difficulty ratings for the monadic- and dyadic-predicate problems.

Table 12.8 (the dyadic part). Inspection of Fig. 12.1 suggests that a three-parameter model would provide a good fit to the data, with Schemas 1, 2, 8, 11, and 13 constituting a relatively easy set of inferences; Schemas 3ii and 14 a set of intermediate difficulty; and Schemas 4ii, 6, and 7 a relatively difficult set of schemas. In this three-parameter model, the weights are .59, .99, and 1.50 for the easiest, intermediate, and most difficult schema sets, respectively. (This three-parameter model leads to the same rank ordering of weights for the monadic and dyadic sets.) The correlation between difficulties predicted by the three-parameter model and obtained mean difficulties = .93, which is identical to the correlation for the 10-parameter model. The three-parameter model thus is both more parsimonious than the 10-parameter model and of equal predictive ability.

Inspection of the three sets of schemas for the three-parameter model reveals what makes a schema relatively easy or difficult. The five schemas that constitute the easiest set (i.e., Schemas 1, 2, 8, 11, and 13) each present only a single logic particle from among *and*, *if*, *some*, and *not*. The two schemas that constitute the set of intermediate difficulty (i.e., Schemas 3ii and 14) both present the particle *or* (and in the case of Schema 14, the particle *not*). The three schemas of the most difficult set (i.e., Schemas 4ii, 6, and 7) are characterized by presentation of multiple particles, that is, Schemas 6 and 7 present *or* and *if*, Schema 4ii presents *and* and *not*. The perceived relative difficulties of the schemas thus can be summarized as follows: Application of a schema with only one particle from among *and*, *if, some,* and *not* is perceived as relatively effortless; application of a schema with *or* is perceived to take somewhat more effort, and application of a schema that presents two particles is perceived as relatively more difficult.

GENERAL DISCUSSION

The theory predicted that because the problems are soluble on the basic schemas and the DRR, there should be relatively few errors overall, and almost none on one-step problems. This clearly was the case, and once again the mental-logic approach has been able to construct a set of logical-reasoning problems that people solve routinely.

Perceived difficulty ratings were used to estimate the amount of cognitive effort expended to solve a problem. It was predicted that the number of reasoning steps required to solve a problem (predicted on the DRR) would correlate with observed mean difficulty ratings, and this also clearly was the case, thus providing support for the claim that the problems were solved in the way described by the theory. It was predicted that perceived problem difficulty would correlate even more strongly with the sum of the difficulties of the schemas applied in the reasoning steps to solve each problem (as predicted by the DRR). This prediction was highly confirmed.

These findings present a challenge to a large number of researchers who have concluded that people have few, or no, logical skills for reasoning, but rather that human reasoning is governed by content-specific processes (e.g., Cheng & Holyoak, 1985; Cosmides, 1989) or by an assortment of heuristics and reasoning biases (e.g., Evans et al., 1993). It is not obvious how any content-dependent process could account for the success found on the problems presented here, nor is it obvious what sort of heuristics or reasoning biases could explain the apparently logical responses to these problems.

The results reported here also present a challenge to two theories that ought, in principle, to be able to account for such findings. Rips's (1994) competing mental-logic theory has some difficulty in representing at least some of the problems, and leads to quite complicated lines of reasoning on many of them, as discussed in chapter 11. The mental-models theorists (e.g., Johnson-Laird & Byrne, 1991) have as yet provided only a partial description of how propositions of the sort presented in these problem would be represented by models, and we can only estimate how models theory would procede. That it is unclear how the models theory would represent the premises of these problems is illustrated by considering the premise *There are no square wooden beads,* from Problem 36 of Appendix 1. Johnson-Laird and Byrne (1991, p. 120) stated that a universal negative proposition, for example, *None of the athletes is a baker,* will be represented as:

```
[a]
   [b]
. . .
```

Application of this structure to *There are no square wooden beads* is problematic. Note that one cannot simply add one line to the model, as such:

```
[b]
   [w]
       [s]
. . .
```

because to do so would preclude the possibility of there being a wooden bead, or a square bead, or a square wooden thing that is not a bead, and these possibilities clearly should be allowed. Indeed, the appropriate model would seem to include six explicit representations:

```
[b]
   [w]
       [s]
           [b w]
                 [b s]
                      [w s]
. . .
```

We do not presume here to say how the models theory should decide how to represent such premises, but until its advocates provide a specification it remains unclear how, or whether, they will account for these findings. They will need to account both for why these problems were solved so routinely, and for the high correlations both between perceived problem difficulties and the number of reasoning steps, and between perceived problem difficulties and the sum of the weights for the schemas used in the solution. Precisely how any competing theory would accomplish this is unclear, so it is incumbent on them to show why they can be considered as serious alternatives.

APPENDIX I
The Test Problems With Monadic Predicates

Control-True Problems

1. All the beads are red

   ? All the beads are red?
2. The blue beads are wooden

   ? The blue beads are wooden?
3.* The metal beads are not round

   ? The metal beads are not round?

Control-False Problems

4. All the beads are square

   ? Some of the beads are not square (13)
5. Some of the beads are metal

   ? None of the beads are metal? (13)
6. All the beads are triangular or round

   ? Some of the beads are neither triangular nor round? (14)
7. Some of the beads are either plastic or wooden

   ? None of the beads are wooden and none are plastic? (14)
8.* Some of the beads are not triangular

   ? All the beads are triangular? (13)
9.* None of the beads are plastic

   ? Some of the beads are plastic? (13)
10.* Some of the beads are neither green nor square

   ? All the beads are green or square? (14)
11.* None of the beads are metal and none are wooden

   ? Some of the beads are either metal or wooden? (14)

One-Step Problems

12. The blue beads are either square or round

   ? The blue beads that are not square are round? (True) (3ii)
13. There are no green square beads
   There are some square beads

   ? The square beads are not green? (True) (4ii)

(Continued)

APPENDIX 1
(Continued)

14.	All the beads are red All the beads are metal ? All the beads are red metal beads?	(True) (1)
15.	All the beads are round and wooden ? All the beads are wooden?	(True) (2)
16.	None of the beads are round Some of the beads are wooden ? The wooden beads are not round?	(True) (8)
17.	All the beads are plastic or wooden The plastic beads are red The wooden beads are red ? All the beads are red?	(True) (6)
18.	All the beads are green or blue The green beads are plastic The blue beads are wooden ? All the beads are plastic or wooden?	(True) (7)
19.	The blue beads are metal ? Some of the beads are metal?	(True) (11)
20.	All the beads are round Some of the beads are red ? The red beads are round?	(True) (8)
One-Step With Contradiction		
21.	All the beads are blue All the beads are plastic ? Some of the beads are not blue plastic beads?	(False) (1 + 13)
22.	All the beads are square wooden beads ? Some of the beads are not wooden?	(False) (2 + 13)
23.	All the beads are blue or green The blue beads are plastic The green beads are plastic ? Some of the beads are not plastic?	(False) (6 + 13)
24.	All the beads are metal or wooden The wooden beads are red The metal beads are blue ? Some of the beads are neither red nor blue?	(False) (7 + 14)

(Continued)

25. None of the beads are metal None of the beads are plastic	
? Some of the beads are either plastic or metal?	(False) (1 + 14)
26. Some of the beads are green plastic beads	
? None of the beads are green?	(False) (2 + 13)
27. All the beads are square or round	
? Some of the green beads are neither square nor round?	(False) (8 + 14)
28.* All the plastic beads are round or triangular	
? Some of the plastic beads that are not round are not triangular?	(False) (3ii + 13)
29.* The red beads are triangular	
? None of the beads are triangular?	(False) (11 + 13)
30.* None of the beads are round	
? The square beads are round?	(False) (11 + 13)
Multistep Problems	
31. All the beads are triangular or square None of red beads are square	
? Some of the red beads are not triangular?	(False) (3ii + 8 + 13)
32. There are no green wooden beads The round beads are wooden	
? Some of the round beads are green?	(False) (4ii + 8 + 13)
33. There are no red metal beads The square beads are metal Every bead is either red or green	
? The square beads are green?	(True) (4ii + 8 + 3ii + 8)
34. All the beads are plastic or metal The blue beads are not plastic	
? The blue beads are metal?	(True) (3ii + 8)
35. There are no red plastic beads The square beads are plastic	
? The square beads are not red?	(True) (4ii + 8)
36. All the beads are triangular or square The green beads are not triangular There are no square wooden beads	
? Some of the beads are not wooden?	(True) (3ii + 8 + 4ii + 8 + 11)

(Continued)

APPENDIX 1
(Continued)

37.	All the beads are metal or wooden The triangular beads are blue and not wooden ? The triangular beads are blue metal beads?	(True) (2 + 3ii + 8 + 2 +1)
38.	There are no green triangular beads The wooden beads are green ? The wooden beads are green and not triangular?	(True) (4ii + 8 + 1)
39.	None of the beads are wooden The blue beads are not square ? All the beads are square or wooden?	(False) (8 + 1 + 11 + 14)
40.	The plastic beads are either blue or green None of the beads are blue ? Some of the beads are green?	(True) (8 + 3ii + 11)
41.	The blue beads are either metal or plastic None of the beads are metal ? The blue beads are plastic?	(True) (8 + 3ii)
42.	All the beads are green and none of them are square wooden beads ? The wooden beads are not square?	(True) (2 + 4ii)
43.	All the beads are wooden or metal The wooden beads are square The metal beads are triangular ? The beads that are not triangular are square?	(True) (7 + 3ii)
44.	There are no green round beads The plastic beads are round ? Some of the beads are not green?	(True) (4ii + 8 + 11)
45.	All the beads are green or blue The green round beads are plastic The blue round beads are plastic ? The round beads are plastic?	(True) (8 + 6)
46.	All the beads are green or red The green beads are triangular The red beads are triangular ? Some of the wooden beads are not triangular?	(False) (6 + 8 + 13)
47.	All the beads are blue or red There are no blue square beads ? Some of the beads that are not red are square?	(False) (3ii + 4ii + 8 + 13)

(Continued)

APPENDIX 1
(Continued)

48. There are no red wooden beads
All the triangular beads are wooden
Every bead is either red or green

? Some of the triangular beads are not green? (False) (4ii + 8 + 3ii + 8 + 13)

49. All the beads are green
Some of the triangular beads are plastic beads

? None of the triangular beads are green plastic beads? (False) (8 + 1 + 13)

50. All the beads are plastic or wooden
The wooden beads are red
The plastic beads are blue
Some of the beads are round

? None of the round beads are red and none are blue? (False) (7 + 8 + 14)

51. None of the beads are wooden
The round beads are not plastic

? Some of the round beads are either wooden or plastic? (False) (8 + 1 + 14)

52. All the beads are red or green
The green beads are round plastic beads
The red beads are wooden beads

? Some of the beads are neither plastic nor wooden? (False) (2 + 7 + 14)

53. All the beads are triangular or square
The triangular beads are red and metal
The square beads are metal

? Some of the beads are not metal? (False) (2 + 6 + 13)

54. The square beads are not green
The square beads are not red

? All the beads are green or red? (False) (1 + 11 + 14)

55.* There are no blue plastic beads
The blue beads are square

? The blue beads are square and not plastic? (True) (4ii + 1)

56.* All the beads are square
The plastic beads are red

? The plastic beads are red square beads? (True) (8 + 1)

57.* All the beads are wooden or metal
The wooden beads are red
The metal beads are green
The square beads are not red

? The square beads are green? (True) (7 + 3ii + 8)

(Continued)

APPENDIX 1
(Continued)

58.* All the beads are plastic	
The blue beads are round	
? None of the beads are round plastic beads?	(False) (8 + 1 + 13)
59.* All the beads are blue	
The square beads are wooden	
? Some of the beads are blue wooden beads?	(True) (8 + 1 + 11)
60.* There are no green metal beads	
The round beads are metal and not red	
? The round beads are not green?	(True) (2 + 4ii + 8)
61.* All the beads are metal	
Some of the beads are green	
? Some of the beads are green metal beads?	(True) (8 + 1 + 11)
62.* All the beads are plastic or wooden	
Some of the round beads are not plastic	
There are no blue wooden beads	
? Some of the beads are not blue?	(True) (3ii + 8 + 4ii + 8 + 11)
63.* There are no square metal beads	
Some of the red beads are metal	
Every bead is either square or triangular	
? None of the red beads are triangular?	(False) (4ii + 8 + 3ii + 8 + 13)
64.* All the beads are plastic or wooden	
? The round beads that are not plastic are wooden?	(True) (8 + 3ii)

Note. *Problems that were not included in Study 1 of Experiment 1 but were included in Studies 2 to 4. For Problems 12 to 64, at the end of each problem, in parentheses, the predicted response (True or False) is shown. For Problems 4 to 64, the predicted line of reasoning is shown in parentheses.

APPENDIX 2
Test Problems With Dyadic Predicates

Control-True Problems	
1. All the children like the red beads	
? All the children like the red beads?	
2. The girls got some wooden beads	
? The girls got some wooden beads?	
3. None of the girls played with Tom	
? None of the girls played with Tom?	
Control-False Problems	
4. All the boys found some square beads in their bags	
? A few boys did not find any square beads in their bags?	(13)
5. Tom likes some of the beads	
? Tom likes none of the beads?	(13)
6. All the children got either wooden beads or metal beads	
? Some of the children got neither wooden beads nor metal beads?	(14)
7. Many of the boys played with Heather or other girls	
? None of the boys played with Heather and none played with other girls?	(14)
8. The girls did not find some kind of beads in their bags	
? The girls found all the kinds of beads in their bags?	(13)
9. None of the children like the triangular beads	
? One of the children likes the triangular beads?	(13)
10. Lisa did not get many red beads and John did not get them either	
? Either Lisa or John got all the red beads?	(14)
11. Jennifer played with none of the boys and Karen played with none of them either	
? Either Jennifer or Karen played with some of the boys?	(14)
One-Step Problems	
12. Every boy either found a few metal beads or got some plastic beads	
? The boys who did not find any metal beads got some plastic beads?	(True) (3ii)

(Continued)

APPENDIX 2
(Continued)

13. Johnson does not like the blue round beads	
The round beads that Johnson likes are not blue?	(True) (4ii)
14. The boys got the red beads The girls played with the boys	
? The boys got the red beads and the girls played with (them)?	(True) (1)
15. Many of the girls played with the boys and they like the green beads	
? The boys played with many of the girls?	(True) (2)
16. Sam and Harry found (none of the) no square beads in their bags	
? Sam and Harry did not find any square metal beads?	(True) (8)
17. All the girls played with either Tom or John Each of the girls who played with Tom got a wooden bead Each of the girls who played with John got a wooden bead	
? Every girl got a wooden bead?	(True) (6)
18. All the boys like either the metal beads or the wooden beads The boys who like the metal beads played with Mary The boys who like the wooden beads played with Linda	
? All the boys played with either Mary or Linda?	(True) (7)
19. All the boys played with those girls who found red beads	
? All the boys played with some children who found red beads?	(True) (11)
20. All the children found some triangular beads	
? The boys all found some triangular beads?	(True) (8)
One-Step With Contradiction Problems	
21. The girls like all the red beads The girls like all the wooden beads	
The girls do not like red square wooden beads?	(False) (1 + 13)
22. The boys all got green beads and the girls played with them	
? Many of the boys did not get green beads?	(False) (2 + 13)

(Continued)

23. The boys all played with either Jennifer or Heather The boys who played with Jennifer like the metal beads The boys who played with Heather like the metal beads	
? Some of the boys do not like the metal beads?	(False) (6 + 13)
24. The girls all got either red beads or blue beads The girls who got red beads played with Tom The girl who got blue beads played with Bill	
? Some of the girls played with neither Tom nor Bill? (? Some of the girls neither played with Tom nor played with Bill?)	(False) (7 + 14)
25. None of the girls played with Jim None of the girls played with Mark	
? Some of the girls played with either Jim or Mark?	(False) (1 + 14)
26. The girls found many green beads and the boys played with them	
? The girls found no green beads?	(False) (2 + 13)
27. All the children like either the wooden or the metal beads	
? Some of the girls like neither the wooden nor the metal beads?	(False) (8 + 14)
28. The boys all got either square beads or triangular beads	
? Some of the boys who got no square beads did not get triangular beads either?	(False) (3ii + 13)
29. The boys played with Mary	
? None of the children played with Mary?	(False) (8 + 9)
30. The girls found the red triangular beads	
? The girls did not find any red beads?	(False) (11 + 13)
Multistep Problems	
31. All the children like either round beads or square beads None of the girls like the square beads	
? A few of the girls do not like the round beads?	(False) (3ii + 8 + 13)
32. The boys found no green plastic beads The triangular beads that the boys found are green	
? Some of the triangular beads that the boys found are plastic?	(False) (4ii + 8 + 13)

(Continued)

APPENDIX 2
(Continued)

33. There are no boys who like both the red beads and the blue beads Tom and John like the blue beads The boys either like the red beads or played with Jennifer ? Tom and John played with Jennifer?	(True) (4ii + 8 + 3ii + 8)
34. All the children got either wooden beads or plastic beads Many of the children did not get plastic beads ? Many of the children got wooden beads?	(True) (3ii + 8)
35. None of the girls played with both Harry and Jim The girls who like the round beads played with Jim ? The girls who like the round beads did not play with Harry?	(True) (4ii + 8)
36. The children all found either triangular or square beads None of the boys found square beads The boys who like the metal beads did not find both triangular beads and metal beads ? Some of the children did not find any metal beads?	(True) (3ii + 8 + 4ii + 8 + 11)
37. All the boys like either the wooden beads or the metal beads The boys who played with Linda do not like metal beads and they like square beads ? The boys who played with Linda like the square and the wooden beads?	(True) (2 + 3ii + 8 + 2 + 1)
38. None of the children got both green beads and triangular beads The girls all got green beads ? The girls got green beads and got no triangular beads?	(True) (4ii + 8 + 1)
39. None of the children found blue beads The girls did not find square beads All the children found either blue beads or square beads?	(False) (8 + 1 + 11 + 14)
40. The boys got either plastic or metal beads None of the children got metal beads ? Some of the children got plastic beads?	(True) (8 + 3ii + 11)
41. The girls all played with either Steve or Johnson None of the children played with Johnson ? The girls all played with Steve?	(True) (8 + 3ii)

(Continued)

APPENDIX 2
(Continued)

42. The boys got some of the wooden beads and they got no square wooden beads	
? The wooden beads that the boys got are not square?	(True) (2 + 4ii)
43. The girls played with either Mike or Bill The girls who played with Mike found triangular beads The girls who played with Bill found round beads	
? The girls who found no triangular beads found round beads?	(True) (7 + 3ii)
44. There are no boys who played with both Karen and Heather The boys who played with Jennifer also played with Heather	
? Some of the boys did not play with Karen?	(True) (4ii + 8 + 11)
45. All the beads are either red or blue The boys got all the red round beads The boys got all the blue round beads	
? The boys got all the round beads?	(True) (8 + 6)
46. All the beads are either plastic or metal The girls found all the plastic beads The girls found all the metal beads	
? There are some of the triangular beads that the girls did not find?	(False) (6 + 8 + 13)
47. The girls played with either Johnson or Bill There are no girls who played with both Johnson and Tom	
? Some of the girls who did not play with Bill played with Tom?	(False) (3ii + 4ii + 8 + 13)
48. The children found no red wooden beads The beads that the boys found are all wooden beads Every beads is either red or blue	
? Some of the beads that the boys found are not blue?	(False) (4ii + 8 + 3ii + 8 + 13)
49. All the children like the green beads Some of the boys like the plastic beads	
? None of the boys like the green plastic beads?	(False) (8 + 1 + 13)
50. All the beads are plastic or wooden The girls got the plastic beads The boys got the wooden beads Some of the beads are round beads	
? Neither the boys nor the girls got round beads?	(False) (7 + 8 + 14)

(Continued)

51. None of the children like the green beads The girls do not like the metal beads ? Some of the girls like either the green beads or the metal beads?	(False) (8 + 1 + 14)
52. All the beads were found by either the boys or the girls The beads that the boys found are square plastic beads The beads that the girls found are wooden beads ? Some of the beads are neither plastic nor wooden?	(False) (2 + 7 + 14)
53. All the beads were found by either the boys or the girls The beads that the boys found are red metal beads The beads that the girls found are metal beads ? Some of the beads are not metal?	(False) (2 + 6 + 13)
54. Lisa and the boys do not like the square beads Lisa and the boys do not like the triangular beads ? All the children like either the square or the triangular beads?	(False) (1 + 11 + 14)
55. None of the girls played with both John and Jim John played with the girls who got red beads ? The girls who got red beads played with John and did not play with Jim?	(True) (4ii + 1)
56. All the beads are square The girls found the red beads ? The girls found the red square beads?	(True) (8 + 1)
57. All the children got either wooden or metal beads The wooden beads are red The metal beads are green None of the boys got red beads ? The boys all got green beads?	(True) (7 + 3ii + 8)
58. All the beads are plastic The girls like the red beads None of the children like the red plastic beads?	(False) (8 + 1 + 13)
59. All the beads are blue The boys got the square beads Some of the children got the blue square beads?	(True) (8 + 1 + 11)
60. There are no round metal beads The beads that Tom and Jennifer found are all metal beads and they found no red beads ? Tom and Jennifer found no round beads?	(True) (2 + 4ii + 8)

(Continued)

APPENDIX 2
(Continued)

61. All the beads are wooden Some of the boys found the green beads ?Some of the children found the green wooden beads?	(True) (8 + 1 + 11)
62. All the children like either the plastic or the wooden beads Some of the girls do not like the plastic beads There are no blue wooden beads ? Some of the beads are not blue?	(True) (3ii + 8 + 4ii + 8 + 11)
63. None of the children found square metal beads The beads that some of the boys found are metal Every bead is either square or triangular ? None of the boys found triangular beads?	(False) (4ii + 8 + 3ii + 8 + 13)
64. All the boys played with either Karen, Jennifer, or Heather A few of the boys who played with neither Jennifer nor Heather played with Karen?	(True) (8 + 3ii)

Note. In the parentheses at the end of each problem are indicated the predicted evaluation and the schemas required to solve the problem.

The left-hand side is the problem premise and the right-hand side the problem conclusion.

Schema 38 is closely related to the propositional schema:

p OR q; NEG-p / q

(See Table 11.3, Schema 3.) This propositional schema has a possible alternative form that is normally valid:

TABLE 11.3
Schemas of the Proposed Mental Predicate Logic

For each schema, the propositional-logic version, if there is one, is stated first, then the corresponding predicate-logic version in bold type, sometimes with possible alternative or additional forms. Premises are given first, followed by "/", followed by the conclusion entailed. In the case of bidirectional inferences and paraphrases "≡" replaces "/." After each schema, its status in the reasoning program is indicated: Core, Feeder, or Indirect. Core schemas apply whenever the subjects or argument terms of the premises present sets to which they can apply, or when there is a topic set that fulfills the indicated role. Feeder schemas apply when they enable a core inference to be made or a conclusion to be matched or contradicted. (See The Direct Reasoning Routine and the text discussion of the reasoning program for further details.)

Alongside several schemas there is a parallel schema that interchanges the roles of S_1 and S_2, for example, alongside #2 the parallel schema would have the conclusion "S_1[q X]." The parallel schemas are obvious and are omitted for the sake of compactness. (In the final version of the logic "&" and "OR" will be treated as n-ary rather than binary, as in prior versions of the propositional logic, and only one schema will be necessary. I have kept to the binary versions of "&" and "OR" for readability.)

Notation. "S[All X]" means that the X or Xs all satisfy the condition S. "[Some X]" indicates some unspecified X or Xs. "[α]" indicates an argument of any form whatever that is not in the scope of a negation. "[q X]" means X modified by any quantifier, for example, *all, each, many, few, some*, etc. (but not *no, none, not any*); "[q X]" includes pro-forms with quantified antecedents (e.g., "[PRO-All X]," "[PRO-Some X]").

"[PRO-α]" is a pro-form whose antecedent is α. "S_1[All X] OR S_2[PRO-All X]" means that each X is either S_1 or S_2; "S_1[Some X] & S_2[PRO-Some X]" means that some X or Xs are S_1 and the same X or Xs are also S_2. Read "[All X: S[PRO]]" as "all the Xs that are such that they satisfy S," that is, "the Xs that are S."

"F" and "C_x" represent classifier predicates.
"E[. . .]" means "There exists [. . .]."
Read "[X] ⊆ [Y]," where [X] is singular, as "X is a Y."

In "NEG S[All X]" [All X] is outside the scope of the negation. "[–. . .–]" means that "[. . .]" is inside the scope of a negation, for example, "NEG S[–All X–]" means "it is not the case that all the Xs are S"; similarly, "NEG S[–Some X–]" means that no Xs are S ("it is not the case that some X is S"), as opposed to "NEG S[Some X]" "Some X is not S." Thus NEG S[Some X] ≡ NEG S[–All X–].

(Continued)

NOTE. A TABLE OF SCHEMAS FROM CHAPTER 11.
(BRAINE, 1998)

TABLE 11.3
(Continued)

Primary Schemas

1. p; q / p & q (*Feeder*)
   S_1[All X]; S_2[All X] / S_1[All X] & S_2[PRO-All X] (*Feeder*)
   The boys wore blue jeans; The girls played with the boys / The boys wore blue jeans and the girls played with them.

Note. The conclusion can be realized in surface structure in various ways. In particular, "[PRO-All X]" is usually not realized when "[All X]" and "[PRO-All X]" would occupy the same argument position in surface structure; instead, the predicates are conjoined, for example, "The boys wore blue jeans and were played with by the girls."

2. p & q / p (*Feeder*)
   S_1[q X] & S_2[PRO-q X] / S_2[q X] (*Feeder*)
   Many of the boys wore blue jeans and the girls played with them / The girls played with many of the boys.
3. p OR q; NEG p / q (*Core*)
   (i) **S_1[All X] OR S_2[PRO-All X]; NEG S_2[α]; [α] ⊆ [X] / S_1[α]** (*Core; [α] may be Topic*)
   (ii) **S_1[All X] OR S_2[PRO-All X] / S_2[All X: NEG S_1[PRO]]** (Requires E[Some X: NEG S_1[PRO]]) (*Feeder*)
   (i) The boys either played with girls or fought with girls; Tom and Dick did not play with girls / Tom and Dick fought with girls.
   (ii) The boys either played with girls or fought with girls / The boys who did not play with girls fought with girls.
4. NEG (p & q); p / NEG q (*Core*)
   (i) **NEG E[–Some X: S_1[PRO] & S_2[PRO]–]; S_2[α]; [α] ⊆ [X] / NEG S_1(α)** (*Core; [α] may be Topic*)
   (ii) **NEG (S_1[All X] & S_2[PRO-All X]) / NEG S_2[All X: S_1[PRO]]** (Requires E[Some X: S_1[PRO]] (*Feeder*)
   (i) There were no boys who wore sandals and blue jeans; The boys that played with Mary wore blue jeans / The boys that played with Mary did not wear sandals.
   (ii) The boys did not wear sandals with blue jeans / The boys that wore blue jeans did not wear sandals.
5. IF p OR q THEN r; p / r (*Core*)
   No predicate logic schema that directly corresponds. See note following Schema 17.
6. p OR q; IF p THEN r; IF q THEN r / r (*Core*)
   S_1[All X] OR S_2[PRO-All X]; S_3[All X: S_1[PRO]]; S_3[All X: S_2[PRO]] / S_3[All X] (*Core*)
   All the cars in the lot have stickers or the guards tow them away; The cars that have stickers are Toyotas; The cars that the guards tow away are Toyotas / All the cars in the lot are Toyotas.
7. p OR q; IF p THEN r; IF q THEN s / r OR s (*Core*)
   S_1[All X] OR S_2[PRO-All X]; S_3[All X: S_1[PRO]]; S_4[All X: S_2[PRO]] / S_3[All X] OR S_4[PRO-All X] (*Core*)
   The cars in the lot have stickers or the guards tow them away; The cars that have stickers are Datsuns; The cars that the guards tow away are Toyotas / The cars in the lot are all Toyotas or Datsuns.
8. IF p THEN q; p / q (Modus Ponens) (*Core*)
   (i) S[X]; [α] ⊆ [X] / S[α] (*Core; [α] may be Topic*)
   (ii) NEG S[–Some X–]; [α] ⊆ [X] / NEG S[α] (*Core; [α] may be Topic*)
   (i) The girls all wore red jeans / The girls in sneakers wore red jeans.
   (ii) None of the boys wore striped shirts / Sam and Harry did not wear striped shirts.

Note. Version (ii) is referred to in the text as Negative Instantiation.

TABLE 11.3
(Continued)

9. Given a chain of reasoning of the form:

   Suppose p

   - - -

   - - -

   q

   One can conclude: IF p THEN q (*Normally Indirect*)

   Given a chain of reasoning of the form:

   Suppose S_1[Some X]

   - - -

   - - -

   S_2[PRO-Some X]

   One can conclude: S_2[Any X: S_1[PRO]] (*Indirect*)

Notes. (i) The propositional schema is the Schema for Conditional Proof. (ii) The Preliminary Procedure of the Direct Reasoning Routine (Table 3) incorporates a routine use of this schema. (iii) The predicate-logic schema applies only for languages with a quantifier like English *any*.

10. No propositional logic version.
   (i) **S[Any X]; E[Some X: S[PRO]] / S[All X]** (*Core*)
   (ii) **If S_1[Some X] then S_2[PRO-Some X]; E[Some X: S_1[PRO]] / S_2[All X: S_1[PRO]]** (*Core*)
   (i) Any child in sandals wore a spotted shirt; one or more children wore sandals / All the children in sandals wore spotted shirts.
   (ii) If a child wore sandals they wore a spotted shirt; one or more children wore sandals / All the children in sandals wore spotted shirts.

Note. Schema 10 corresponds to Universal Generalization of standard predicate logic. Version (i) would not be applicable for languages without a quantifier like English *any*. For languages like English one might call the predicate-logic version of Schema 9 "Any-Introduction" and this schema "Any-Elimination."

11. No propositional logic version.
   S[α]; [α] ⊆ [X] / S[Some X] (*Feeder*)
   Many of the girls in spotted shirts wore red jeans / Some of the girls wore red jeans.
   All the girls played with boys in green jeans / All the girls played with children in green jeans.

Note. Corresponds to Existential Generalization of standard logic.

Primary schemas concerning the logic of negation and incompatibility:

12. NEG (NEG p) ≡ p (*Core from left to right, feeder from right to left*)

Notes. (i) Although formally a paraphrase schema, it almost always operates from left to right. (ii) For simple propositions NEG is realized in surface structure by verb negation, for compound propositions by *It is false (not true) that* . . . . (iii) No predicate-logic schema is needed that is additional to the propositional schema. Some pertinent double-negative cancellations are managed as follows: NEG (NEG S[Some X]) is assumed to be mapped by Rule 23(ii) to NEG (NEG S[–All X–]) and then the propositional rule converts it to S[X] (e.g., "It is not true that some of the boys did not play with Susan" is paraphrased to "It is not true that not all the boys played with Susan" and then converted by the propositional schema to "All the boys played with Susan").

13. p; NEG p / INCOMPATIBLE (*Core*)
   (i) **S[All X]; NEG S[q X] / INCOMPATIBLE** (*Core*)
   (ii) **S[q X]; NEG S[All X] / INCOMPATIBLE** (*Core*)
   (i) The boys are all wearing sneakers; Some of the boys are not wearing sneakers / INCOMPATIBLE.
   (ii) Some of the boys are wearing sneakers; None of the boys are wearing sneakers / INCOMPATIBLE

(Continued)

TABLE 11.3
(Continued)

& NEG q / INCOMPATIBLE (*Core*)
R S_2[PRO-All X]; NEG S_1[q X] & NEG S_2[PRO-q X] / INCOMPATIBLE

: S_2[PRO-q X]; NEG S_1[All X] & NEG S_2[All X] / INCOMPATIBLE

had stickers or the guards towed them away; Some of the cars did not
:s and the guards did not tow them away / INCOMPATIBLE.
boys wore a striped or a spotted shirt; None of the boys wore a striped
one wore a spotted shirt / INCOMPATIBLE.
of reasoning of the form:

3LE
de: NEG p (*Normally Indirect*)
ation Procedure of the Direct Reasoning Routine (Table 11.4) incorpo-
use of this schema to generate a response of "False." (ii) There is no
ng.

Paraphrases and Truisms

il logic version.
Some X: S[PRO]] (*Feeder*)
ls wore red jeans = There were girls in red jeans.
il logic version.
= S[q_1 X] & S[q_2 Y] (*Feeder*)
s well as many of the girls in red jeans played with boys / Jane and
h boys and many of the girls in red jeans did too.
may be the same quantifier. (ii) This schema could be regarded as a
on of Propositional Schemas 5 and 6, in that these jointly guarantee the
R q THEN r = IF p THEN r & IF q THEN r, which generalizes to the

il logic version.
[All Z] ⊆ [X] OR [PRO-All Z] ⊆ [Y] (*Feeder*)
ise boys and girls = Every child is a boy or a girl.
with Schema 13 defines the relation of the set connective "+" to
nd OR. This schema also serves the propositional logic as an
iema.
il logic version.
$_2$[PRO-Some X] = S_1[Some X: S_2[PRO]] = S_2[Some X: S_1[PRO]] (*Feeder*)
en jeans and she played with John = A girl who played with John wore
girl in green jeans played with John.
il logic version.
⊆ [F] (*Feeder*)
] = [α] ⊆ [F: S[PRO]] (*Feeder*)
⊆ [C_α: S[PRO]]

TABLE 11.3
(Continued)

21. No propositional logic version.
    [X: S[PRO]] = [X] ≡ S[All X]
    The boys in sandals were all the boys there were ≡ All the boys wore sandals.
22. NEG (p OR q) ≡ NEG p & NEG q (*Feeder*)
    NEG (S_1[q X] OR S_2[PRO-q X]) ≡ NEG S_1[q X] & NEG S_2[PRO-q X] (*Feeder*)
    Many of the girls did not wear sandals nor did they wear sneakers ≡ Many of the girls did not wear sandals and they did not wear sneakers.

Note. NEG (p or q) would have a typical surface realization as *neither . . . nor.*

23. No propositional logic version.
    (i) **NEG S[All X] ≡ NEG S[~Some X~]** (*Feeder*)
    (ii) **NEG S[Some X] ≡ NEG S[~All X~]** (*Feeder*)
    (i) "None of the girls wore sneakers" illustrates both representations.
    (ii) Some of the girls did not wear sneakers ≡ Not all of the girls wore sneakers.

Truisms (Axioms)

24. **S[q X: S[PRO]]**
25. **[q X: S[PRO]] ⊆ [X]**
26. **[q X] ⊆ [X + Y]**
27. **[q X] ⊆ [C_x]**
28. **[([X: S_1[PRO]]: S_2[PRO]]: S_3[PRO] . . .] = [X: S_1[PRO] & S_2[PRO] & S_3[PRO] & . . .]**

39. p OR q / IF NEG-p THEN q

Schema 38 is the universal closure of 39, in the following sense: If we were to take the left-hand side of 38 and instantiate in the normal manner of standard predicate logic, we would obtain an expression such as "S_1a OR S_2a"; 39 could be applied to this expression to obtain "IF NEG S_1a THEN S_2a"; applying universal generalization, and then rendering the result in the notation developed earlier, we would obtain the right-hand side of 38. Thus Schema 38 can be considered a predicate-logic version of 39. In the same way, it can be anticipated that most propositional logic schemas will have predicate logic versions.

Table 11.3 presents the inference schemas proposed for mental predicate logic, together with the propositional logic schemas that have been presented and discussed in prior work (see earlier chapters). Schemas 1 to 9, 12 to 15, and 22 are propositional logic schemas; most of these have predicate logic versions that are presented alongside the propositional versions. In each case the propositional logic version is presented first; below it the predicate logic version is presented in bold type, and then an example of the predicate logic version of the inference is given. Sometimes there appear to be two candidate predicate logic versions, and in that case both are given and illustrated. Some of the schemas are followed by comments.

The other schemas exist only in predicate logic versions. Schema 11 provides for existential generalization. Schemas 12 to 15 are the primary