Physical Epistemic Logic

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Academic Ethics

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The necessity for applying to Philosophy the words of Christ, “Let the dead bury their dead; arise, and follow Me.” The whole of the history of philosophy becomes a battlefield covered with the bones of the dead; it is a kingdom not merely formed of dead and lifeless individuals, but of refuted and spiritually dead systems, since each has killed and buried the other. (Hegel, *Lectures on the History of Philosophy*)

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The genuine teacher will take good care not to use his position at the lectern to promote any particular point of view, whether explicitly or by suggestion. (Max Weber, *Science as a Vocation*)

My Choice

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Therefore at last, I think I had better concentrate on concrete work and technical issues.

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the law of natural consistency vs. the law of energy conservation

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Physicalism

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Under such assumption, we introduce $\approx$ relation, which should also be S5, to connect all the possible worlds that agree on all the (both known and unknown) physical laws.

A world may be epistemically possible but physically impossible; physically possible but epistemically impossible.

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Bundle Modality

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bundle modality by Y. Wang et al.: $\exists x\Box$

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where $i \in \mathbf{I}$, $p \in \mathbf{P}$. We define $\mathcal{G}_i\varphi$ as the abbreviation of $\mathcal{K}_i\mathcal{A}\varphi$.

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where $i \in \mathbf{I}$, $p \in \mathbf{P}$. We define $\mathcal{G}_i\varphi$ as the abbreviation of $\mathcal{K}_i\mathcal{A}\varphi$.
When **I** is a singleton, without loss of generality we always denote $\mathbf{I} = \{i\}$, and we further omit this i in the subscripts of $\mathcal{K}_i$ and $\mathcal{G}_i$ so as to simply write $\mathcal{K}$ and $\mathcal{G}$, which will not cause any unpleasant confusion as no other agent except i is present.

Model

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Definition 3 (Model)

- S is a non-empty set of epistemically possible worlds.
- For every $i \in \mathbf{I}$, $\sim_i$ is an equivalence relation over S.
- $V : S \to \mathcal{P}(\mathbf{P})$.
- $\approx$ is an equivalence relation over S representing the (physical) possibilities.

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Let $[s]_i$ and $[s]_\mathcal{A}$ be the equivalence classes generated by s with respect to $\sim_i$ and $\approx$, respectively. Let $[s]^i_\mathcal{A}$ be $\bigcup_{t \in [s]_i} [t]_\mathcal{A}$. Intuitively $[s]^i_\mathcal{A}$ is the set of worlds which are reachable by the sequential composition of $\sim_i$ and $\approx$ ($\approx \circ \sim_i$). Note that $\approx \circ \sim_i$ is *NOT* an equivalence relation, although both $\sim_i$ and $\approx$ are.

Semantics

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Definition 4 (Semantics)

$$
\begin{array}{lcl}
\mathcal{M}, s \vDash p & \iff & p \in V(s) \\
\mathcal{M}, s \vDash \neg\varphi & \iff & \mathcal{M}, s \nvDash \varphi \\
\mathcal{M}, s \vDash \varphi \wedge \psi & \iff & \mathcal{M}, s \vDash \varphi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \vDash \mathcal{K}_i\varphi & \iff & \mathcal{M}, t \vDash \varphi \text{ for all } t \in [s]_i \\
\mathcal{M}, s \vDash \mathcal{A}\varphi & \iff & \mathcal{M}, t \vDash \varphi \text{ for all } t \in [s]_\mathcal{A}
\end{array}
$$

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\end{array}$$

Given the semantics, it is not hard to see that:

$$\mathcal{M}, s \vDash \mathcal{G}_i\varphi \iff \mathcal{M}, t \vDash \varphi \text{ for all } t \in [s]^i_{\mathcal{A}}$$

System SELA

System SELA

Definition 5 (System SELA)

Axioms			
TAUT	all axioms of propositional logic		
DISTK	$\mathcal{K}_i\varphi \wedge \mathcal{K}_i(\varphi \to \psi) \to \mathcal{K}_i\psi$	DISTA	$\mathcal{A}\varphi \wedge \mathcal{A}(\varphi \to \psi) \to \mathcal{A}\psi$
T	$\mathcal{K}_i\varphi \to \varphi$	TA	$\mathcal{A}\varphi \to \varphi$
4	$\mathcal{K}_i\varphi \to \mathcal{K}_i\mathcal{K}_i\varphi$	4A	$\mathcal{A}\varphi \to \mathcal{A}\mathcal{A}\varphi$
5	$\neg\mathcal{K}_i\varphi \to \mathcal{K}_i\neg\mathcal{K}_i\varphi$	5A	$\neg\mathcal{A}\varphi \to \mathcal{A}\neg\mathcal{A}\varphi$
Rules			
MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$		
NECK	$\dfrac{\varphi}{\mathcal{K}_i\varphi}$	NECA	$\dfrac{\varphi}{\mathcal{A}\varphi}$

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DISTK	$\mathcal{K}_i\varphi \wedge \mathcal{K}_i(\varphi \to \psi) \to \mathcal{K}_i\psi$	DISTA	$\mathcal{A}\varphi \wedge \mathcal{A}(\varphi \to \psi) \to \mathcal{A}\psi$
T	$\mathcal{K}_i\varphi \to \varphi$	TA	$\mathcal{A}\varphi \to \varphi$
4	$\mathcal{K}_i\varphi \to \mathcal{K}_i\mathcal{K}_i\varphi$	4A	$\mathcal{A}\varphi \to \mathcal{A}\mathcal{A}\varphi$
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Rules			
MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$		
NECK	$\dfrac{\varphi}{\mathcal{K}_i\varphi}$	NECA	$\dfrac{\varphi}{\mathcal{A}\varphi}$

Proposition 6

The following formula is provable in SELA:

$$\mathcal{K}_i\varphi \to \mathcal{G}_i\widehat{\mathcal{G}}_j\varphi$$

Language **ELG**

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Definition 7 (Language ELG)

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{G}_i\varphi$$

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Language **ELG**

Given a countable nonempty set $\mathbf{P}$ of basic proposition letters and a countable non-empty set $\mathbf{I}$ of agents, the formulae of the epistemic language with $\mathcal{G}_i$ operator is constructed as follows:

Definition 7 (Language ELG)

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{G}_i\varphi$$

where $i \in \mathbf{I}$, $p \in \mathbf{P}$.

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System SKNKA

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Definition 8 (System SKNKA)

Axioms			
TAUT	all axioms of propositional logic		
DISTK	$\mathcal{K}\varphi \wedge \mathcal{K}(\varphi \to \psi) \to \mathcal{K}\psi$	DISTG	$\mathcal{G}\varphi \wedge \mathcal{G}(\varphi \to \psi) \to \mathcal{G}\psi$
T	$\mathcal{K}\varphi \to \varphi$	GK	$\mathcal{G}\varphi \to \mathcal{K}\varphi$
4	$\mathcal{K}\varphi \to \mathcal{K}\mathcal{K}\varphi$	4G	$\mathcal{G}\varphi \to \mathcal{K}\mathcal{G}\varphi$
5	$\neg\mathcal{K}\varphi \to \mathcal{K}\neg\mathcal{K}\varphi$	5G	$\neg\mathcal{G}\varphi \to \mathcal{K}\neg\mathcal{G}\varphi$
		BG	$\mathcal{K}\varphi \to \mathcal{G}\widehat{\mathcal{G}}\varphi$
Rules			
MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$		
NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$	NECG	$\dfrac{\varphi}{\mathcal{G}\varphi}$

Observations

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Proposition 9

The following formula is provable in **SKNKA**:

$$\mathcal{K}(\bigvee_{1\leqslant a\leqslant n}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\mathcal{G}\psi_b) \to \mathcal{G}(\bigvee_{1\leqslant a\leqslant n}\widehat{\mathcal{G}}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\psi_b),$$

$$m, n \in \omega, m > 0 \textit{ or } n > 0$$

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Proposition 9

The following formula is provable in SKNKA:

$$\mathcal{K}\Big(\bigvee_{1\leqslant a\leqslant n}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\mathcal{G}\psi_b\Big) \to \mathcal{G}\Big(\bigvee_{1\leqslant a\leqslant n}\widehat{\mathcal{G}}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\psi_b\Big),$$

$$m, n \in \omega, m > 0 \text{ or } n > 0$$

Example 10

Consider the set of **ELG**-formulae $\Gamma = \{\varphi \to \mathcal{K}\varphi \mid \varphi \in \mathbf{ELG}\} \cup \{\widehat{\mathcal{G}}\mathcal{G}p, \widehat{\mathcal{G}}\neg p\}$. Γ is consistent. However, in order to satisfy Γ, in the model we must need different worlds satisfying exactly the same **ELG**-formulae.

Backward Lemma for the Single-agent Case

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Let s be a possible world and $\Theta(s)$ be an MCS.

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Definition 11 ($K(s)$)

$$K(s) = \{\mu \mid \mu \text{ is an MCS}, \forall \mathcal{K}\varphi \in \Theta(s), \varphi \in \mu\}.$$

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$$G(s) = \{\nu \mid \nu \text{ is an MCS}, \nu \notin K(s), \forall \mathcal{G}\varphi \in \Theta(s), \varphi \in \nu\}.$$

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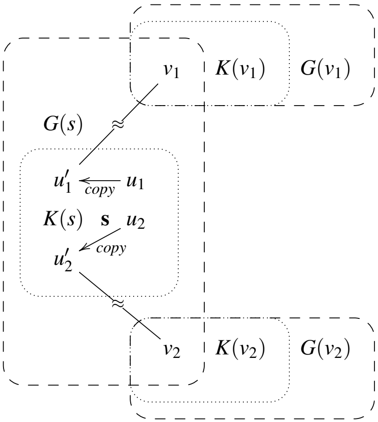
Lemma 13 (Backward Lemma)

$\forall \nu \in G(s), \exists \mu \in K(s)$, *such that* $\forall \mathcal{G}\psi \in \nu$, $\psi \in \mu$.

Strong Completeness for the Single-agent Case

Strong Completeness for the Single-agent Case



System MSKNKA

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Definition 14 (System MSKNKA)

Axioms			
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KGP	$\mathcal{K}_i(\bigvee_{1\leqslant a\leqslant n}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\mathcal{G}_{j_b}\psi_b) \to \mathcal{G}_i(\bigvee_{1\leqslant a\leqslant n}\widehat{\mathcal{G}}_{k_a}\varphi_a \vee \bigvee_{1\leqslant b\leqslant m}\psi_b)$, $m, n \in \omega, m > 0$ or $n > 0, i, j_b, k_a \in \mathbf{I}$		
Rules			
MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$		
NECK	$\dfrac{\varphi}{\mathcal{K}_i\varphi}$	NECG	$\dfrac{\varphi}{\mathcal{G}_i\varphi}$

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Let s be a possible world, $\Theta(s)$ be an MCS, and $i \in \mathbf{I}$ be a fixed agent.

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Definition 15 ($K(s)$)

$$K(s) = \{\mu \mid \mu \text{ is an MCS}, \forall \mathcal{K}_i\varphi \in \Theta(s), \varphi \in \mu\}.$$

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Lemma 17 (Backward Lemma)

$\forall \nu \in G(s)$, $\exists \mu \in K(s)$, such that $\forall k \in \boldsymbol{I}$, $\forall \mathcal{G}_k\varphi \in \nu$, $\varphi \in \mu$, and that $\forall j \in \boldsymbol{I}$, $\forall \mathcal{G}_j\psi \in \mu$, $\psi \in \nu$.

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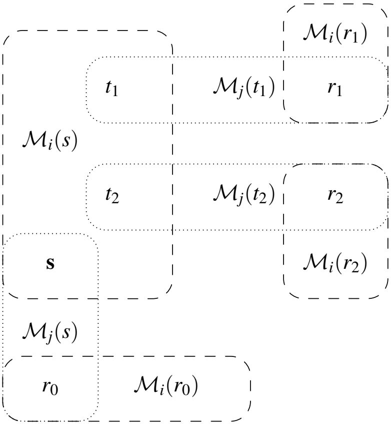

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Other Unpublished Results

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- When $\approx$ relation is not S5.

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- Belief (KD45) frames.

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- Decidability.

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Future Work

Future Work

- Axiomatization for confluent frames with multiple agents.

Future Work

- Axiomatization for confluent frames with multiple agents.
- Better approach to public announcement.

Future Work

- Axiomatization for confluent frames with multiple agents.
- Better approach to public announcement.
- When $\sim_i$ relation is not S5.

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Paper Link

You can find my paper at https://arxiv.org/abs/1905.10976.

Important Notation

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	Section 2	Section 3
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Model	$\sim_i$	$\sim_{\mathcal{K}}$
	$\approx$	$\sim_{\mathcal{G}}$
	V	U

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Also note that unlike in Section 2, $\mathcal{P}$ is defined as specific meaning in Section 3 and thus does not represent power set. (Actually this was a fault of mine. But we do indeed run out of letters.)

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Practically, a dependent variable y is usually influenced by thousands of independent factors $x_1, x_2, \ldots$ in a complicated way, such that it is impossible to obtain a function to precisely determine the value of y.

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relation with independence logic (Väänänen 2007; Grädel and Väänänen 2013; Galliani and Väänänen 2014; Galliani 2017; Galliani 2012; Hodges 1997), with Halpern's (Halpern 2016) and Jingzhi Fang's work

Contents

Language

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For a fixed countable set of propositions **P**, and a fixed countable set of variables A, the language **EDL** of dependence epistemic logic is defined recursively as:

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Definition 18 (Language EDL)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}\varphi \mid \mathcal{G}\varphi \mid \mathcal{D}_g(X, Y) \mid \mathcal{D}_l(X, Y)$$

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Definition 18 (Language EDL)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}\varphi \mid \mathcal{G}\varphi \mid \mathcal{D}_g(X, Y) \mid \mathcal{D}_l(X, Y)$$

where $p \in \mathbf{P}$, and X as well as Y are finite subsets of A. $\mathcal{D}_g(X, Y)$ reads as Y depends on X globally, while $\mathcal{D}_l(X, Y)$ reads as Y depends on X locally. When some property applies to both $\mathcal{D}_g$ and $\mathcal{D}_l$, we will simply omit the subscript and write as $\mathcal{D}$ for convenience, and the omitting is also similar for other notations derived from $\mathcal{D}$. If $X = \{x\}$, we will also denote $\mathcal{D}(\{x\}, Y)$ as $\mathcal{D}(x, Y)$, and likely for Y if $Y = \{y\}$.

Model

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A dependence epistemic model $\mathcal{M}$ is $\langle S, U, A^+, V, \sim_{\mathcal{K}}, \sim_{\mathcal{G}} \rangle$:

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Definition 19 (Model)

- S is a set of possible worlds.
- U: $S \times \mathbf{P} \to \{0, 1\}$.
- $A^+ \supseteq A$ is a countable set of variables.
- V: $S \times A^+ \to \mathbb{N}$.
- $\sim_{\mathcal{K}}$ is an equivalence relation over S.
- $\sim_{\mathcal{G}}$ is an equivalence relation over S.

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Definition 19 (Model)

- S is a set of possible worlds.
- U: $S \times \mathbf{P} \to \{0, 1\}$.
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- $\sim_{\mathcal{G}}$ is an equivalence relation over S.

For any proposition $p \in \mathbf{P}$, it may have its corresponding variable $\overline{p} \in A^+$. If so, we then stipulate that $\forall s \in S$, $V(s, \overline{p}) = U(s, p)$. Examples later on.

Semantics

Semantics

$\forall s, t \in S, \forall$ subset $X \subseteq A^+, X_s = X_t$ iff $\forall x \in X, V(s,x) = V(t,x)$.

Semantics

$$\forall s, t \in S, \forall \text{ subset } X \subseteq A^+, X_s = X_t \text{ iff } \forall x \in X, V(s,x) = V(t,x).$$

Definition 20 (Semantics)

$$
\begin{array}{lcl}
\mathcal{M}, s \vDash \top & \iff & \text{always} \\
\mathcal{M}, s \vDash p & \iff & U(s,p) = 1 \\
\mathcal{M}, s \vDash \neg\varphi & \iff & \mathcal{M}, s \nvDash \varphi \\
\mathcal{M}, s \vDash (\varphi \wedge \psi) & \iff & \mathcal{M}, s \vDash \varphi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \vDash \mathcal{K}\varphi & \iff & \forall t \in S, t \sim_{\mathcal{K}} s, \mathcal{M}, t \vDash \varphi \\
\mathcal{M}, s \vDash \mathcal{G}\varphi & \iff & \forall t \in S, t \sim_{\mathcal{G}} s, \mathcal{M}, t \vDash \varphi \\
\mathcal{M}, s \vDash \mathcal{D}_g(X,Y) & \iff & \exists u, v \in S, u \sim_{\mathcal{G}} v \sim_{\mathcal{G}} s, \\
 & & (A^+ \backslash (X \cup Y))_u = (A^+ \backslash (X \cup Y))_v, \\
 & & X_u \neq X_v, Y_u \neq Y_v \\
\mathcal{M}, s \vDash \mathcal{D}_l(X,Y) & \iff & \exists t \in S, t \sim_{\mathcal{G}} s, \\
 & & (A^+ \backslash (X \cup Y))_t = (A^+ \backslash (X \cup Y))_s, \\
 & & X_t \neq X_s, Y_t \neq Y_s
\end{array}
$$

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$$\mathcal{D}_g(X, Y) \leftrightarrow \neg\mathcal{G}\neg\mathcal{D}_l(X, Y)$$

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$$\mathcal{D}_g(X, Y) \leftrightarrow \neg\mathcal{G}\neg\mathcal{D}_l(X, Y)$$

Definition 21 (Language EDG)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}\varphi \mid \mathcal{G}\varphi \mid \mathcal{D}_g(X, Y)$$

Contents

An Open Door

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p denotes the door is open now, q the agent possesses the key, and r the agent is able to enter the room. Suppose perfect knowledge.

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$$\begin{array}{ccccccc} s : p, q, r & & p, \neg q, r & & \neg p, q, r & & \neg p, \neg q, \neg r \\ \overline{p} = 1 & & \overline{p} = 1 & & \overline{p} = 0 & & \overline{p} = 0 \\ \overline{q} = 1 & \text{—}\mathcal{G}\text{—} & \overline{q} = 0 & \text{—}\mathcal{G}\text{—} & \overline{q} = 1 & \text{—}\mathcal{G}\text{—} & \overline{q} = 0 \\ \overline{r} = 1 & & \overline{r} = 1 & & \overline{r} = 1 & & \overline{r} = 0 \end{array}$$

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$$\begin{array}{ccccccc} s: p, q, r & & p, \neg q, r & & \neg p, q, r & & \neg p, \neg q, \neg r \\ \bar{p}=1 & & \bar{p}=1 & & \bar{p}=0 & & \bar{p}=0 \\ \bar{q}=1 & \text{—}\mathcal{G}\text{—} & \bar{q}=0 & \text{—}\mathcal{G}\text{—} & \bar{q}=1 & \text{—}\mathcal{G}\text{—} & \bar{q}=0 \\ \bar{r}=1 & & \bar{r}=1 & & \bar{r}=1 & & \bar{r}=0 \end{array}$$

$\mathcal{M}, s \vDash \mathcal{K}\mathcal{D}_g(\bar{p}, \bar{r})$ and $\mathcal{M}, s \vDash \mathcal{K}\neg\mathcal{D}_l(\bar{p}, \bar{r})$. The former says that the agent knows whether he can enter the room is somewhat related to whether the door is open – if he did not possess the key. The latter says that under the present situation, since he does possess the key, he knows that if this precondition is kept unchanged, then he could still enter the room even if the door was closed, providing us with a fancy way to express counterfactual assumptions.

Non-perfect Experiment

Non-perfect Experiment

There are two independent variables x and y which may influence the dependent variable z. However, we cannot control or even measure y, only knowing that it will be either 1 or 2 in every experiment. We have done this experiment twice. When $x = 1$, $z = 1$. When $x = 2$, $z = 2$.

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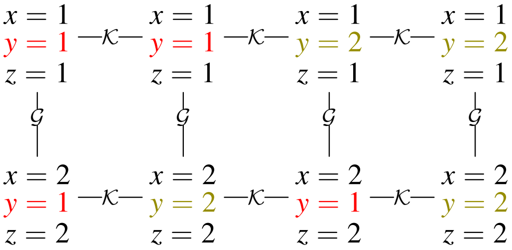

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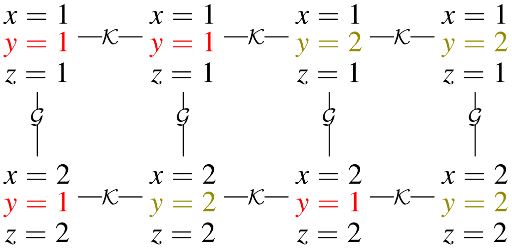

We cannot conclude that z depends on x, as the change of z may be brought about by the change of y. $\forall s \in M, \mathcal{M}, s \nvDash \mathcal{K}\mathcal{D}_g(x, z)$.

Non-perfect Experiment (Cont.)

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If we have further done the third experiment, and when $x = 3, z = 3$. Now can we conclude that z depends on x?

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$$\begin{array}{c}x=1\\y=1\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=1\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=1\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=1\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=2\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=2\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=2\\z\underset{\mathcal{G}}{=}1\end{array}\mathcal{K}\begin{array}{c}x=1\\y=2\\z\underset{\mathcal{G}}{=}1\end{array}$$

$$\begin{array}{c}x=2\\y=1\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=1\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=2\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=2\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=1\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=1\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=2\\z\underset{\mathcal{G}}{=}2\end{array}\mathcal{K}\begin{array}{c}x=2\\y=2\\z\underset{\mathcal{G}}{=}2\end{array}$$

$$\begin{array}{c}x=3\\y=1\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=2\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=1\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=2\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=1\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=2\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=1\\z=3\end{array}\mathcal{K}\begin{array}{c}x=3\\y=2\\z=3\end{array}$$

Non-perfect Experiment (Cont.)

If we have further done the third experiment, and when $x = 3$, $z = 3$. Now can we conclude that z depends on x?

$$\begin{array}{ccccccccccccccc}
x=1 & & x=1 & & x=1 & & x=1 & & x=1 & & x=1 & & x=1 & & x=1 \\
y=1 & \mathcal{K} & y=1 & \mathcal{K} & y=1 & \mathcal{K} & y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=2 & \mathcal{K} & y=2 & \mathcal{K} & y=2 \\
z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 & & z \underset{\mathcal{G}}{=} 1 \\
x=2 & & x=2 & & x=2 & & x=2 & & x=2 & & x=2 & & x=2 & & x=2 \\
y=1 & \mathcal{K} & y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=2 & \mathcal{K} & y=1 & \mathcal{K} & y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=2 \\
z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 & & z \underset{\mathcal{G}}{=} 2 \\
x=3 & & x=3 & & x=3 & & x=3 & & x=3 & & x=3 & & x=3 & & x=3 \\
y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=1 & \mathcal{K} & y=2 & \mathcal{K} & y=1 & \mathcal{K} & y=2 \\
z=3 & & z=3 & & z=3 & & z=3 & & z=3 & & z=3 & & z=3 & & z=3
\end{array}$$

There must be at least two experiments in which y is the same, so we can only explain the difference between z as caused by the difference between x. $\forall s \in M, \mathcal{M}, s \vDash \mathcal{KD}_g(x, z)$.

Judging a Case

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It might seem that $\mathcal{D}(\{a, b\}, c)$ is very similar to $\mathcal{D}(a, c) \vee \mathcal{D}(b, c)$. Actually they are *NOT*.

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It might seem that $\mathcal{D}(\{a,b\},c)$ is very similar to $\mathcal{D}(a,c) \vee \mathcal{D}(b,c)$. Actually they are *NOT*.

Charles got killed in a tragedy (c), which was related to Alan having done something (a) and/or Bob having done something (b). We consider two kinds of scenarios.

Judging a Case (Cont.)

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Firstly, suppose that either a or b could happen to cause c, and only one of them could have happened to be c's indeed cause. However, on the current world s we are yet not sure whether a or b happened.

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$$
\begin{array}{ccccc}
s : a, \neg b, c & & \neg a, b, c & & \neg a, \neg b, \neg c \\
\overline{a} = 1 & \overset{\mathcal{G},\mathcal{K}}{\text{———}} & \overline{a} = 0 & \overset{\mathcal{G}}{\text{———}} & \overline{a} = 0 \\
\overline{b} = 0 & & \overline{b} = 1 & & \overline{b} = 0 \\
\overline{c} = 1 & & \overline{c} = 1 & & \overline{c} = 0
\end{array}
$$

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Firstly, suppose that either a or b could happen to cause c, and only one of them could have happened to be c's indeed cause. However, on the current world s we are yet not sure whether a or b happened.

$$\begin{array}{ccccc} s: a, \neg b, c & & \neg a, b, c & & \neg a, \neg b, \neg c \\ \overline{a} = 1 & \overset{\mathcal{G},\mathcal{K}}{\text{———}} & \overline{a} = 0 & \overset{\mathcal{G}}{\text{———}} & \overline{a} = 0 \\ \overline{b} = 0 & & \overline{b} = 1 & & \overline{b} = 0 \\ \overline{c} = 1 & & \overline{c} = 1 & & \overline{c} = 0 \end{array}$$

$\mathcal{M}, s \vDash \mathcal{K}\mathcal{D}_l(\{\overline{a}, \overline{b}\}, \overline{c}) \wedge \mathcal{K}(\mathcal{D}_l(\overline{a}, \overline{c}) \vee \mathcal{D}_l(\overline{b}, \overline{c}))$. It is within our knowledge that not only the whole group event $\{a, b\}$ is related to c, but also either a or b itself is alone related to c, namely, their influences on c can be separated in concept. Hence, unless we obtain further evidence to determine whether Alan or Bob was the real criminal, by presumption of innocence neither of them can be sentenced guilty.

Judging a Case (Cont.)

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Secondly, suppose b was a direct consequence of a, e.g., let a denote that Alan compelled Bob to kill Charles, by threaten or mind control. In other words, we restrict ourselves to worlds on which $a \to b$ holds.

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\overline{a} = 1 & \overset{\mathcal{G}}{\text{———}} & \overline{a} = 0 & \overset{\mathcal{G}}{\text{———}} & \overline{a} = 0 \\
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Physically, b should be the only direct cause of c, as $\mathcal{G}(b \leftrightarrow c)$ holding everywhere. But to our little surprise $\mathcal{M}, s \nvDash \mathcal{KD}_l(\overline{b}, \overline{c})$. In fact, $\mathcal{M}, s \vDash \mathcal{KD}_l(\{\overline{a}, \overline{b}\}, \overline{c}) \wedge \mathcal{K}(\neg\mathcal{D}_l(\overline{a}, \overline{c}) \wedge \neg\mathcal{D}_l(\overline{b}, \overline{c}))$, a contrast against the former scenario that $\mathcal{M}, s \vDash \mathcal{KD}_l(\{\overline{a}, \overline{b}\}, \overline{c}) \wedge \mathcal{K}(\mathcal{D}_l(\overline{a}, \overline{c}) \vee \mathcal{D}_l(\overline{b}, \overline{c}))$.

Judging a Case (Cont.)

Judging a Case (Cont.)

We not only know that c locally depends on $\{a, b\}$ as a whole, but also know that this dependency should be viewed as an entirety instead of conceptually separable, and therefore, both Alan and Bob should be responsible for Charles' death. Further considering that $\mathcal{M}, s \vDash \mathcal{KG}(a \to b)$, a legal and rational sentence ought to be that Alan is the principal criminal while Bob is the coerced criminal, which precisely captures the meanings of all the formulae mentioned above.

Contents

Evidence

Evidence

Definition 22 ($\Delta(u, v)$)

$$\Delta(u,v) = \begin{cases} \{x \mid x \in A, V(u,x) \neq V(v,x)\}, & \text{if } (A^+ \backslash A)_u = (A^+ \backslash A)_v \\ \emptyset, & \text{otherwise} \end{cases}$$

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Definition 23 (Evidence)

For any three sets X, Y and Z, X is called an evidence of $\{Y, Z\}$, iff $X \cap Y \neq \emptyset$, $X \cap Z \neq \emptyset$, and $X \subseteq Y \cup Z$.

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Definition 23 (Evidence)

For any three sets X, Y and Z, X is called an evidence of $\{Y, Z\}$, iff $X \cap Y \neq \emptyset$, $X \cap Z \neq \emptyset$, and $X \subseteq Y \cup Z$.

Lemma 24 (Evidence Lemma I)

$$\begin{aligned} \mathcal{M}, s \vDash \mathcal{D}_g(X,Y) \quad &\Longleftrightarrow \quad \exists u, v \in S, u \sim_{\mathcal{G}} v \sim_{\mathcal{G}} s, \\ & \qquad \Delta(u,v) \textit{ is an evidence of } \{X, Y\} \\ \mathcal{M}, s \vDash \mathcal{D}_l(X,Y) \quad &\Longleftrightarrow \quad \exists t \in S, t \sim_{\mathcal{G}} s, \\ & \qquad \Delta(t,s) \textit{ is an evidence of } \{X, Y\} \end{aligned}$$

$\mathcal{P}(s)$

$\mathcal{P}(s)$

Definition 25 ($\mathcal{P}(s)$)

$$\mathcal{P}_g(s) = \{\text{nonempty finite set } \Delta(u,v) \mid u,v \in S, u \sim_{\mathcal{G}} v \sim_{\mathcal{G}} s\}$$
$$\mathcal{P}_l(s) = \{\text{nonempty finite set } \Delta(t,s) \mid t \in S, t \sim_{\mathcal{G}} s\}$$

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$$\mathcal{P}_l(s) = \{\text{nonempty finite set } \Delta(t,s) \mid t \in S, t \sim_{\mathcal{G}} s\}$$

Lemma 26 (Evidence Lemma II)

$$\mathcal{M}, s \vDash \mathcal{D}_g(X,Y) \iff \exists W \in \mathcal{P}_g(s), W \textit{ is an evidence of } \{X,Y\}$$
$$\mathcal{M}, s \vDash \mathcal{D}_l(X,Y) \iff \exists W \in \mathcal{P}_l(s), W \textit{ is an evidence of } \{X,Y\}$$

Generative

Generative

Definition 27 (Generative)

$\forall s \in S$, any nonempty finite set $X \subseteq A$ is called generative from $\mathcal{P}(s)$, iff for any two finite sets $Y, Z \subseteq A$, such that X is an evidence of $\{Y, Z\}$, there exists $W \in \mathcal{P}(s)$, such that W is an evidence of $\{Y, Z\}$.

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Theorem 28 (Equivalence Theorem I)

For any two pointed models $\mathcal{M}, s$ and $\mathcal{M}', t$, they satisfy exactly the same $\mathcal{D}(X, Y)$ formulae iff:

- *Zig: $\forall X \in \mathcal{P}(s), X$ is generative from $\mathcal{P}(t)$.*
- *Zag: $\forall X \in \mathcal{P}(t), X$ is generative from $\mathcal{P}(s)$.*

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$\mathcal{R}(s)$

$\mathcal{R}(s)$

Definition 29 ($\mathcal{R}(s)$)

$$\mathcal{R}(s) = \{X \mid X \text{ is generative from } \mathcal{P}(s)\}$$

$\mathcal{R}(s)$

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$$\mathcal{R}(s) = \{X \mid X \text{ is generative from } \mathcal{P}(s)\}$$

Theorem 30 (Equivalence Theorem II)

For any two pointed models $\mathcal{M}, s$ and $\mathcal{M}', t$, they satisfy exactly the same $\mathcal{D}(X, Y)$ formulae iff $\mathcal{R}(s) = \mathcal{R}(t)$.

Generative Lemma

Generative Lemma

Lemma 31 (Generative Lemma)

$\forall s \in S$, for any nonempty finite set $X \subseteq A$, X is generative from $\mathcal{P}(s)$ iff:

- *if $|X| = 1$, then $X \in \mathcal{P}(s)$.*
- *if $|X| \geqslant 2$, then $\forall Y \subset X$, $Y \neq \emptyset$, there exists $W \in \mathcal{P}(s)$, such that W is an evidence of $\{Y, X \backslash Y\}$.*

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Generative Theorem

Generative Theorem

Theorem 32 (Generative Theorem)

$\forall s \in S$, for any nonempty finite set $X \subseteq A$, we define $\Sigma(s,X) = \{Y \mid Y \in \mathcal{P}(s), Y \subseteq X\}$, then:

$$
\begin{aligned}
& X \textit{ is generative from } \mathcal{P}(s) \\
\iff\ & \bigcup \Sigma(s,X) = X, \forall Z \subset X \textit{ such that } Z \neq \emptyset, \\
& \exists Y \in \Sigma(s,X) \textit{ such that } Y \cap Z \neq \emptyset \wedge Y \cap (X \backslash Z) \neq \emptyset \\
\iff\ & \bigcup \Sigma(s,X) = X, \forall \Gamma \subset \Sigma(s,X) \textit{ such that } \Gamma \neq \emptyset, \\
& (\bigcup \Gamma) \cap (\bigcup (\Sigma(s,X) \backslash \Gamma)) \neq \emptyset \\
\iff\ & \bigcup \Sigma(s,X) = X, \\
& \forall Y_1, Y_2 \in \Sigma(s,X),\ \textit{define } \mathcal{R} Y_1 Y_2 \textit{ iff } Y_1 \cap Y_2 \neq \emptyset, \\
& \textit{then } Y_1 \textit{ is connected to } Y_2 \textit{ by a chain of } \mathcal{R} \textit{ relations.}
\end{aligned}
$$

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Generative Theorem (Cont.)

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The last equivalent condition in Theorem 32 is to say, we can construct an undirected graph over $\mathcal{P}(s)$ by its elements' intersection relation, and all the generative sets are exactly union of some connected finite subgraph. This provides us with a clear picture and an intuitive understanding about where every generative set comes from and what $\mathcal{R}(s)$ looks like. Hence given $\mathcal{P}(s)$, there is an explicit algorithm to calculate all the generative Xs so as to obtain $\mathcal{R}(s)$.

Bisimulation

Bisimulation

Definition 33 (Bisimulation)

A nonempty binary relation $Z \subseteq S \times S'$ is $Z : \mathcal{M} \underline{\leftrightarrow} \mathcal{M}'$ iff:

- If sZs', then $\forall p \in \mathbf{P}$, $U(s,p) = U(s',p)$.
- If sZs', then $\mathcal{R}_g(s) = \mathcal{R}_g(s')$.
- If sZs', then $\mathcal{R}_l(s) = \mathcal{R}_l(s')$.
- Zig for $\mathcal{K}$: if sZs' and $s \sim_{\mathcal{K}} t$, then $\exists t' \in S'$ such that tZt' and $s' \sim_{\mathcal{K}} t'$.
- Zig for $\mathcal{G}$: if sZs' and $s \sim_{\mathcal{G}} t$, then $\exists t' \in S'$ such that tZt' and $s' \sim_{\mathcal{G}} t'$.
- Zag for $\mathcal{K}$: if sZs' and $s' \sim_{\mathcal{K}} t'$, then $\exists t \in S$ such that tZt' and $s \sim_{\mathcal{K}} t$.
- Zag for $\mathcal{G}$: if sZs' and $s' \sim_{\mathcal{G}} t'$, then $\exists t \in S$ such that tZt' and $s \sim_{\mathcal{G}} t$.

Bisimulation (Cont.)

Bisimulation (Cont.)

Theorem 34 (Hennessy-Milner Theorem)

For any two m-saturated models $\mathcal{M}$ *and* $\mathcal{M}'$, $\forall s \in S$, $\forall s' \in S'$, $\mathcal{M}, s \underline{\leftrightarrow} \mathcal{M}', s'$ *iff* $\mathcal{M}, s \leftrightsquigarrow \mathcal{M}', s'$.

Contents

Naïve Axioms

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Although these naïve axioms indeed look very similar to those in independence logic (Galliani and Hella 2013), they alone are away from being complete. The good news is that, we can instead find some conciser axioms, which entirely grasp the full properties of modality $\mathcal{D}$ itself, and from which all the above sound axioms can surely be deduced.

$\mathcal{Q}(X)$

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Definition 35 ($\mathcal{Q}(X)$)

$$\mathcal{Q}(X) ::= \begin{cases} D(X,X), & |X| = 1 \\ \bigwedge_{Y \subset X, Y \neq \emptyset} D(Y, X \backslash Y), & |X| \geqslant 2 \end{cases}$$

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Recall Lemma 31, readers should be aware that this $\mathcal{Q}(X)$ precisely depicts the minimum necessary $\mathcal{D}(Y,Z)$ formulae, such that X is an evidence of $\{Y,Z\}$.

Axiomatization

Axiomatization

Theorem 36 (Strongly Complete Axiomatization for **EDG**)

TAUT	*all instances of tautologies*
MP	*from* φ *and* $\varphi \to \psi$ *infer* ψ
NEC for $\mathcal{K}$	*from* φ *infer* $\mathcal{K}\varphi$
DIST for $\mathcal{K}$	$\mathcal{K}(\varphi \to \psi) \to (\mathcal{K}\varphi \to \mathcal{K}\psi)$
T for $\mathcal{K}$	$\mathcal{K}\varphi \to \varphi$
4 for $\mathcal{K}$	$\mathcal{K}\varphi \to \mathcal{K}\mathcal{K}\varphi$
5 for $\mathcal{K}$	$\neg\mathcal{K}\varphi \to \mathcal{K}\neg\mathcal{K}\varphi$
NEC for $\mathcal{G}$	*from* φ *infer* $\mathcal{G}\varphi$
DIST for $\mathcal{G}$	$\mathcal{G}(\varphi \to \psi) \to (\mathcal{G}\varphi \to \mathcal{G}\psi)$
T for $\mathcal{G}$	$\mathcal{G}\varphi \to \varphi$
4 for $\mathcal{G}$	$\mathcal{G}\varphi \to \mathcal{G}\mathcal{G}\varphi$
5 for $\mathcal{G}$	$\neg\mathcal{G}\varphi \to \mathcal{G}\neg\mathcal{G}\varphi$
Q for $\mathcal{D}$	$\mathcal{D}(X, Y) \leftrightarrow \bigvee_{X' \subseteq X, Y' \subseteq Y, X', Y' \neq \emptyset} \mathcal{Q}(X' \cup Y')$, *given* $X, Y \neq \emptyset$
E for $\mathcal{D}$	$\mathcal{D}(\emptyset, X) \leftrightarrow \mathcal{D}(X, \emptyset) \leftrightarrow \bot$
4 for $\mathcal{D}_g$	$\mathcal{D}_g(X, Y) \to \mathcal{G}\mathcal{D}_g(X, Y)$

Contents

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- Decidability.
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- Axiomatization when possible values are finite, e.g., 2-value dependency.
- More modalities, e.g., $\mathcal{K}v$, $\mathcal{K}hw$, into this framework.

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Thanks for listening!

Bibliographies I

Hodges, Wilfrid (1997). "Compositional Semantics for a Language of Imperfect Information". In: *Logic Journal of the IGPL* 5.4, pp. 539–563.

Väänänen, Jouko (2007). *Dependence Logic: a New Approach to Independency Friendly Logic*. Cambridge: Cambridge University Press.

Galliani, Pietro (2012). "Inclusion and Exclusion Dependencies in Team Semantics – On Some Logics of Imperfect Information". In: *Annals of Pure and Applied Logic* 163.1, pp. 68–84.

Galliani, Pietro and Lauri Hella (2013). "Inclusion Logic and Fixed Point Logic". In: *Computer Science Logic 2013 (CSL 2013)*. Vol. 23. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, pp. 281–295.

Bibliographies II

- Grädel, Erich and Jouko Väänänen (2013). "Dependence and Independence". In: *Studia Logica* 101.2, pp. 399–410.
- Wang, Yanjing and Jie Fan (2013). "Knowing That, Knowing What, and Public Communication: Public Announcement Logic with Kv Operators". In: *Proceedings of International Joint Conference on Artificial Intelligence 2013*. AAAI Press, pp. 1147–1154.
- Galliani, Pietro and Jouko Väänänen (2014). "On Dependence Logic". In: *Johan van Benthem on Logic and Information Dynamics: Outstanding Contributions to Logic, 5*. Switzerland: Springer International Publishing, pp. 101–119.
- Wang, Yanjing and Jie Fan (2014). "Conditionally Knowing What". In: *Proceedings of Advances in Modal Logic 2014*. Vol. 10. College Publications, pp. 569–587.

Bibliographies III

- Ding, Yifeng (2016a). *Axiomatization and Complexity of Modal Logic with Knowing-what Operator on Model Class K*. http://arxiv.org/abs/1609.07684.
- – (2016b). "Epistemic Logic with Functional Dependency Operator". In: *Studies in Logic* 9.4, pp. 55–84.
- Gu, Tao and Yanjing Wang (2016). "“Knowing Value” Logic as a Normal Modal Logic". In: *Proceedings of Advances in Modal Logic 2016*. Vol. 11. College Publications, pp. 362–381.
- Halpern, Joseph Y. (2016). *Actual Causality*. Cambridge, Massachusetts: The MIT Press.
- Eijck, Jan van, Malvin Gattinger, and Yanjing Wang (2017). "Knowing Values and Public Inspection". In: *Proceedings of Indian Conference on Logic and Its Applications 2017*. Springer, pp. 77–90.

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Bibliographies IV

Galliani, Pietro (2017). *Dependence Logic*. https://plato.stanford.edu/entries/logic-dependence/.

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