Epistemic Modelling and Protocol Dynamics

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A classic example

Muddy Children - the setting

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- After exactly k requests to step forward, the k dirty children suddenly do so (assuming they are honest and perfect reasoners).

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The language of *Public Announcement Logic* (PAL [Pla89, GG97]) is defined as follows:

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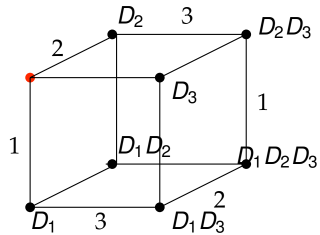
It is interpreted on S5 models $\mathcal{M} = (S, \{\sim_i\}_{i\in\mathbf{I}}, V)$:

$$\begin{array}{|rcl|} \mathcal{M}, s \vDash K_i\phi & \Leftrightarrow & \text{for all } t \text{, if } s \sim_i t \text{ then } \mathcal{M}, t \vDash \phi \\ \mathcal{M}, s \vDash [!\psi]\phi & \Leftrightarrow & \text{if } \mathcal{M}, s \vDash \psi \text{ then } \mathcal{M}|_\psi, s \vDash \phi \end{array}$$

where $\mathcal{M}|_\psi = (S', \{\sim'_i\}_{i\in\mathbf{I}}, V')$ with $S' = \{s \in S \mid \mathcal{M}, s \vDash \psi\}$, $\sim'_i = \sim_i \cap (S' \times S')$, and $V' = V|_{S'}$

When there are 3 dirty children...



"At least one of you is dirty!"
Announcement: $\psi_0 = D_1 \vee D_2 \vee D_3$

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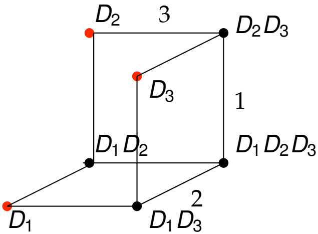

No one steps forward.
Announcement: $\psi_1 = \neg K_1 D_1 \wedge \neg K_2 D_2 \wedge \neg K_3 D_3$

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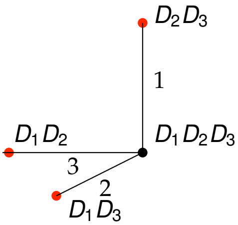

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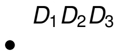

Now all the children know that they are dirty.

$\mathcal{M}, (D_1 D_2 D_3) \vDash [!\psi_0][\psi_1][\psi_1] K_1 D_1 \wedge K_2 D_2 \wedge K_3 D_3$

Is it really perfect?

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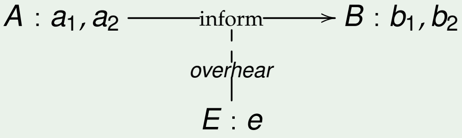
- Where was the father after the first announcement?

Is it really perfect?

Example (Procedural information is important)



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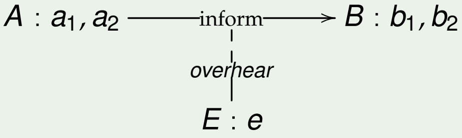

A “promising protocol” for this scenario is that A announces the disjunction of his actual hand (say 01) with all the different combinations of the remaining cards, so he would announce “I have 01 or 23 or 24 or 34.”

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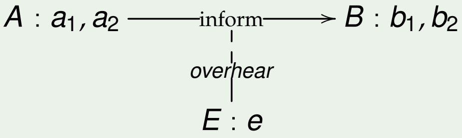

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It can go wrong if we assume...

- The goal of the protocol is commonly known.
- The procedure to generate the announcement is commonly known.

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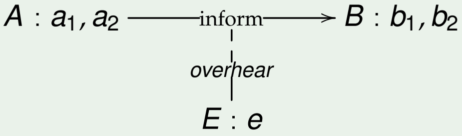

A "promising protocol" for this scenario is that *A* announces the disjunction of his actual hand (say 01) with all the different combinations of the remaining cards, so he would announce "I have 01 or 23 or 24 or 34."

"If you know the protocol and it is assumed to be correct, then it may turn incorrect!"

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- Where was the father?

Is it really perfect?

- Where was the father?
- How to build a suitable model?

Is it really perfect?

- Where was the father?
- How to build a suitable model?
- What if there are a lot of children?

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- Dynamic (epistemic) logics with protocol announcements

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Protocols and their functions

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Protocols: in a very broad sense

Procedural rules that govern our everyday life

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- Let us know what to do (do *a* **then** do *b* or *c*).

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Functions of protocols

- Let us know what to do (do *a* **then** do *b* or *c*).
- Let us know the meaning of actions (**if** *p* **then** do a).

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Francois De La Rochefoucauld

True love is like ghosts, which everybody talks about and few have seen.

Protocols and their functions

Francois De La Rochefoucauld

True love is like ghosts, which everybody talks about and few have seen.

However...

You can actually show this ghost without seeing it or understanding what it is.

Protocol dynamics

This is how we did it in the past

Protocol dynamics

Until something evil came in...

Häagen-Dazs: “Love her, take her to Häagen-Dazs”.

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Until something evil came in...

Häagen-Dazs: “Love her, take her to Häagen-Dazs”.

The announcement of the slogan makes it commonly known that buying an icecream shows your love.

Protocol dynamics

Even “better”: a true “father” can change his mind.

Protocol dynamics

Current situation

Protocol announcement logic

Previous research

Protocols are not specified in the logical languages in the existing work of Dynamic Epistemic Logic and Epistemic Temporal Logic [HY09, vBGHP09, Hos09, HF89, PR03]

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To specify protocols and their dynamics explicitly!

Our logics are based on:

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To specify protocols and their dynamics explicitly!

Our logics are based on:

- Propositional Dynamic Logic (PDL [FL79])
- Public Announcement Logic (PAL [Pla89, GG97])

Protocol announcement logic

The first language $\mathtt{PDL}^!$

The formulas of $\mathtt{PDL}^!$ are built from a set of basic proposition letters **P** and a finite set of atomic action symbols **Σ** as follows:

$$\begin{array}{rcl} \phi & ::= & \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid [\pi]\phi \mid [!\pi]\phi \\ \pi & ::= & \mathbf{1} \mid \mathbf{0} \mid a \mid \pi \cdot \pi \mid \pi + \pi \mid \pi^* \end{array}$$

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The language of regular expressions

$$\mathcal{L}(\mathbf{0}) = \emptyset \qquad \mathcal{L}(\mathbf{1}) = \{\epsilon\} \qquad \mathcal{L}(a) = \{a\}$$

$$\mathcal{L}(\pi \cdot \pi') = \{wv \mid w \in \mathcal{L}(\pi), v \in \mathcal{L}(\pi')\}$$

$$\mathcal{L}(\pi + \pi') = \mathcal{L}(\pi) \cup \mathcal{L}(\pi')$$

$$\mathcal{L}(\pi^*) = \{\epsilon\} \cup \bigcup_{n>0}(\mathcal{L}(\underbrace{\pi \cdots \pi}_{n}))$$

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Example (The language of regular expressions)

$(a + (b \cdot c))^* = \{\epsilon, a, bc, abc, aaa, bcbca \ldots\}$

The semantics

input derivative: $\backslash w$

The language of the $\pi\backslash w$ of a regular expression π is defined as $\mathcal{L}(\pi\backslash w) = \{v \mid wv \in \mathcal{L}(\pi)\}$.

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Example (Deriving the remaining protocol)

$$(a + (b \cdot c))^*\backslash b = (a\backslash b + (b \cdot c)\backslash b) \cdot (a + b \cdot c)^* =$$
$$(\mathbf{0} + (\mathbf{1} \cdot c)) \cdot (a + b \cdot c)^* = c \cdot (a + (b \cdot c))^*$$

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\mathcal{M}, s \vDash_\pi [\pi']\phi & \Leftrightarrow & \text{for all } (w, s') : \text{ if } w \in \mathcal{L}(\pi'), w \propto \pi, s \xrightarrow{w} s' \\
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\mathcal{M}, s \vDash_\pi [!\pi']\phi & \Leftrightarrow & \mathcal{M}, s \vDash \langle \pi' \rangle \top \implies \mathcal{M}, s \vDash_{\pi'} \phi
\end{array}
$$

where the mode $\mathbf{\Sigma}^*$ stands for the *universal protocol* $(a_0 + a_1 + \cdots + a_n)^*$ if $\mathbf{\Sigma} = \{a_0, a_1, \ldots, a_n\}$.

The semantics

$\mathcal{M}, s \models_{\pi} [\pi']\phi$	$\Leftrightarrow$	for all (w, s') : if $w \in \mathcal{L}(\pi'), w \propto \pi, s \xrightarrow{w} s'$ then $\mathcal{M}, s' \models_{\pi \backslash w} \phi$
$\mathcal{M}, s \models_{\pi} [!\pi']\phi$	$\Leftrightarrow$	$\mathcal{M}, s \models \langle \pi' \rangle \top \Longrightarrow \mathcal{M}, s \models_{\pi'} \phi$

Example

The semantics

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Example

Consider the following model $\mathcal{M}$:

$s \vDash [!(a{\cdot}c+b{\cdot}d)][a+b](\neg\langle d\rangle\top \wedge \langle c\rangle\top \wedge [!(c+d)]\langle d\rangle\top)$

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$$\mathcal{M}, s \models_\pi [\pi']\phi \quad \Leftrightarrow \quad \text{for all } (w, s') : \text{ if } w \in \mathcal{L}(\pi'), w \propto \pi, s \xrightarrow{w} s' \text{ then } \mathcal{M}, s' \models_{\pi \backslash w} \phi$$

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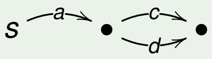

$s \models_{a \cdot c + b \cdot d} [a + b](\neg \langle d \rangle \top \wedge \langle c \rangle \top \wedge [!(c + d)]\langle d \rangle \top)$

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Example

Consider the following model $\mathcal{M}$:

$s \vDash_c (\neg\langle d \rangle \top \wedge \langle c \rangle \top \wedge [!(c + d)]\langle d \rangle \top)$

The semantics

$$
\begin{array}{lcl}
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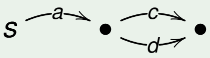

$s \models_{c+d} \langle d \rangle \top$

Expressivity

Theorem

$\mathtt{PDL}^{!}$ is equally expressive as test-free $\mathtt{PDL}$.

Expressivity

$$\mathcal{M}, s \vDash_\pi [\pi']\phi \quad \Leftrightarrow \quad \text{for all } (w, s') : \text{ if } w \in \mathcal{L}(\pi'), w \propto \pi, s \xrightarrow{w} s' \text{ then } \mathcal{M}, s' \vDash_{\pi \backslash w} \phi$$

Proof.

By a translation:

$$\begin{array}{rcl}
t(\phi) & = & t_{\Sigma^*}(\phi) \\
t_\pi(p) & = & p \\
t_\pi(\neg\phi) & = & \neg t_\pi(\phi) \\
t_\pi(\phi_1 \wedge \phi_2) & = & t_\pi(\phi_1) \wedge t_\pi(\phi_2) \\
t_\pi([\pi']\phi) & = & \bigwedge_{i=0}^{k}([\theta_i] t_{\pi \backslash \pi_i}(\phi)) \\
t_\pi([!\pi']\phi) & = & \langle \pi' \rangle \top \rightarrow t_{\pi'}(\phi)
\end{array}$$

The idea behind $t_\pi([\pi']\phi)$: partition $\{w \mid w \propto \pi \text{ and } w \in \mathcal{L}(\pi')\}$ by their consequences. □

Other variations

Introducing $[!\pi(x)]$

$$\mathcal{M}, s \vDash_\pi [!\pi'(x)]\phi \quad \Leftrightarrow \quad (\mathcal{M}, s \vDash \langle \pi'(\pi) \rangle \top \implies \mathcal{M}, s \vDash_{\pi'(\pi)} \phi)$$

We can then concatenate, add, insert and repeat protocols by announcing $x \cdot \pi'$, $x + \pi'$, $\pi' + x$, and x^* respectively.

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We can then concatenate, add, insert and repeat protocols by announcing $x \cdot \pi'$, $x + \pi'$, $\pi' + x$, and x^* respectively.

Refinement operator $[!(a/\pi')]$

$$\mathcal{M}, s \vDash_\pi [!(a/\pi')]\phi \quad \Leftrightarrow \quad \mathcal{M}, s \vDash \langle \pi[a/\pi'] \rangle \top \implies \mathcal{M}, s \vDash_{\pi[a/\pi']} \phi$$

Public event logic

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The language $\mathtt{PDL}^{!?_b}$

$$\begin{array}{lll} \phi & ::= & \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid [\pi']\phi \mid [!\pi]\phi \mid K_i\phi \\ \pi & ::= & ?\phi_b \mid a \mid \pi \cdot \pi \mid \pi + \pi \mid \pi^* \end{array}$$

π is test-free.

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π is test-free. A *uniform guarded string* over finite sets **P** and **Σ** is a sequence $\rho a_1 \rho a_2 \rho \ldots \rho a_n \rho$ for some $\rho \subseteq \mathbf{P}$ [Koz01].

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π is test-free. A *uniform guarded string* over finite sets **P** and **Σ** is a sequence $\rho a_1 \rho a_2 \rho \ldots \rho a_n \rho$ for some $\rho \subseteq \mathbf{P}$ [Koz01].

Example

If $\mathbf{P} = \{p\}$ then $\mathcal{L}_g(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c) = \{\{p\}a\{p\}b\{p\}, \emptyset a \emptyset c \emptyset\}$

Public event logic

The language $\texttt{PDL}^{!?_b}$

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Example

$\mathcal{L}_g((?p \cdot a \cdot b + ?\neg p \cdot a \cdot c + b) \backslash a) = \{\{p\}b\{p\}, \emptyset c \emptyset\} = \mathcal{L}_g(?p \cdot b + ?\neg p \cdot c)$.

The semantics

Now we interpret $\mathtt{PDL}^{!?_b}$ on the $S5$ models $(S, \{\sim_i\}_{i \in \mathbf{I}}, V)$ as follows:

$$
\begin{array}{rcl}
\mathcal{M}, s \vDash \phi & \Leftrightarrow & \mathcal{M}, s \vDash_{\mathbf{\Sigma}^*} \phi \\
\mathcal{M}, s \vDash_\pi p & \Leftrightarrow & p \in V(s) \\
\mathcal{M}, s \vDash_\pi \neg\phi & \Leftrightarrow & \mathcal{M}, s \nvDash_\pi \phi \\
\mathcal{M}, s \vDash_\pi \phi \wedge \phi' & \Leftrightarrow & \mathcal{M}, s \vDash_\pi \phi \text{ and } \mathcal{M}, s \vDash_\pi \phi' \\
\mathcal{M}, s \vDash K_i\phi & \Leftrightarrow & \text{for all } t\text{: } s \sim_i t \implies \mathcal{M}, t \vDash \phi \\
\mathcal{M}, s \vDash_\pi [\pi']\phi & \Leftrightarrow & \text{for all } w \in \mathcal{L}(\pi') : \text{ if } \mathcal{M}, s \vDash \phi^w_\pi \\
 & & \text{ then } \mathcal{M}|_{\phi^w_\pi}, s \vDash_{\pi \backslash w} \phi \\
\mathcal{M}, s \vDash_\pi [!\pi']\phi & \Leftrightarrow & \text{ if } (\exists w : w = \rho v \in \mathcal{L}_g(\pi') \text{ and } V(s) = \rho) \\
 & & \text{ then } \mathcal{M}, s \vDash_{\pi'} \phi
\end{array}
$$

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 & & \text{then } \mathcal{M}|_{\phi^w_\pi}, s \vDash_{\pi\backslash w} \phi \\
\mathcal{M}, s \vDash_\pi [!\pi']\phi & \Leftrightarrow & \text{if } (\exists w : w = \rho v \in \mathcal{L}_g(\pi') \text{ and } V(s) = \rho) \\
 & & \text{then } \mathcal{M}, s \vDash_{\pi'} \phi
\end{array}
$$

where:

$\phi^w_\pi = \bigvee\{\phi_\rho \mid v = \rho a_1 \rho a_2 \rho \cdots \rho a_k \rho, \mathcal{L}_p(v) = w, v \propto_g \pi\}$.

The semantics

Now we interpret $\mathtt{PDL}^{!?b}$ on the $S5$ models $(S, \{\sim_i\}_{i\in\mathbf{I}}, V)$ as follows:

$$
\begin{array}{rcl}
\mathcal{M}, s \vDash \phi & \Leftrightarrow & \mathcal{M}, s \vDash_{\mathbf{\Sigma}^*} \phi \\
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\mathcal{M}, s \vDash_\pi \neg\phi & \Leftrightarrow & \mathcal{M}, s \nvDash_\pi \phi \\
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\mathcal{M}, s \vDash K_i\phi & \Leftrightarrow & \text{for all } t\text{: } s \sim_i t \implies \mathcal{M}, t \vDash \phi \\
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\end{array}
$$

where:
$\phi^w_\pi = \bigvee\{\phi_\rho \mid v = \rho a_1 \rho a_2 \rho \cdots \rho a_k \rho, \mathcal{L}_p(v) = w, v \propto_g \pi\}$.For example, let $\pi = ?p \cdot a \cdot b + ?\neg p \cdot a \cdot c$, $w = a$, then $\phi^w_a = p \vee \neg p$.

The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \overset{1}{—} t : \neg p$$

The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \overset{1}{\text{——}} t : \neg p$$

$$\mathcal{M}, s \vDash [!(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c)][a](\neg K_1 p \wedge [b]K_1 p)$$

The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \overset{1}{—} t : \neg p$$

$\mathcal{M}, s \models [!(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c)][a](\neg K_1 p \wedge [b]K_1 p)$
$\iff \mathcal{M}, s \models_{?p \cdot a \cdot b + ?\neg p \cdot a \cdot c} [a](\neg K_1 p \wedge [b]K_1 p)$

The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \overset{1}{\text{——}} t : \neg p$$

$$\mathcal{M}, s \vDash [!(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c)][a](\neg K_1 p \wedge [b]K_1 p)$$
$$\iff \mathcal{M}, s \vDash_{?p \cdot a \cdot b + ?\neg p \cdot a \cdot c} [a](\neg K_1 p \wedge [b]K_1 p)$$
$$\iff \mathcal{M}|_{p \vee \neg p}, s \vDash_{?p \cdot b + ?\neg p \cdot c} (\neg K_1 p \wedge [b]K_1 p)$$

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$$\iff \mathcal{M}, s \vDash_{?p \cdot b + ?\neg p \cdot c} \neg K_1 p \text{ and } \mathcal{M}, s \vDash_{?p \cdot b + ?\neg p \cdot c} [b]K_1 p$$

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The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \overset{1}{—} t : \neg p$$

$\mathcal{M}, s \vDash [!(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c)][a](\neg K_1 p \wedge [b]K_1 p)$

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$\iff \mathcal{M}, s \vDash \neg K_1 p$ and $\mathcal{M}, s \vDash_{\mathrm{PAL}} [!p]K_1 p$

The semantics

Example

Consider the following model $\mathcal{M}$:

$$s : p \text{ —1— } t : \neg p$$

$\mathcal{M}, s \vDash [!(?p \cdot a \cdot b + ?\neg p \cdot a \cdot c)][a](\neg K_1 p \wedge [b]K_1 p)$

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$\iff \mathcal{M}, s \vDash \neg K_1 p$ and $\mathcal{M}|_p, s \vDash_{?p} K_1 p$

$\iff \mathcal{M}, s \vDash \neg K_1 p$ and $\mathcal{M}, s \vDash_{\text{PAL}} [!p]K_1 p$

Consider the Häagen-Dazs protocol: $\pi_{H\text{-}D} = ?p_{love} \cdot a_{buy}$, $[!\pi_{H\text{-}D}][a_{buy}]K_i p_{love}$ is valid. However, buying an ice cream without the announcement $!\pi_{H\text{-}D}$ does not mean anything: $[a_{buy}]K_i p_{love}$ is not valid.

Expressivity

Theorem

$\mathtt{PDL}^{!?_b}$ is equally expressive as $\mathtt{PAL}$ on S5 models.

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Theorem

$\mathtt{PDL}^{!?_b}$ is equally expressive as $\mathtt{PAL}$ on S5 models.

Proof.

We can define the following translation from $\mathtt{PDL}^{!?_b}$ to $\mathtt{PAL}$:

$$
\begin{array}{rcl}
t(\phi) & = & t_{\boldsymbol{\Sigma}^*}(\phi) \\
t_\pi(p) & = & p \\
t_\pi(\neg\phi) & = & \neg t_\pi(\phi) \\
t_\pi(\phi_1 \wedge \phi_2) & = & t_\pi(\phi_1) \wedge t_\pi(\phi_2) \\
t_\pi(K_i\phi) & = & K_i t_\pi(\phi) \\
t_\pi([\pi']\phi) & = & \bigwedge\{[!\psi_j]t_{\theta_j}(\phi) \mid \mathcal{L}(\pi_j) \cap \mathcal{L}(\pi') \neq \emptyset\} \\
t_\pi([!\pi']\phi) & = & \chi_{\pi'} \to t_{\pi'}(\phi)
\end{array}
$$

□

Expressivity

Theorem

$\mathtt{PDL}^{!?_b}$ is equally expressive as $\mathtt{PAL}$ on S5 models.

Note that $[!p](K_i p \wedge [!q]q)$ can be reinterpreted in $\mathtt{PDL}^{!?_b}$ as

$$[!(?p \cdot a + ?q \cdot b)^*][a](K_i p \wedge [b]q).$$

Thus we can separate actions from their meanings.

Composing models: Joint work with van Eijck and Sietsma

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Aristotle said...

Love is composed of a single soul inhabiting two bodies.

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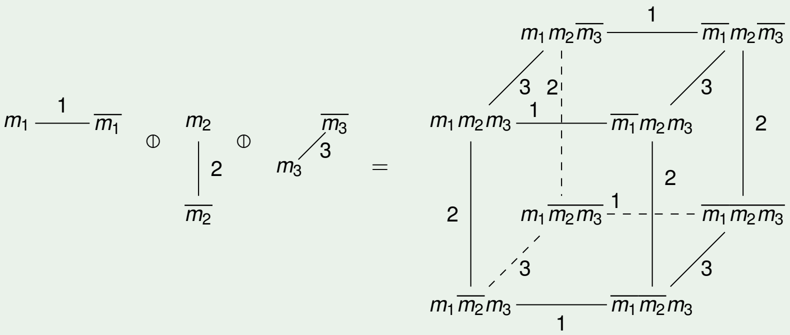
Definition (Merging Composition of "Partial" Kripke Models)

Given two models with the same set of agents $\mathbf{I}$: $\mathcal{M} = (S, \mathbf{P}, \mathbf{I}, \sim, V)$ and $\mathcal{N} = (T, \mathbf{P}', \mathbf{I}, \sim', V')$, the *merging composition* $\mathcal{M} \oplus \mathcal{N}$ is given by $(S'', \mathbf{P} \cup \mathbf{P}', \mathbf{I}, \sim'', V'')$, where:

- $S'' = \{(s, t) \mid s \in S, t \in T, V(s) \cap \mathbf{P}' = V'(t) \cap \mathbf{P}\}$,
- $(s, s') \sim_i'' (t, t')$ iff $s \sim_i t$ and $s' \sim_i' t'$,
- $V''(s, t) = V(s) \cup V'(t)$.

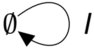
Composing models

Example (Composing Muddy Children)



Composing models

We define the *unit* model $\mathcal{E}$ as the model $(\{s\}, \emptyset, \mathbf{I}, \sim, V)$ where $V(s) = \emptyset$ and $\sim_i = \{(s, s)\}$ for any i. In a picture:



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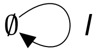

Theorem

Kripke models with the same set of agents form a commutative monoid under the $\oplus$ operation, with total bisimilarity as the appropriate equality notion. In particular, we have:

$$
\begin{array}{rcl}
\mathcal{E} \oplus \mathcal{M} & \underline{\leftrightarrow} & \mathcal{M} \\
\mathcal{M} \oplus \mathcal{E} & \underline{\leftrightarrow} & \mathcal{M} \\
\mathcal{M} \oplus (\mathcal{N} \oplus \mathcal{K}) & \underline{\leftrightarrow} & (\mathcal{M} \oplus \mathcal{N}) \oplus \mathcal{K} \\
\mathcal{M} \oplus \mathcal{N} & \underline{\leftrightarrow} & \mathcal{N} \oplus \mathcal{M}
\end{array}
$$

Composing models

The commutative monoid yields the algebraic preordering $\leq$ on the class of Kripke models with different vocabularies:

$$\mathcal{M} \leq \mathcal{N} \text{ iff there is a } \mathcal{K} \text{ with } \mathcal{M} \oplus \mathcal{K} \leftrightarrows \mathcal{N}.$$

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Given two models $\mathcal{M}$ and $\mathcal{N}$ such that $\mathbf{P}_{\mathcal{M}} \subseteq \mathbf{P}_{\mathcal{N}}$, a left-simulation between $\mathcal{M}$ and $\mathcal{N}$ is a relation $R \subseteq S_{\mathcal{M}} \times S_{\mathcal{N}}$ such that sRt implies that the following hold:

Restricted Invariance $V_{\mathcal{M}}(s) = V_{\mathcal{N}}(t) \cap \mathbf{P}_{\mathcal{M}}$;

Zag If for some $i \in \mathbf{I}$ there is a $t' \in S_{\mathcal{N}}$ with $t \xrightarrow{i} t'$ then there is a $s' \in S_{\mathcal{M}}$ with $s \xrightarrow{i} s'$ and $s'Rt'$.

Composing models

Theorem

For any models $\mathcal{M}, \mathcal{N}$ with arbitrary vocabularies:

$$\mathcal{M} \leq \mathcal{N} \implies \mathcal{M} \leftleftharpoons \mathcal{N}$$

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Theorem

Let $\mathcal{M}$ be a propositionally differentiated model. Then

$$\mathcal{M} \leq \mathcal{N} \iff \mathcal{M} \leftleftharpoons \mathcal{N}$$

Composing models

Theorem

If $\mathcal{M}, s \leq \mathcal{N}, t$ then all formulas ϕ in the diamond fragment of $\mathit{PDL}_{\mathbf{P}_{\mathcal{M}}, \mathbf{I}}$ are preserved from right to left under left simulation: if $\mathcal{N}, t \models \phi$ then $\mathcal{M}, s \models \phi$. Equivalently, the box fragment of $\mathit{PDL}_{\mathbf{P}_{\mathcal{M}}, \mathbf{I}}$ is preserved from left to right under left simulation.

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If $\mathcal{M}, s \leq \mathcal{N}, t$ then all formulas ϕ in the diamond fragment of $PDL_{\mathbf{P}_{\mathcal{M}}, \mathbf{I}}$ are preserved from right to left under left simulation: if $\mathcal{N}, t \vDash \phi$ then $\mathcal{M}, s \vDash \phi$. Equivalently, the box fragment of $PDL_{\mathbf{P}_{\mathcal{M}}, \mathbf{I}}$ is preserved from left to right under left simulation.

Theorem

If a pointed model $(\mathcal{M}, s)$ is decomposable into models $(\mathcal{M}_0, s_0), \ldots, (\mathcal{M}_n, s_n)$ with disjoint vocabularies $\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_n$, then for any i: $\mathcal{M}_i, s_i \leftrightarroweq_{\mathbf{P}_i} \mathcal{M}, s$. Therefore for any ϕ in $PDL_{\mathbf{P}_i, \mathbf{I}} : \mathcal{M}_i, s_i \vDash \phi \iff \mathcal{M}, s \vDash \phi$.

Composing models

We say $\mathcal{M}$ is *locally generated* if, for every agent i, there is a non-empty set of boolean formulas Φ_i (the set of local observables) based on $\mathbf{P}_{\mathcal{M}}$ such that:

$$\text{for all } s, s' \in S_{\mathcal{M}}, s \sim_i s' \text{ iff for all } \varphi \in \Phi_i, \mathcal{M}, s \models \varphi \Leftrightarrow \mathcal{M}, s' \models \varphi$$

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Theorem (Decomposition by agents)

Given a set of agent $\mathbf{I} = \{1, 2, \ldots n\}$. If $\mathcal{M} = (S, \mathbf{P}, \mathbf{I}, \sim, V)$ is locally generated w.r.t. $\Phi_1, \ldots, \Phi_n$, then there are models $\mathcal{M}_1, \ldots, \mathcal{M}_n$ and $\mathcal{M}_0$ such that:

- $\mathcal{M} \leftrightarrows (\mathcal{M}_0 \oplus \mathcal{M}_1 \oplus \cdots \oplus \mathcal{M}_n)$;
- $|S_{\mathcal{M}_j}| \leq |S|$ *and* $\mathcal{M}_i$ *is bisimulation contracted model;*
- $\mathbf{P}_{\mathcal{M}_j} = \{p \in \mathbf{P}_{\mathcal{M}} \mid p \text{ appears in } \Phi_j\}$ *for* $j > 0$;

Composing models

Theorem (Decomposition by issues)

Given a set of agent $\mathbf{I} = \{1, 2, \ldots n\}$ and a set of proposition letters $\mathbf{P} = \{p_1, \ldots, p_k\}$, if $\mathcal{M} = (S, \mathbf{P}, \mathbf{I}, \sim, V)$ is locally generated by $\Phi_1, \ldots, \Phi_n$ such that Φ_i only contains atomic propositions (i.e., $\Phi_i \subseteq \mathbf{P}$), then there are models $\mathcal{M}_1, \ldots, \mathcal{M}_k$ and $\mathcal{M}_0$ such that:

- $\mathcal{M} \leftrightarrows (\mathcal{M}_0 \oplus \mathcal{M}_1 \oplus \cdots \oplus \mathcal{M}_k)$;
- $\mathbf{P}_{\mathcal{M}_j} = \{p_j\}$ *for* $j > 0$ *and* $\mathbf{P}_0 = \mathbf{P}$;
- $|S_{\mathcal{M}_j}| = 2$ *for* $j > 0$

Composing models

Definition (Model expansion)

Given $\mathcal{M}$ we define the *expansion* of $\mathcal{M}$ w.r.t. vocabulary $\mathbf{P}'$ as follows: $\mathcal{M} \triangleleft \mathbf{P}' = \mathcal{M} \oplus \mathcal{M}^{\mathbf{I}}_{\mathbf{P}'}$.

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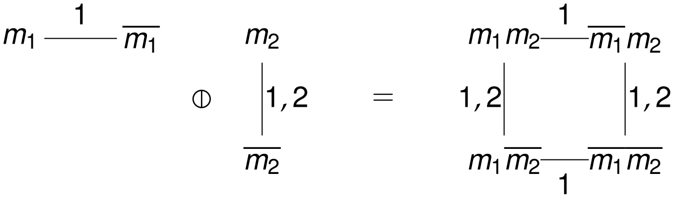

Composing models

Definition (Extended Product Update)

Given a static model $\mathcal{M}$ and an event model $\mathcal{A}$ for the same set of agents $\mathbf{I}$. Let $X = \mathbf{P}_{\mathcal{A}} - \mathbf{P}_{\mathcal{M}}$. Then the extended product update $\mathcal{M} \circledcirc \mathcal{A}$ is the static model defined by $(\mathcal{M} \triangleleft X) \otimes \mathcal{A}$.

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Theorem

When $\mathcal{A}$ is propositionally differentiated:
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Theorem

Let $\mathcal{A} \oplus \mathcal{B}$ be the composition of event models:
$\mathcal{M} \circledcirc (\mathcal{A} \oplus \mathcal{B}) \leftrightarrows (\mathcal{M} \circledcirc \mathcal{A}) \oplus (\mathcal{M} \circledcirc \mathcal{B})$.

Counting models: joint work with Siestma

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All happy families are happy alike, all unhappy families are unhappy in their own way.

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My question is...

How many different kinds of unhappiness are there?

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In other words...

Given a finite set of *formulas*, how many *different* models are there?

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To be more precise, we consider:

- modal μ–calculus formulas
- bisimulation as the equivalence notion between models
- only *image-finite* models

Counting models

Modal μ-calculus [Koz83]

$$\phi \quad ::= \quad \top \mid \bot \mid X \mid p \mid \bar{p} \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle a \rangle \phi \mid [a]\phi \mid \mu X.\phi \mid \nu X.\phi$$

Very expressive: $\mu X.\Box X$ which expresses well-foundedness. It is shown in [vBI08] that `Mu` is closed under product update.

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A μ-automaton A on set of basic propositions **P** and set of basic actions **Σ** is a tuple: $A = (Q, B, q_0, \rightarrow_{OR}, \rightarrow_{BR}, L, \Omega)$.

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μ automata recognizes (infinite) trees. Let $\mathcal{L}(A)$ be the language of A, i.e., the set of trees which are accepted by A.

Counting models

Theorem ([JW95])

For each μ–automaton there is an equivalent $\mathtt{Mu}$–*formula. For each* $\mathtt{Mu}$–*formula there is an equivalent μ–automaton .*

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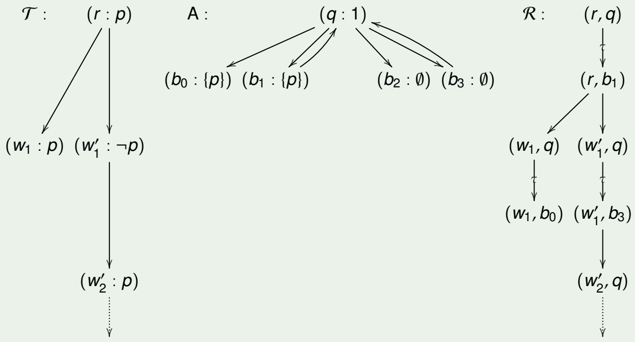
Example ($\mu X.\Box X$ and its μ-automata)



Counting models

Theorem

Let A be a μ-automaton. Then the following are equivalent:

1. $|\mathcal{L}(A)|_{/\underline{\leftrightarrow}} = 2^{\aleph_0}$,
2. $|\mathcal{L}(A)|_{/\underline{\leftrightarrow}} > \aleph_0$,
3. $\mathcal{L}(A)$ *contains a tree with infinite non-bisimilar subtrees (non-B-regular tree).*

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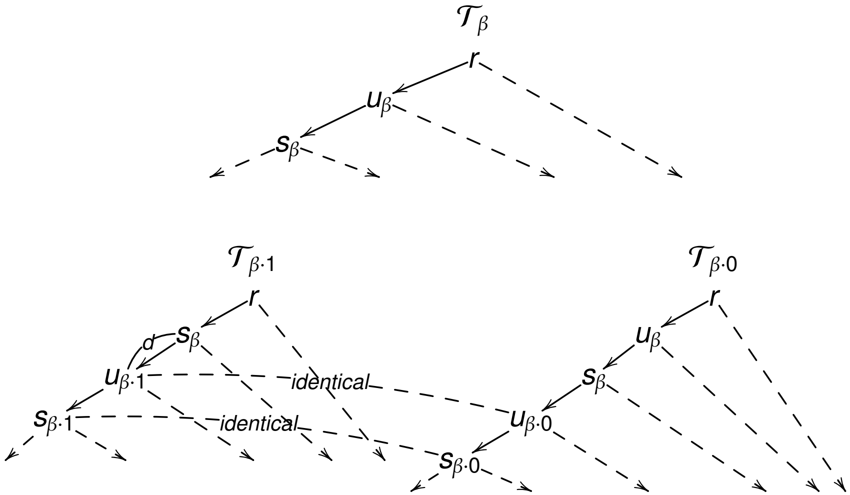
3. Show that all these tree are accepted by A.

Counting models



Counting models

Lemma

Given a μ–automaton A, if $\mathcal{L}(A)$ is countable up to bisimulation then $|alive(\mathcal{L}(A))|_{/\underline{\leftrightarrow}}$ is finite.

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Theorem (Normal form of countable languages)

Given a μ–automaton A, if $\mathcal{L}(A)$ is countable up to bisimulation, then it can be represented by

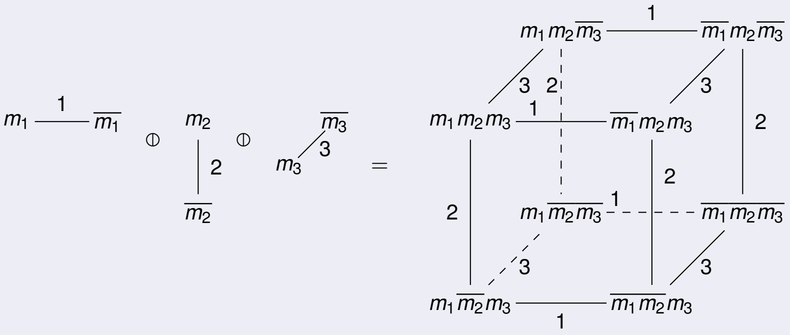
$$F_n[x_1\backslash\mathcal{T}_1, \ldots, x_1\backslash\mathcal{T}_n]$$

for some $n < \omega$, $\{\mathcal{T}_1, \ldots, \mathcal{T}_n\} \subseteq alive(\mathcal{L}(A))$, and some $F_n \subseteq \mathfrak{T}_n$ which is recognizable by an finite automaton B on finite trees in $\mathfrak{T}_n$.

Model Abstraction

Model Abstraction

State space explosion: e.g., n children, 2^n states.



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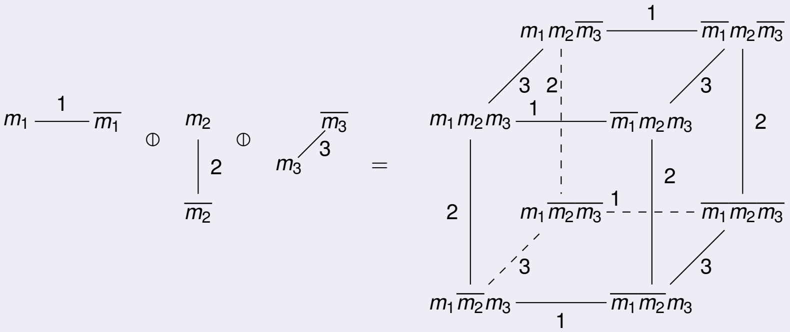

Good representations may help.

More than 10^{20} states can be handled by *symbolic model checking* [BCM$^+$92].

Model Abstraction

However, it is never enough...

Model Abstraction

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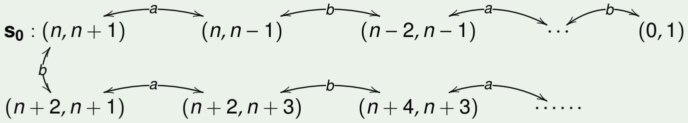
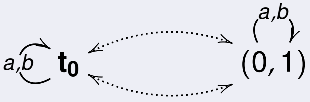
Example (Guessing the other number)

a and b are given two natural numbers n and $n+1$ respectively. They are told that what they have are two consecutive natural numbers, but they do not know who has the bigger one. We can build the following model (suppose n is an even number):



Model Abstraction

Making the models smaller by abstraction



Clearly, we may lose some information...

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Idea: safely reason about the big models at their small abstractions.

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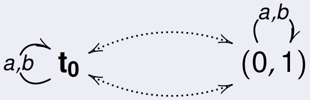

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Model abstraction

Idea: safely reason about the big models at their small abstractions.
"Most general [technique to reduce state space] and flexible" [CGL94]. Can be fully automated [CGJ+03].

Abstraction w.r.t. PAL: with Dechesne & Orzan

Definition (Kripke Modal Labelled Transition System [HJS01])

A *Kripke Modal Labelled Transition System* (KMLTS) is a tuple $\mathcal{M} = (S, \mathbf{P}, \mathbf{\Sigma}, \dashrightarrow, \rightarrow, V)$ where:

- $\dashrightarrow$ is a set of transitions of the form $s \overset{i}{\dashrightarrow} s'$ where $i \in \mathbf{\Sigma}$;
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We require that $\rightarrow \subseteq \dashrightarrow$.

The signature of $\mathcal{M}$: $(\mathbf{P}, \mathbf{\Sigma})$ is also important.

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Recall the formulas of the Public Announcement Logic:

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Refinement and Abstraction

A relation linking the abstract to the more refined

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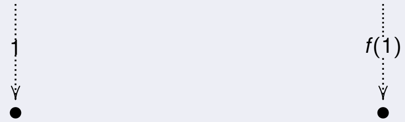

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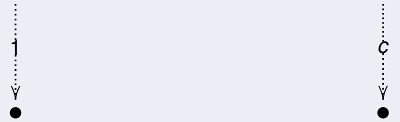

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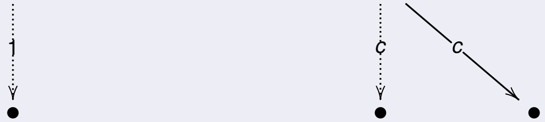

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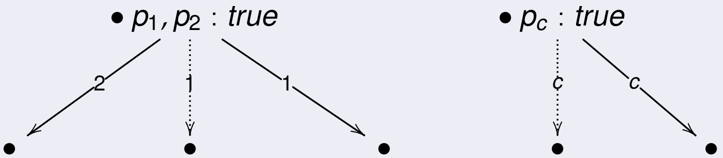

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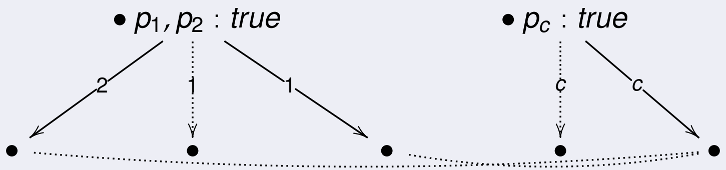
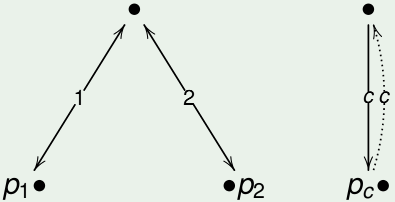


Model Abstraction

Example

Example of a KMLTS $\mathcal{M}$ and a f, g–abstraction of it where $f(1) = f(2) = c; g(p_1) = g(p_2) = p_c$.



Desired Property

Notation

Given two pointed models $(\mathcal{M}, s), (\mathcal{N}, t)$, and two formulas ϕ, ψ, we say $[\![\psi]\!]^{\mathcal{M},s} \leq [\![\phi]\!]^{\mathcal{N},t}$ if the following hold:

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Safe reasoning

$(\mathcal{N}, t) \Subset_{f,g} (\mathcal{M}, s)$ implies for all $\phi \in \mathtt{PAL}_{\mathbf{I}',P'}$:
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Translation of the formulas

Definition (Translation of formulas)

Given signatures $(\mathbf{I}', P'), (\mathbf{I}, P)$, and surjective functions $f : \mathbf{I}' \to \mathbf{I}, g : P' \to P$, we define the translation of an $\mathtt{PAL}_{\mathbf{I}',P'}$-formula ϕ into an $\mathtt{PAL}_{\mathbf{I},P}$-formula $\ulcorner \phi \urcorner_{f,g}$ inductively as follows:

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\ulcorner \psi_1 \wedge \psi_2 \urcorner_{f,g} & = & \ulcorner \psi_1 \urcorner_{f,g} \wedge \ulcorner \psi_2 \urcorner_{f,g} \\
\ulcorner K_{i'} \psi \urcorner_{f,g} & = & K_{f(i')} \ulcorner \psi \urcorner_{f,g} \\
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Example

$\ulcorner [p \wedge q \wedge r]K_1 p \vee K_2 q \urcorner_{f,g} = [P \wedge R]K_A P$ with $f(1) = f(2) = A$; $g(p) = g(q) = P$ and $g(r) = R$.

Logical Characterization

Lemma

$(\mathcal{N}, t) \Subset_{f,g} (\mathcal{M}, s)$ implies $(\mathcal{N}|_{\chi}, t) \Subset_{f,g} (\mathcal{M}|_{\ulcorner \chi \urcorner_{f,g}}, s)$ under certain condition.

Logical Characterization

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$(\mathcal{N}, t) \Subset_{f,g} (\mathcal{M}, s)$ *implies* $(\mathcal{N}|_{\chi}, t) \Subset_{f,g} (\mathcal{M}|_{\ulcorner \chi \urcorner_{f,g}}, s)$ *under certain condition.*

Theorem

$(\mathcal{N}, t) \Subset_{f,g} (\mathcal{M}, s)$ *implies for all* $\phi \in \mathtt{PAL}_{I', P'}$:
$[\![\ulcorner \phi \urcorner_{f,g}]\!]^{\mathcal{M},s} \leq [\![\phi]\!]^{\mathcal{N},t}$.

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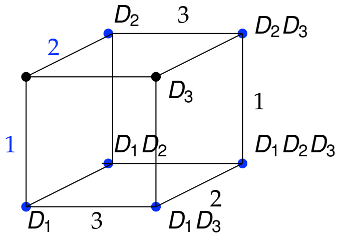
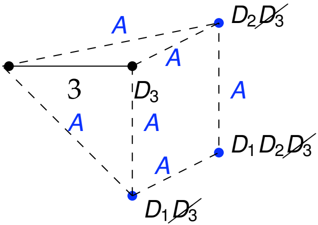
Theorem

If for every formula $\phi \in PAL_{I',P'}$: $[\![\ulcorner\phi\urcorner_{f,g}]\!]^{\mathcal{M},s} \leq [\![\phi]\!]^{\mathcal{N},t}$ then $(\mathcal{N}, t) \Subset_{f,g} (\mathcal{M}, s)$ (Image finitness assumed).

Muddy Children - Abstraction of n=3 case





Abstractions of the Muddy Children for $n = 3$ children. $f(1) = f(2) = A, f(3) = 3$ and $g = Id$. $\not{D}_3$ means proposition D_3 has valuation $\perp$ in the current state.

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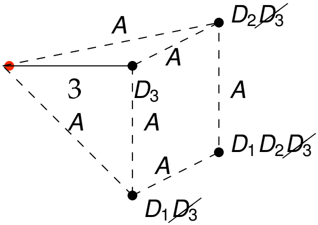
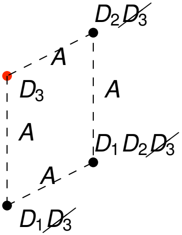


First announcement: $\ulcorner D_1 \vee D_2 \vee D_3 \urcorner_{f,g} = D_1 \vee D_2 \vee D_3$.

Muddy Children - Abstraction of n=3 case



$\ulcorner K_3 D_3 \urcorner_{f,g} = K_3 D_3$ holds at world D_3. Announcement can be made if more than one child is dirty:
$\ulcorner \neg K_1 D_1 \wedge \neg K_2 D_2 \wedge \neg K_3 D_3 \urcorner_{f,g} = \neg K_A D_1 \wedge \neg K_A D_2 \wedge \neg K_3 D_3$.

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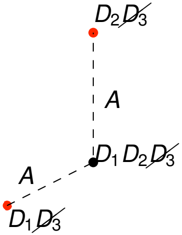

$\ulcorner K_1 D_1 \urcorner_{f,g} = K_A D_1$ holds at world $D_1 \not{D}_3$ and $\ulcorner K_1 D_2 \urcorner_{f,g} = K_A D_2$ holds at $D_2 \not{D}_3$. If all the three children are dirty then announce: $\ulcorner \neg K_1 D_1 \wedge \neg K_2 D_2 \wedge \neg K_3 D_3 \urcorner_{f,g} = \neg K_A D_1 \wedge \neg K_A D_2 \wedge \neg K_3 D_3$.

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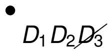
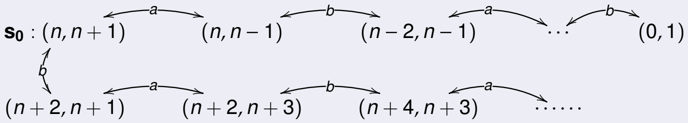


$\ulcorner K_1 D_1 \wedge K_2 D_2 \urcorner_{f,g} = K_A D_1 \wedge K_A D_2$ holds at the only world.

Model Abstraction

Recall the *Guessing Number* example



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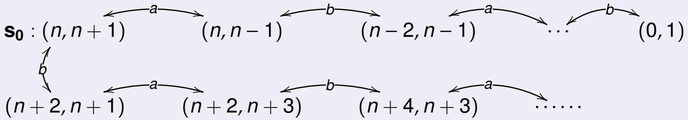
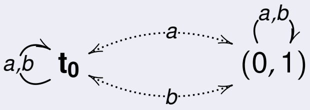


Can we do better than this?



Not enough *must* transitions for evaluating existential formulas (properties of reachability) e.g., $\langle(a+b)^*\rangle has_a 0$.

Model Abstraction

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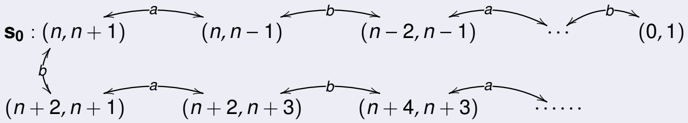
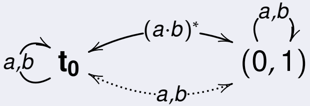


Yes! *Accelerated Modal LTS* [EvdP06]



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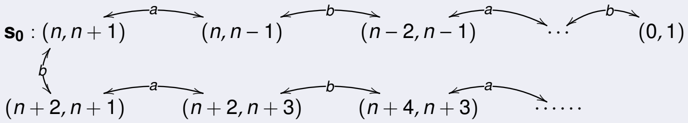

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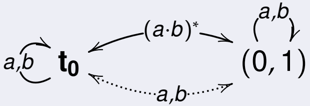

$\langle\pi\rangle\phi$ is true at s if there is a *must*-path $s \xrightarrow{\pi_1} s_1 \xrightarrow{\pi_2} \cdots \xrightarrow{\pi_n} s_n$ to a state where ϕ is true, such that $\mathcal{L}(\pi_1 \cdots \pi_n) \subseteq \mathcal{L}(\pi)$. e.g, $\langle(a+b)^*\rangle has_a 0$ is true on the above abstract model.

PDL on Accelerated Models: with Chen & van de Pol

Question: how to model check PDL on AMLTS?

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Recall the semantics:

$$\mathcal{M}, s \models \langle \pi \rangle \phi \quad \Leftrightarrow \quad \text{there exists a path } s \xrightarrow{\pi_1} s_1 \xrightarrow{\pi_2} \cdots \xrightarrow{\pi_n} s_n : \mathcal{M}, s_n \models \phi \text{ and } \mathcal{L}(\pi_1 \cdots \pi_n) \subseteq \mathcal{L}(\pi)$$

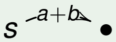
PDL on Accelerated Models: with Chen & van de Pol

Question: how to model check PDL on AMLTS?

Recall the semantics:

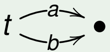
$$\mathcal{M}, s \vDash \langle\pi\rangle\phi \quad \Leftrightarrow \quad \text{there exists a path } s \xrightarrow{\pi_1} s_1 \xrightarrow{\pi_2} \cdots \xrightarrow{\pi_n} s_n : \; \mathcal{M}, s_n \vDash \phi \text{ and } \mathcal{L}(\pi_1 \cdots \pi_n) \subseteq \mathcal{L}(\pi)$$

Example



$\mathcal{M}, s \vDash \langle a+b+c \rangle \top$

$\mathcal{M}, s \nvDash \langle a \rangle \top$



$\mathcal{M}, t \vDash \langle a+b+c \rangle \top$

$\mathcal{M}, t \vDash \langle a \rangle \top \wedge \langle b \rangle \top$

A straightforward idea which is hard to implement: reduce the new to the old.

PDL on Accelerated Models

Regular Expression Rewriting [CDGLV02]

Given a regular expression π, rewrite π, if possible, by a set of other regular expressions $\mathcal{E} = \{\pi_0, \pi_1, \ldots \pi_n\}$.

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Theorem

([CDGLV02]) There is an essentially optimal algorithm to compute the maximal $\mathcal{E}$-rewriting of a given π w.r.t a given set $\mathcal{E}$ in 2-EXPTIME*.*

$\mathtt{PDL}$ on Accelerated Models

Definition (Rewriting $\mathtt{PDL}$ formula w.r.t. an accelerated model)

Given an AKM $\mathcal{M}$ and a $\mathtt{PDL}_{\mathbf{\Sigma}}$ formula ϕ, let $\langle\rangle_{\mathcal{M}}$ be the set of labels in $\mathcal{M}$, $\Re_{\mathcal{M}}(\phi)$ is the rewriting of ϕ in the language $\mathtt{PDL}_{\mathbf{\Sigma}_{\langle\rangle_{\mathcal{M}}}}$ defined by:

- $\Re_{\mathcal{M}}(\langle\pi\rangle\psi) = \langle\widehat{\pi_{\langle\rangle_{\mathcal{M}}}}\rangle\Re_{\mathcal{M}}(\psi)$.

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Theorem

For any pointed AKM $\mathcal{M}, s$ and any $\mathtt{PDL}_{\mathbf{\Sigma}}$ formula ϕ,

$$\mathcal{M}, s \vDash \phi \iff \ulcorner\mathcal{M}\urcorner, s \Vdash \Re_{\mathcal{M}}(\phi).$$

PDL on Accelerated Models

We also give a direct model checking algorithm by using the following proposition:

Theorem

Given a pointed AKM $\mathcal{M} = (S, \mathbf{\Sigma}, \rightarrow, V, s_0)$ *and* $T \subseteq S$*, if* $T = \{t \mid \mathcal{M}, t \vDash \phi\}$*, then we have:*

$$\mathcal{M}, s_0 \models \langle\pi\rangle\phi \iff \mathcal{L}(\mathcal{M} \otimes_T A_\pi) \neq \emptyset$$

where A_π *denotes the deterministic automaton corresponding to* π*.*

PDL on Accelerated Models

Theorem

We can reduce the problem of the existence of the non-empty rewriting to model checking problem.

$$\textit{there is a non-empty rewriting of } \pi \textit{ w.r.t. } \mathcal{E} \iff$$

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Theorem

Model checking PDL on AKM is EXPSPACE*-complete.*

$\mathtt{PDL}$ on Accelerated Models

Definition

Rewriting of a $\mathtt{PDL}^+_{\mathbf{\Sigma}}$ formula Given a $\mathtt{PDL}^+_{\mathbf{\Sigma}}$ formula ϕ, let $\langle\rangle_\phi$ be the set $\{\pi \mid \langle\pi\rangle \text{ appears in } \phi\}$, $\Re(\phi)$ is the rewriting of ϕ in the language $\mathtt{PDL}^+_{\mathbf{\Sigma}_{\langle\rangle_\phi}}$ defined by:

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- $\Re([\pi]\psi) = [\widehat{\pi_{\langle\rangle_\phi}}]\Re(\psi)$.

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Theorem

Given a $\mathtt{PDL}^{+}_{\mathbf{\Sigma}}$ formula ϕ, ϕ is satisfiable on an AKM $\iff$ $\Re(\phi)$ is satisfiable on a standard Kripke model.

Please contact me if you are interested in solving them...

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Thank you very much for your attention!

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