乌鸦怪论

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1. 天下乌鸦一般黑吗？
2. 归纳逻辑与乌鸦怪论

Contents

请验证：偶数背面必定是红色

白粉笔与黑乌鸦

$$\neg R(x) \wedge \neg B(x) \ \text{confirms} \ \forall x(\neg B(x) \rightarrow \neg R(x))$$

$$\frac{\forall x(R(x) \rightarrow B(x)) \leftrightarrow \forall x(\neg B(x) \rightarrow \neg R(x))}{\neg R(x) \wedge \neg B(x) \ \text{confirms} \ \forall x(R(x) \rightarrow B(x))}$$

乌鸦怪论

H: All ravens are black.

$\bar{H}$: All non-black objects are non-ravens.

H': Half the ravens are black.

D: A randomly selected raven is black.

N: A randomly selected non-black object is non-raven.

乌鸦怪论——世界太大太大太大

$$P(D|H) = P(N|H) = 1 \approx P(N|H') < 1$$

$$P(D|H') = 1/2$$

$$P(D) \neq P(N) \Rightarrow P(H|D) \neq P(\bar{H}|N) \quad \text{even if} \quad P(H) = P(\bar{H})$$

$$\frac{P(H|N)}{P(H'|N)} = \frac{P(N|H)}{P(N|H')} \times \frac{P(H)}{P(H')} \approx \frac{P(H)}{P(H')}$$

$$\frac{P(H|D)}{P(H'|D)} = \frac{P(D|H)}{P(D|H')} \times \frac{P(H)}{P(H')} = 2\frac{P(H)}{P(H')}$$

如何确证——天下乌鸦一般黑？

Problem 1—0 prior

Suppose θ is the percentage of ravens that are black.

"All ravens are black" $\equiv$ $\theta = 1$.

or, 1^∞ or, $\forall x : R(x) \to B(x)$

$$P[\theta = 1] = \int_1^1 P[H_\theta] d\theta = 0$$

$$\Downarrow$$

$$P[\theta = 1 | 1^n] = \frac{P[1^n | \theta = 1] P[\theta = 1]}{P[1^n]} = 0$$

先验信念咋赋？无差别原则)-:-(

Problem 2—明天太阳会升起11111···

$$\left.\begin{array}{l} \int_0^1 P[H_\theta]d\theta = 1 \\ \forall \theta, \theta' : \; P[H_\theta] = P[H_{\theta'}] \end{array}\right\} \Longrightarrow \forall \theta : \; P[H_\theta] = 1$$

$$\begin{aligned} P[x] &= \int_0^1 P[x|H_\theta]P[H_\theta]d\theta \\ &= \int_0^1 \theta^s(1-\theta)^f d\theta \\ &= B(s+1, f+1) \\ &= \frac{\Gamma(s+1)\Gamma(f+1)}{\Gamma(s+f+2)} = \frac{s!f!}{(s+f+1)!} \end{aligned}$$

Problem 2—明天太阳会升起11111···

$$P[x_{1:n}] = \frac{n!}{(n+1)!} = \frac{1}{n+1}$$

$$P[1^k|1^n] = \frac{P[1^{n+k}]}{P[1^n]} = \frac{n+1}{n+k+1}$$

$$P[1^\infty|1^n] = \lim_{k\to\infty} \frac{n+1}{n+k+1} = 0$$

Solution 1—Soft Hypothesis 绝对真理X

$$H_\varepsilon = \{\theta : \theta \in (1-\varepsilon, 1]\}$$

$$P[H_\varepsilon] = \int_{1-\varepsilon}^{1} P[H_\theta] d\theta = \varepsilon > 0$$

$$\begin{aligned} P[H_\varepsilon | 1^n] &= \int_{1-\varepsilon}^{1} P[H_\theta | 1^n] d\theta \\ &= \int_{1-\varepsilon}^{1} \frac{P[1^n | H_\theta] P[H_\theta]}{P[1^n]} d\theta \\ &= \int_{1-\varepsilon}^{1} (n+1)\theta^n d\theta \\ &= \theta^{n+1}\Big|_{1-\varepsilon}^{1} = 1 - (1-\varepsilon)^{n+1} \xrightarrow{n\to\infty} 1 \end{aligned}$$

Solution 2—ad hoc

$$P[H_\theta] = \frac{1}{2}(1 + \delta(1 - \theta))$$

where δ is the Dirac-δ function with the sifting property

$$\int f(\theta)\delta(\theta - a)\, d\theta = f(a) \tag{1}$$

Solution 2—ad hoc

$$
\begin{aligned}
P[x] &= \int_0^1 P[x|H_\theta]P[\theta]d\theta \\
&= \int_0^1 \theta^s(1-\theta)^f \cdot \frac{1}{2}(1+\delta(1-\theta))d\theta \\
&= \frac{1}{2}\int_0^1 \theta^s(1-\theta)^f(1+\delta(\theta-1))d\theta && [\delta \text{ is even}] \\
&= \frac{1}{2}\left(\frac{s!f!}{(s+f+1)!} + 1^s \cdot (1-1)^f\right) && [\text{sifting property(1)}] \\
&= \frac{1}{2}\left(\frac{s!f!}{(s+f+1)!} + \delta_{f,0}\right)
\end{aligned}
$$

Solution 2—ad hoc

$$P[1^n] = \frac{1}{2}\left(\frac{n!0!}{(n+1)!} + 1\right) = \frac{1}{2} \cdot \frac{n+2}{n+1}$$

$$P[1^k|1^n] = P[1^k|1^n] = \frac{P[1^{n+k}]}{P[1^n]} = \frac{n+k+2}{n+k+1} \cdot \frac{n+1}{n+2}$$

$$P[1^\infty|1^n] = \lim_{k\to\infty} P[1^k|1^n] = \frac{n+1}{n+2} \xrightarrow{n\to\infty} 1$$

先知啊

Contents

2. 归纳逻辑与乌鸦怪论
   - Carnap's Inductive Logic
   - Solomonoff's Prior in Carnap's Inductive Logic
   - How to confirm "all ravens are black"?

$\mathscr{L}$ contains countable constants $\mathscr{C}$ and m monadic predicates $\mathscr{R} = \{R_1, R_2, \ldots, R_m\}$ with no function symbols nor equality.

Definition (**Probability on Sentences**)

A probability on sentences is a non-negative function $w \colon \mathscr{S} \to [0, 1]$ such that

P_1. $\vDash \psi \Longrightarrow w(\psi) = 1$

P_2. $\psi_1 \vDash \neg\psi_2 \Longrightarrow w(\psi_1 \vee \psi_2) = w(\psi_1) + w(\psi_2)$

P_3. $w(\exists x \psi(x)) = \lim_{n \to \infty} w\left(\bigvee_{i=1}^{n} \psi(a_i)\right)$

一些性质

Theorem

(i) $w(\neg\phi) = 1 - w(\phi)$

(ii) $\vDash \neg\phi \Longrightarrow w(\phi) = 0$

(iii) *The following are equivalent:*

  (a) $w(\phi) = 1 \Longrightarrow \vDash \phi$

  (b) $w(\phi) = 0 \Longrightarrow \vDash \neg\phi$

(iv) $\phi \vDash \psi \Longrightarrow w(\phi) \leq w(\psi)$

(v) $\vDash \phi \leftrightarrow \psi \Longrightarrow w(\phi) = w(\psi)$

(vi) $w(\phi) + w(\psi) = w(\phi \wedge \psi) + w(\phi \vee \psi)$

Extension Theorem

Theorem (Extension Theorem)

For any probability function over quantifier-free sentences $w : \mathscr{S}_{QF} \to [0,1]$ satisfying P_1, P_2, w has an unique extension to $w^+ : \mathscr{S} \to [0,1]$ satisfying P_1, P_2, P_3.

卡尔纳普——归纳逻辑

卡尔纳普的可能世界

Let $Q_i \equiv \bigwedge_{j=1}^{m} \pm R_j$ for $1 \leq i \leq 2^m := r$. $\mathscr{Q} = \{Q_1, \cdots, Q_{2^m}\}$ is a 2^m-fold classification system of some Universe with domain $\mathscr{C}$.

state description $h: a_i \mapsto Q_{h_i}$

$$\bigwedge_{i=1}^{n} Q_{h_i}(a_i)$$

structure description

$$\{n_i : 1 \leq i \leq r\}$$

where $n_i := |h\restriction_{\{1,\ldots,n\}}{}_{-1}(i)|$

怎么找合适的w？

for any permutation σ of $\mathbb{N}^+$,

$$w(\psi(a_1,\dots,a_n)) = w(\psi(a_{\sigma(1)},\dots,a_{\sigma(n)})) \qquad \text{(Ex)}$$

for any permutation τ of $\{1,2,\dots,r\}$,

$$w\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = w\left(\bigwedge_{i=1}^{n} Q_{\tau(h_i)}(a_i)\right) \qquad \text{(Ax)}$$

sufficient postulate

$$w\left(Q_j(a_{n+1}) \,\middle|\, \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = f_j(n_j, n) \qquad \text{(SP)}$$

怎么找合适的w？

Principle Ex asserts that $w\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right)$ depends only on the vector $\langle n_{h_i} : 1 \leq i \leq n\rangle$, so that it is independent on the order of observing the individuals, while in the presence of Ex, principle Ax asserts that $w\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right)$ depends only on $\{n_i : 1 \leq i \leq r\}$, and $w(Q_i(a_1)) = 1/r$ for all $1 \leq i \leq r$.

Carnap's λ-continuum

Theorem

Suppose language $\mathscr{L}$ has at least two predicates i.e. $m \geq 2$, then the probability function w on $\mathscr{L}$ satisfies Ex, SP if and only if $w = \mathfrak{c}_\lambda$ for some $0 \leq \lambda \leq \infty$.

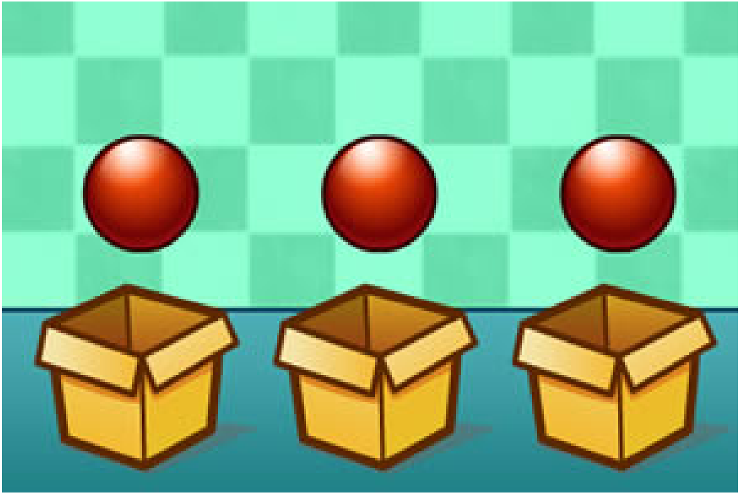
Namely,

$$f_i(n_i, n) = \frac{n_i + \lambda\gamma_i}{n + \lambda}$$

where $\gamma_i = f_i(0,0)$ *and* $\lambda = \dfrac{f_i(0,1)}{f_i(0,0) - f_i(0,1)}$.

By adding Ax, $\forall i : \gamma_i = \dfrac{1}{r}$.

别急，让我们慢慢来~ 扔个球先~



无差别原则

(A) All state descriptions have equal weight.

(B) All structure descriptions have equal weight.

Given n individuals, there are r^n possible state descriptions and

$$\binom{n+r-1}{r-1}$$

possible structure descriptions.

（A）状态描述无差别X

According to (A),

$$m^{\dagger}\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \frac{1}{r^n}$$

$$\mathfrak{c}^{\dagger}\left(Q_j(a_{n+1}) \,\middle|\, \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \frac{m^{\dagger}\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \wedge Q_j(a_{n+1})\right)}{m^{\dagger}\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right)} = \frac{\frac{1}{r^{n+1}}}{\frac{1}{r^n}} = \frac{1}{r}$$

（B）结构描述无差别✓

According to (B),

$$m^*(n_1, \ldots, n_r) = \frac{1}{\binom{n+r-1}{r-1}}$$

一个结构描述= $\binom{n}{n_1,\ldots,n_r}$ 状态描述

$$m^*\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \frac{m^*(n_1, \ldots, n_r)}{\binom{n}{n_1,\ldots,n_r}} = \frac{1}{\binom{n+r-1}{r-1}\binom{n}{n_1,\ldots,n_r}}$$

degree of confirmation

$$\mathfrak{c}^*\left(Q_j(a_{n+1})\,\middle|\,\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \frac{m^*\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \wedge Q_j(a_{n+1})\right)}{m^*\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right)} = \frac{n_j+1}{n+r}$$

回到Carnap's λ-continuum——$\mathfrak{c}^*$ 的推广

Suppose $(Q_1, \cdots, Q_r)$ are defined so that they have different relative widths γ_i such that $\sum_{i=1}^{r} \gamma_i = 1$, Carnap's λ-continuum is

$$\mathfrak{c}_\lambda\left(Q_j(a_{n+1}) \,\middle|\, \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \frac{n_j + \lambda\gamma_j}{n+\lambda} = \frac{n}{n+\lambda}\frac{n_j}{n} + \frac{\lambda}{n+\lambda}\gamma_j$$

从条件概率到状态描述上

Suppose $\vec{a} = (a_1, \ldots, a_n)$, and the state description of $\vec{a}$ is $\Theta(\vec{a}) \equiv \bigwedge_{i=1}^{n} Q_{h_i}(a_i)$, which can be assigned a degree of confirmation.

$$\begin{aligned}
\mathfrak{c}_\lambda(\Theta(\vec{a})) &= \mathfrak{c}_\lambda\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) \\
&= \prod_{i=0}^{n-1} \mathfrak{c}_\lambda\left(Q_{h_{i+1}}(a_{i+1}) \,\middle|\, \bigwedge_{j=0}^{i} Q_{h_j}(a_j)\right) \\
&= \prod_{i=0}^{n-1} \frac{i_{h_{i+1}} + \lambda\gamma_{h_{i+1}}}{i+\lambda}
\end{aligned}$$

$\gamma_i := 1/r$ invariant under xxx principle, $\mathfrak{c}_r = \mathfrak{c}^*$

$$
\begin{aligned}
\mathfrak{c}_\lambda(\Theta(\vec{a})) &= \mathfrak{c}_\lambda\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) \\
&= \prod_{j=1}^{n} \mathfrak{c}_\lambda\left(Q_{h_j}(a_j) \,\middle|\, \bigwedge_{i=j+1}^{n} Q_{h_i}(a_i)\right) \\
&= \prod_{j=1}^{n}\left(\frac{|h\restriction_{\{j+1,\dots,n\}}{}_{-1}(h_j)| + \frac{\lambda}{r}}{n-j+\lambda}\right) \\
&= \frac{\prod_{i=1}^{r}\prod_{j=0}^{n_i-1}\left(j+\frac{\lambda}{r}\right)}{\prod_{j=0}^{n-1}(j+\lambda)}
\end{aligned} \tag{2}
$$

回顾卡尔纳普“degree of confirmation”及其三原则

1. 无视时间顺序；无视因果
2. 无差别原则作用于结构描述
3. 乌鸦怪论！白粉笔不影响n_j
4. 不能确证“天下乌鸦一般黑”（后面证明）

2. 归纳逻辑与乌鸦怪论
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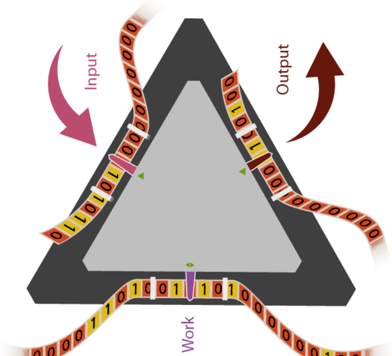
所罗门诺夫

State Description

$$h_{1:n} \quad \text{determine} \quad \bigwedge_{i=1}^{n} Q_{h_i}(a_i)$$

$$p \quad \text{determine} \quad h$$



Universal Probability

Definition (Universal Probability)

$$\mathfrak{c}_M\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \sum_{p:U(p)=h_{1:n}*} 2^{-|p|}$$

Where U is a universal monotone Turing machine.

Universal Probability

$$\mathfrak{c}_M\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \sum_p 2^{-|p|}[\![U(p) = h_{1:n}*]\!]$$

$$= \lim_{n\to\infty} \frac{\sum_{p:|p|\le n} 2^{n-|p|}\left[\!\left[\bigwedge_{i=1}^{n} Q_{\langle U(p)\rangle_i}(a_i) \equiv \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right]\!\right]}{2^n}$$

$$\approx \lim_{n\to\infty} \frac{\left|\left\{p : |p| = n \;\&\; \bigwedge_{i=1}^{n} Q_{\langle U(p)\rangle_i}(a_i) \equiv \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right\}\right|}{2^n}$$

$$\textbf{probability} = \frac{|\text{consistent universes}|}{|\text{all possible universes}|}$$

Normalization

$$\mathfrak{c}'_M(\top) := 1$$

$$\mathfrak{c}'_M\left(\bigwedge_{i=1}^{t} Q_{h_i}(a_i)\right) := \mathfrak{c}'_M\left(\bigwedge_{i=1}^{t-1} Q_{h_i}(a_i)\right) \frac{\mathfrak{c}_M\left(\bigwedge_{i=1}^{t} Q_{h_i}(a_i)\right)}{\sum_{1\leq k\leq r} \mathfrak{c}_M\left(\bigwedge_{i=1}^{t-1} Q_{h_i}(a_i) \wedge Q_k(a_t)\right)}$$

$$= \frac{\mathfrak{c}_M\left(\bigwedge_{i=1}^{t} Q_{h_i}(a_i)\right)}{\mathfrak{c}_M(\top)} \prod_{i=1}^{t} \frac{\mathfrak{c}_M\left(\bigwedge_{j=1}^{i-1} Q_{h_j}(a_j)\right)}{\sum_{1\leq k\leq r} \mathfrak{c}_M\left(\bigwedge_{j=1}^{i-1} Q_{h_j}(a_j) \wedge Q_k(a_i)\right)}$$

一些性质

for any state description Θ, Θ',

(i) $\mathfrak{c}'_M(\Theta(a_1, \ldots, a_n)) \geq 0$

(ii) $\mathfrak{c}'_M(\top) = 1$

(iii)

$$\mathfrak{c}'_M(\Theta(a_1, \ldots, a_n)) = \sum_{\Theta'(a_1,\ldots,a_{n+1}) \vDash \Theta(a_1,\ldots,a_n)} \mathfrak{c}'_M(\Theta'(a_1, \ldots, a_{n+1}))$$

For any quantifier-free sentence $\psi(\vec{a})$, let

$$\mathfrak{c}'_M(\psi(\vec{a})) := \sum_{\Theta(\vec{b}) \vDash \psi(\vec{a})} \mathfrak{c}'_M\left(\Theta(\vec{b})\right)$$

where $\vec{b}$ is sufficiently large that all of the $\vec{a}$ are amongst $\vec{b}$, and $\bigvee_{\Theta(\vec{b}) \vDash \psi(\vec{a})} \Theta(\vec{b})$ is the *full disjunctive normal form* of $\psi(\vec{a})$.

$$\psi(\vec{a}) \equiv \bigvee_{\Theta(\vec{b}) \vDash \psi(\vec{a})} \Theta(\vec{b}) \qquad \text{(DNF)}$$

“degree of confirmation” and “convergence theorem”

$$c'_M(\psi_H|\psi_E) = \frac{\mathfrak{c}'_M(\psi_E \wedge \psi_H)}{\mathfrak{c}'_M(\psi_E)}$$

Theorem (Convergence Theorem)

$$\mathfrak{c}'_M\left(\bigwedge_{i=l+1}^{t} Q_{h_i}(a_i) \,\middle|\, \bigwedge_{i=1}^{l} Q_{h_i}(a_i)\right) \xrightarrow{l\to\infty} \mu\left(\bigwedge_{i=l+1}^{t} Q_{h_i}(a_i) \,\middle|\, \bigwedge_{i=1}^{l} Q_{h_i}(a_i)\right)$$

Problem

$$\mathfrak{c}'_M(\psi|\phi) \xrightarrow{?} \mu(\psi|\phi)$$

对比$\mathfrak{c}^*$与$\mathfrak{c}'_M$ 无差别原则的作用对象不同

With zero-knowledge ($n = 0$),

Carnap $\mathfrak{c}_\lambda(Q_j(a_1)) = \dfrac{0 + \lambda\gamma_j}{0 + \lambda} = \gamma_j$

Solomonoff $\mathfrak{c}'_M(Q_{h_1}(a_1)) = \dfrac{\mathfrak{c}_M(Q_{h_1}(a_1))}{\sum_{1\le j\le r} \mathfrak{c}_M(Q_j(a_1))}$

with sufficient experiences (n large enough),

Carnap $\mathfrak{c}_\lambda\left(Q_j(a_{n+1}) \,\middle|\, \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) = \dfrac{n_j + \lambda\gamma_j}{n + \lambda} \approx \dfrac{n_j}{n}$

$\mathfrak{c}^*$ vs. $\mathfrak{c}'_M$: 现象的频率VS可能世界的频率

2 归纳逻辑与乌鸦怪论

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全称命题

$$
\begin{aligned}
w(\forall x\psi(x)) &= 1 - w(\exists x\neg\psi(x)) \\
&= 1 - \lim_{n\to\infty} w\left(\bigvee_{i=1}^{n} \neg\psi(a_i)\right) \\
&= \lim_{n\to\infty}\left(1 - w\left(\bigvee_{i=1}^{n} \neg\psi(a_i)\right)\right) \\
&= \lim_{n\to\infty} w\left(\bigwedge_{i=1}^{n} \psi(a_i)\right)
\end{aligned}
$$

怎么确证“天下乌鸦一般黑”？

$$
\begin{aligned}
&\lim_{n\to\infty} \mathfrak{c}'_M\left(\forall x(R(x)\to B(x))\,\middle|\,\bigwedge_{i=1}^{n}(\neg R(a_i)\vee B(a_i))\right)\\
&=\lim_{n\to\infty}\frac{\mathfrak{c}'_M(\forall x(R(x)\to B(x)))}{\mathfrak{c}'_M\left(\bigwedge_{i=1}^{n}(R(a_i)\to B(a_i))\right)}\\
&=\frac{\mathfrak{c}'_M(\forall x(R(x)\to B(x)))}{\lim_{n\to\infty}\mathfrak{c}'_M\left(\bigwedge_{i=1}^{n}(R(a_i)\to B(a_i))\right)}\\
&=\frac{\mathfrak{c}'_M(\forall x(R(x)\to B(x)))}{\mathfrak{c}'_M(\forall x(R(x)\to B(x)))}\\
&=1
\end{aligned}
$$

确保下式成立!

$$\lim_{n\to\infty} w\left(\bigwedge_{i=1}^{n}(R(a_i) \to B(a_i))\right) > 0$$

$\mathfrak{c}_\lambda$ 做不到!

$$
\begin{aligned}
&\mathfrak{c}_\lambda\left(\forall x(R(x)\to B(x))\right)\\
&= \lim_{n\to\infty} \sum_{\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \vDash \bigvee_{j=1}^{2^n}\left(\bigwedge_{i=1}^{n}(\neg R(a_i)/B(a_i))\right)_j} \mathfrak{c}_\lambda\left(\bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right)\\
&= \lim_{n\to\infty} \sum_{\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \vDash \bigvee_{j=1}^{2^n}\left(\bigwedge_{i=1}^{n}(\neg R(a_i)/B(a_i))\right)_j} \prod_{i=0}^{n-1} \mathfrak{c}_\lambda\left(Q_{h_{i+1}}(a_{i+1}) \,\middle|\, \bigwedge_{j=1}^{i} Q_{h_j}(a_j)\right)\\
&= \lim_{n\to\infty} \sum_{\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \vDash \bigvee_{j=1}^{2^n}\left(\bigwedge_{i=1}^{n}(\neg R(a_i)/B(a_i))\right)_j} \prod_{i=0}^{n-1} \frac{i_{h_{i+1}} + \lambda\gamma_{h_{i+1}}}{i+\lambda}\\
&\leq \lim_{n\to\infty} \prod_{i=0}^{n-1} \frac{i + \lambda(1 - \min_{1\leq t\leq r}\gamma_t)}{i+\lambda}\\
&= 0
\end{aligned}
$$

速度决定成败!

The last step follows from

$$\prod_{n\geq 1} a_n = 0 \iff \sum_{n\geq 1}(1 - a_n) = \infty \quad \text{for} \quad \forall n:\ 0 < a_n \leq 1$$

$$\mathfrak{c}'_\lambda\left(Q_j(a_{n+1}) \,\middle|\, \bigwedge_{i=1}^{n} Q_{h_i}(a_i)\right) := \frac{\dfrac{n_j^2 + \lambda\gamma_j}{n^2 + \lambda}}{\sum\limits_{1\leq k\leq r} \dfrac{n_k^2 + \lambda\gamma_k}{n^2 + \lambda}} = \frac{n_j^2 + \lambda\gamma_j}{\sum\limits_{1\leq k\leq r} n_k^2 + \lambda}$$

$$\mathfrak{c}'_\lambda(\forall x(R(x) \to B(x)) > 0$$

但是不合频率，SP也被违反。

$\mathfrak{c}'_M$ 可确证“天下乌鸦一般黑”!

结论：在任何可计算的世界（状态描述h）里（$Km(h_{1:\infty}) < \infty$），只要确实所有乌鸦都是黑的，

$$\bigwedge_{i=1}^{n} Q_{h_i}(a_i) \vDash \bigwedge_{i=1}^{n} (\neg R(a_i)/B(a_i)) \quad \text{for any } n.$$

那么

$$\lim_{n\to\infty} \mathfrak{c}'_M \left(\forall x(R(x) \to B(x)) \,\middle|\, \bigwedge_{i=1}^{n} (\neg R(a_i) \vee B(a_i)) \right) = 1$$

谢谢！

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