Ontology of Structuralism for mathematical philosophy

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Introduction

- What is the ontological problem in mathematics? Do mathematical object exist?
- realism and anti-realism

Introduction: realism

- realism: mathematical objects exist objectively, independent of the mathematician.
- can explain the necessity of mathematics
- lead to an epistemic mystery

Introduction: anti-realism

- ▶ idealism: mathematical objects depend on the mind of human
- ▶ Nominalism: mere linguistic constructions.

An example

In "What Numbers Could Not Be", Benacerraf argues against a Platonist view of mathematics. This argument is based on the point that Ernst Zermelo and John von Neumann give distinct, and completely adequate, identifications of natural numbers with sets.

STRUCTURALISM

main points:

- ▶ The essence of a natural number is its relations to other natural numbers
- ▶ a system is a collection of objects with certain relations among them.
- ▶ structure is the abstract form of a system, highlighting the interrelationships among the objects
- ▶ mathematics is the deductive study of structures as such.

STRUCTURALISM

In mathematics, I claim, we do not have objects with an 'internal' composition arranged in structures, we have only structures. The objects of mathematics, that is, the entities which our mathematical constants and quantifiers denote, are structureless points or positions in structures. As positions in structures, they have no identity or features outside a structure.
(Resnik 1981)

STRUCTURALISM

the status of structures themselves:
Do structures exist as objects in their own right?
a structure is the form of a system, as universal is to subsumed
particular, Our question is whether, and in what sense, structures
themselves exist independent of the systems of objects that
exemplify them.

STRUCTURALISM

the status of structures themselves:

- ▶ ante rem realism: the structures are independent
- ▶ in re realism: no, they are not.

STRUCTURALISM

ante rem realism

- ▶ a natural number is a place in the natural number structure. The latter is the pattern common to all of the models of arithmetic.
  For example, anything at all can 'be' 3-anything can occupy that place in a system exemplifying the natural number structure, $\{\{\{ \emptyset \}\}\}$ or $\{ \emptyset, \{ \emptyset \} , \{ \emptyset, \{ \emptyset \} \} \}$

- ▶ what is the difference between an object and a position in a structure? Sometimes the places of a structure are treated in the context of one or more systems that exemplify the structure. This is the places-are-offices perspective.

STRUCTURALISM

ante rem realism

- there are contexts in which the places of a given structure are treated as objects in their own right. This is the places-are-objects perspective. places-are-objects statements apply to the objects or people that occupy the positions in any system exemplifying the structure.
- each structure exemplifies itself. A regress emerges.

STRUCTURALISM

in re realism

- places-are-objects statements are no more than a convenient rephrasing of corresponding generalizations over systems that exemplify the structure in question. Talk of structures generally is convenient shorthand for talk about systems.
- nominalist proposes that we speak of possible structures rather than structures.