The Outline of A New Solution to The Liar Paradox

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Contents

1 The Problems

Example (1): Strengthened liar:

(1): (1) is not true

What is the value of the sentence?
In the common way, if it is true it will be false, if it isn't true it will be true.

1. (1) = (1) is not true	Given
2. (1) is not true	Hypothesis
3. (1) is not true is not true	1 and 2, by Intersub
4. Not that (1) is not true	3, by (T) and intersu
5. (1) is not true and not that (1) is not true	2 and 4, by conjunc
6. (1) is true	2–5, by reductio ad
7. (1) is true is not true	1 and 6, by Intersub
8. Not that (1) is true	7, by (T) and Intersu
9. (1) is true and not that (1) is true	6 and 8, by conjunc

Example (2)

A: B is not true

B: A is true

Using the similar inference we can get that: if A is true then B is not true and hence A is not true; if A is not true then it is not true that B is not true, so B is true, hence A is true.

Example (3): Curry paradox

K: True(<K>)→the earth is flat

Letting "A↔B" abbreviates "(A→B)∧(B→A)" and "⊥" abbreviates "the earth is flat", we now argue as follows:

1. K↔(True(<K>)→ ⊥)	By construction of K
2. True(<K>)↔(True(<K>)→)⊥	1, by (T) and Intersubstitut
3. True(<K>)→(True(<K>)→ ⊥)	Left to Right of 2
4. (True(<K>)∧True(<K>))→ ⊥	3, by Importation
5. True(<K>)→ ⊥	4, by Intersubstitutivity (ϕ /
6. (True(<K>)→)⊥) → True(<K>)	Right to Left of 2
7. True(<K>)	5 and 6, by modus ponens
8. ⊥	5 and 7, by modus ponens

It looks as if all the liar paradoxes contain self-reference. However, it's not the case, there is another example which shows that we can get a paradox without using self-reference.

Example (4): Yablo's paradox

Imaging an infinite sequence of sentences S_1, S_2, S_3.... Each sentence claims that every subsequent sentence is untrue:

(S_1) for all k>1, S_k is untrue

(S_2) for all k>2, S_k is untrue

(S_3) for all k>3, S_k is untrue

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Example (4): Yablo's paradox

Suppose for contradiction that some S_n is true. Given what S_n says, for all k>n, S_k is untrue. Therefore (a) S_{n+1} is untrue and (b) for all m>k+1, S_m is untrue. By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is untrue. So every sentence S_n in the sequence is untrue. But then the sentences sequent to any given S_n are untrue whence S_n is true after all. Hence for any S_n, S_n is true iff it is not true. Obviously there isn't self-reference in this example, however, a liar-like paradox still appears.

Example (5): Contingent liar

Consider the following statement made by Jones:

(1) Most of Nixon's assertions about Watergate are false.

Suppose, however, Nixon's assertions about Watergate are evenly balanced between the true and the false, except for one problematic case,

(2) Everything Jones says about Watergate is true.

Example (5): Contingent liar

Suppose that (1) is Jones's sole assertion about Watergate. Then (1) and (2) are both paradoxical: they are true if and only if they are false.

However, if Jones says other sentence about Watergate and there are false sentences among them, then (1) is true, and (2) is false, and there is no paradox.

So the contingent liar means whether a sentence is paradox or not depends on environment.

Example (6): Tarski's undefinability theorem

In fact, this example is not a liar paradox. But it is a very important issue about the liar paradox. Nearly every solution to liar's paradox has to be consistent with this theorem.

Theorem

For a logic L, which is rich enough to contain arithmetic, if B(v) is any formula in the language of arithmetic with v as the only free variable, then there is a sentence Q in that language such that $\vdash\rightarrow(\phi\leftrightarrow B(<\phi>))$.

Therefore we cannot define *true* predicate such that ϕ is true iff ϕ, or else we will get $\vdash\rightarrow(\phi\leftrightarrow \mathrm{True}(<\phi>))$

2 Tarski's proposal

Three families of solutions

There are three kinds of solutions to the liar paradox:

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There are three kinds of solutions to the liar paradox:

Next, I will first give an introduction to the Tarski's proposal, and then show some disadvantages of this proposal. Then I will present my proposal which can solve the problems mentioned in the above and avoid the disadvantages of Tarski's proposal.

Tarski's proposal

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- Then he distinguishes object language and meta-language. The truth of the object language can be said only in the meta-language because of the Tarski's undefinability theorem.
- Meta-language is relative, so there is a sequence of meta-languages.
- Besides, he claims that any truth definition must satisfy any substitution of the form: x is true iff ϕ. Here x is the name of the sentence ϕ. This form is called (T) schema or T biconditionals.

Tarski's proposal

- L_0 contains the following primitive predicates: P_1, P_2, P_3... and Q_1, Q_2, Q_3 ...; the functions: f_1, f_2, f_3... and the constants: t_1, t_2, t_3.... The formulas of L_0 are built up by the usual operations of the first-order predicate calculus.

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- L_0 cannot contain its own truth predicate, so a metalanguage L_1 containing a truth predicate T_o is needed to talk about the sentence of L_0 which is true. $L_1 = L_o \cup \{T_0\}$. The formulas of L_1 is defined as usual. The process can be iterated, leading to a sequence $\{L_0, L_1, L_2, L_3, \ldots\}$ of languages, each with a higher truth$_n$ predicate for the preceding language.

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- $\mathscr{L}_{Tarski} = \cup\{\mathscr{L}_n : n < \omega\}$.

Tarski's proposal

The semantics of every language L_i is as that of predicate logic except that $\sigma(T_n(t))=1$ iff $\sigma(t)=\ulcorner\phi\urcorner$ and $\sigma \models \phi$, where $\ulcorner\phi\urcorner$ is the Gödel number of a sentence ϕ of the language L_m and m<n

Instead of using constant t, from now on I use $\overline{\ulcorner\phi\urcorner}$ to refer to the Gödel number of the sentence ϕ. Then the (T) schema is $T_n(\overline{\ulcorner\phi\urcorner})\leftrightarrow \phi$, and it is valid in the arithmetic models.

Objections to Tarski's proposal

The common objections to Tarski's hierarchical approaches to truth is that they fragment the concept of truth. And hence the (T) schema divided into many (T_n) schemas. However, there is only one truth predicate rather than many true$_n$s.

3 My proposal

My proposal

In this section I will give my proposal to the liar paradox This proposal can avoid the objection to Tarski's proposal. In my proposal we have only one Truth predicate and the (T) schema, not (T_n) schemas, hold in some conditions. Besides the semantics here is bivalent rather than many-valued.

3.1 Syntax

Given a limit ordinal $\omega \leqslant \gamma < 2^{\omega}$.
Let $\mathscr{L}_0$ be the language including predicates P_1^1, P_2^1, P_3^1...P_1^n, P_2^n, P_3^n...and constants $c_{\perp 0}^0$, $c_{\perp 1}^0$, $c_{\perp 2}^0$... All the n-place functions are treated as n+1-place predicates in the usual way, so there are no functions. Besides in the syntax there is no common quantifier $\forall$ and $\exists$, instead, there are many $\forall^{\alpha}$s and correspondingly there are many $\exists^{\alpha}$s, $\alpha < \gamma$.

3.1 Syntax

$\mathscr{L}_1$=$\mathscr{L}_0$∪ {$c^1_{\phi 0}$: ϕ is a formula of $\mathscr{L}_0$ and 0 means ϕ is an open formula } ∪ {$c^1_{\phi 1}$: ϕ is a formula of $\mathscr{L}_0$ and 1 means ϕ is a closed formula } ∪ { T }

$\mathscr{L}_{n+1}$=$\mathscr{L}_n$ ∪ {$c^{n+1}_{\phi 0}$: ϕ is a formula of $\mathscr{L}_n$ that doesn't appear before, 0 means ϕ is an open formula } ∪ {$c^{n+1}_{\phi 1}$: ϕ is a formula of $\mathscr{L}_n$ that does not appear before, 1 means ϕ is a closed formula }

For limit ordinal $\alpha < \gamma$, $\mathscr{L}_\alpha = \cup\{\mathscr{L}_\beta : \beta < \alpha\}$

$\mathscr{L}_T = \cup\{\mathscr{L}_\beta :\ \beta < \gamma\}$

3.1 Syntax

Next we define formulas and sentences of $\mathscr{L}_T$ recursively. Since we don't have functions, the terms in our language contain only constants and variables.

3.1 Syntax

Formu_0: $P^i_j c^0_{\perp k_1} \dots c^0_{\perp k_i}$ is a formula
$P^i_j x_{k_1} \dots x_{k_i}$ is a formula
If ϕ is a formula then $\neg\phi$ is a formula
If ϕ and φ are both formulas then $\phi \vee \varphi$ is a formula
If ϕ is a formula then $\forall^\alpha \phi$ is a formula, $\alpha < \gamma$
$\wedge$, $\rightarrow$, $\leftrightarrow$, and $\exists^\alpha$ can be defined as usual.

Formu_{n+1}: If ϕ is a formula in Formu_n then it is a formula in Formu_{n+1}
$Tc^i_{\phi j}$ is a formula where $c^i_{\phi j}$ is a constant in $\mathscr{L}_{n+1}$
$P^i_j c^{h_1}_{\phi k_1} \dots c^{h_i}_{\chi k_i}$ is a formula where $c^{h_1}_{\phi k_1} \dots c^{h_i}_{\chi k_i}$ are constans in $\mathscr{L}_{n+1}$
$P^i_j x_{k_1} \dots x_{k_i}$ is a formula
If ϕ is a formula then $\neg\phi$ is a formula
If ϕ and φ are both formulas then $\phi \vee \varphi$ is a formula
If ϕ is a formula then $\forall^\alpha \phi$ is a formula, $\alpha < \gamma$
$\wedge$, $\rightarrow$, $\leftrightarrow$, and $\exists^\alpha$ can be defined as usual.

For limit $\alpha < \gamma$, $\text{Formu}_\alpha = \cup\{\text{Formu}_\beta: \beta < \alpha\}$
$\text{Formu} = \cup\{\text{Formu}_\alpha : \alpha < \gamma\}$
A sentence is a formula belonging to Formu without free variables.

σ_T=<A_T, I_T, V_T> is any $\mathscr{L}_T$ model,

- For variable x_i, $V_T(x_i)$ is an element in A_T
- For constant $c^{\alpha}_{\phi\Delta}$ $I_T(c^{\alpha}_{\phi\Delta})$ is an element in A_T.
- For any predicate P^i_jwhich is not T, I_T (P^i_j)is a subset of $(A_T)^i$
- **For predicate T, $I_T(T)=\{I_T(c^{\alpha}_{\phi 1})$: $\sigma_T(\phi)=1\}$**
- For any formula $P^i_j c^{h_1}_{\phi k_1} \ldots c^{h_i}_{\phi k_i}$, $\sigma_T \models P^i_j c^{h_1}_{\phi k_1} \ldots c^{h_i}_{\phi k_i}$ iff $<\sigma_T(c^{h_1}_{\phi k_1}) \ldots \sigma_T(c^{h_i}_{\phi k_i})> \in \sigma_T(P^i_j)$
- For any formula $\neg\phi$, $\sigma_T \models \neg\phi$ iff $\sigma_T \not\models \phi$
- For any formula $\phi \vee \varphi$, $\sigma_T \models \sigma \vee \varphi$ iff $\sigma_T \models \phi$ or $\sigma_T \models \varphi$
- **For any formula $\forall^{\alpha}\phi$, $\sigma_T \models \forall^{\alpha}x\phi$ iff for any d∈ M^{α} $\sigma_T \models \phi(x/d)$, where $M^{\alpha}=\max\{M \subseteq A_T$: for any $\beta>\alpha$, $\sigma_T(c^{\beta}_{\phi\Delta}) \notin M\}$**

Axioms:

3.3 Axiomatization

Axioms:

(1) $\phi \rightarrow (\varphi \rightarrow \phi)$

(2) $(\phi \rightarrow (\varphi \rightarrow \chi)) \longrightarrow ((\phi \rightarrow \varphi) \rightarrow (\phi \rightarrow \chi))$

(3) $(\neg\phi \rightarrow \varphi) \rightarrow ((\neg\phi \rightarrow \neg\varphi) \rightarrow \phi)$

(4) $\phi \rightarrow T(c^{\alpha+1}_{\phi 1})$ α is the largest ordinal appears in the ϕ

(5) $\forall^{\alpha} x(\phi \rightarrow \varphi) \rightarrow (\forall^{\alpha} x\phi \rightarrow \forall^{\alpha} x\varphi)$

(6) $\forall^{\alpha} x\phi \rightarrow \forall^{\beta} x\phi$ $\beta \leq \alpha$

(7) $\phi \leftrightarrow \forall^{\alpha} x\phi$ x doesn't appear in the ϕ

(8) $t \equiv t$

(9) $t \equiv t' \rightarrow (Pt_1 \ldots t_{k-1} t t_{k+1} \ldots t_n \rightarrow Pt_1 \ldots t_{k-1} t' t_{k+1} \ldots t_n)$

(10) $\phi(x/t) \leftrightarrow \exists^{\alpha} x\phi(x)$, $t \neq c^{\beta}_{\phi\Delta}$ for any $\beta > \alpha$

(11) $\forall^{\alpha} x_1 \ldots \forall^{\beta} x_n \phi$ ϕ is a formula with one of the form of (1)–(10)

3.3 Axiomatization

Rule:

$\phi, \phi \rightarrow \varphi \vdash \varphi$ (Modus ponens)

Rules can be deduced:

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Rule:

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Rules can be deduced:

If $\Gamma \vdash A$ and $\Gamma \subseteq \Delta$ then $\Delta \vdash A$ (Structural rules)

If $\Gamma \vdash A$, $A \vdash B$ then $\Gamma \vdash B$ (Structural rules)

If Γ, $A \vdash C$, $B \vdash C$ then Γ, $A \vee B \vdash C$ (Disjunction elimination)

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Theorems can be proved:

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Theorems can be proved:

$\vdash \phi \vee \neg \phi$ (Excluded Middle)

$\phi \wedge \neg \phi \vdash \varphi$ (Explosion)

3.3 Axiomatization

Rule:

$\phi, \phi \to \varphi \vdash \varphi$ (Modus ponens)

Rules can be deduced:

If $\Gamma \vdash A$ and $\Gamma \subseteq \Delta$ then $\Delta \vdash A$ (Structural rules)

If $\Gamma \vdash A$, $A \vdash B$ then $\Gamma \vdash B$ (Structural rules)

If $\Gamma, A \vdash C$, $B \vdash C$ then $\Gamma, A \vee B \vdash C$ (Disjunction elimination)

Theorems can be proved:

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The following sentences aren't theorems:

3.3 Axiomatization

Rule:

$\phi, \phi \rightarrow \varphi \vdash \varphi$ (Modus ponens)

Rules can be deduced:

If $\Gamma \vdash$A and $\Gamma \subseteq \Delta$then $\Delta \vdash$A (Structural rules)

If $\Gamma \vdash$A, A$\vdash$B then $\Gamma \vdash$B (Structural rules)

If Γ, A $\vdash$C, B $\vdash$C then Γ, A$\vee$B$\vdash$C (Disjunction elimination)

Theorems can be proved:

$\vdash \phi \vee \neg \phi$ (Excluded Middle)

$\phi \wedge \neg \phi \vdash \varphi$ (Explosion)

The following sentences aren't theorems:

$\forall^{\alpha} x\phi \rightarrow \phi(x/t)$ (t is a term)

$\phi(x/t) \rightarrow \forall^{\alpha} x\phi$ (t is a term)

$\exists^{\alpha} x\phi \rightarrow \phi(x/t)$ (t is a term)

$\phi(x/t) \rightarrow \exists^{\alpha} x\phi$ (t is a term)

3.4 PA*

Because the language $\mathscr{L}_T$ doesn't have functions and the constants are special in my syntax I use the constant $c^0_{\perp 0}$ as the numeral 0, $c^0_{\perp i}$ as the numeral i, the predicate P^2_1 as the consequent relation, P^3_1 as the addition relation, P^3_2 as the multiplication relation. Then the axioms of PA are shown as follows:

(1) $\neg P^2_1 x_i c^0_{\perp 0}$

(2) $P^2_1 x_i x_j \wedge P^2_1 x_k x_j \rightarrow x_i = x_k$

(3) $P^3_1 x_1 c^0_{\perp 0} x_1$

(4) $P^2_1 x_2 x_3 \wedge P^3_1 x_1 x_2 x_4 \wedge P^2_1 x_4 x_5 \rightarrow P^3_1 x_1 x_3 x_5$

(5) $P^3_2 x_1 c^0_{\perp 0} c^0_{\perp 0}$

(6) $P^2_1 x_2 x_3 \wedge P^3_2 x_1 x_2 x_4 \wedge P^3_1 x_4 x_1 x_5 \rightarrow P^3_2 x_1 x_3 x_5$

3.4 PA*

Since the quantifiers change the induction axiom has to be transformed too:

$\forall^{\alpha} Q(Q(c^{0}_{\perp 0}|_{\alpha}) \wedge \forall^{\alpha} x_i \forall^{\alpha} x_j (Qx_i \wedge P^2_1|_{\alpha}(x_i x_j) \rightarrow Qx_j) \rightarrow \forall^{\alpha} x_i Qx_i)$

The $\forall^{\alpha} Q\phi$, as a second order quantified formula, means that for any subset $\sigma_T(Q)$ of M^{α}, ϕ holds, where M^{α}=max{$M \subseteq A_T$: for any $\beta > \alpha, \sigma_T(c^{\beta}_{\phi\Delta}) \notin M$}. $c^{0}_{\perp 0}|_{\alpha}$ is the least number in M^{α}, and $P^2_1|_{\alpha}$ $(x_i x_j)$ means x_j is the least number in M^{α} except x_i.

3.4 PA*

These transformed arithmetic axioms are all valid in the standard model.

Because of the change of the form of the induction axiom the Peano arithmetic is called PA* rather than PA in my system.

Proof. It is straightforward to verify that these axioms are all true, and that the rule preserves truth. ■

3.6 Completeness

The proof of the completeness of this system is almost the same as that of the first order logic. The only differences are the definition of the set of witness and the steps relating to quantified sentences.

It seems that there is no difference between the two schemas except the change of the symbol. However, you will see that the meaning of the change is not only the variation of the symbols but also that the (T^*) schema, not the (T_n) schemas, will be valid under certain condition without the trouble of the liar paradox.

Obviously, according to the explanation of the predicate T in the semantics, the (T^*) schema isn't valid. For example, let τ be a $\mathscr{L}_T$-model. Then let $\tau(c^{\alpha}_{\phi 1})=\tau(c^{\alpha}_{\neg\phi 1})$. If (T^*) schema hold, we will get $\phi \leftrightarrow \neg\phi$ from the (T^*) schema and the explanation of T, which is a contradiction. So we need some restrictions on the (T^*) schema.

Definition 3.6.1

Definition

Let Γ be a set of consistent sentences. We claim that Γ contains witness of $\mathscr{L}_T$ if there is a set D of constants such that for any formula $\varphi(x)$ of $\mathscr{L}_T \cup D$ which has at most one free variable, if $\exists^\alpha x\varphi(x) \in \Gamma$ then there is a constant $d \in D$ such that $\{\varphi(x/d)\} \cup \{d \neq c^{\beta}_{\phi\Delta} : \beta > \alpha\} \subseteq \Gamma$. D is called the set of witness of Γ.

Lemma 3.6.2

Lemma

Let W be a consistent theory of $\mathscr{L}_T$, D be a set of new constants such that $D \cap \mathscr{L}_T = \emptyset$, $\mathscr{L}' = \mathscr{L}_T \cup D$ and $|D| = |\mathscr{L}_T|$. There is a consistent set W' of sentences of $\mathscr{L}'$ such that $W \subseteq W'$ and D is the set of witness of W'.

Proof. Suppose λ=|D|=| $\mathscr{L}_T$ | and D={d_ξ:$\xi < \lambda$}. Let {φ_ξ: $\xi < \lambda$} be the set of formulas of $\mathscr{L}_T \cup$D with at most one free variable. Then we define sets W_ξs, $\xi < \lambda$, as follows:

(1) W_0=W.

(2) $W_{\xi+1} = W_\xi \cup \{\exists^\alpha x \varphi_\xi(x)\} \cup \{ \varphi_\xi(x/d_\eta)\} \cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta > \alpha \}$ if $\{\exists^\alpha x \varphi_\xi(x)\} \cup W_\xi$ is consistent, where d_η is the first constant which doesn't appear in W_ξ and φ_ξ;

$W_{\xi+1} = W_\xi$ if $\{\exists^\alpha x \varphi_\xi(x)\} \cup W_\xi$ is inconsistent.

(3) For limit ξ, $W_\xi = \cup_{\beta<\xi} W_\beta$.

Obviously, $W_0 \subseteq W_1 \subseteq$.... Let $W' = \cup_{\xi<\lambda} W_\xi$.

Next I will prove inductively that W' is consistent.
(1) W_0=W, according to the hypothesis W_0 is consistent.
(2) Suppose W_ξ is consistent. If $\{\exists^\alpha x\varphi_\xi(x)\}\cup W_\xi$ is inconsistent then $W_{\xi+1}= W_\xi$. Hence $W_{\xi+1}$ is consistent. If $\{\exists^\alpha x\varphi_\xi(x)\}\cup W_\xi$ is consistent then $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\}\cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta>\alpha\}$ is consistent. Assume it's not the case i.e. $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\}\cup \{d_\eta \neq c^\beta_{\phi\Delta}: \beta>\alpha\}$ is inconsistent. Because d_η is a new constant there must be $\lambda>\alpha$ such that $W_\xi\cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \forall^\lambda x(x=c^\lambda_{\phi\Delta})$. Since $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \exists^\alpha x\varphi(x)$. Hence we have $W_\xi \cup \{\exists^\alpha x\varphi_\xi(x)\} \vdash \forall^\lambda x(x=c^\lambda_{\phi\Delta})\wedge \exists^\alpha x\varphi(x)$.

However, $\forall^{\lambda}x(x=c^{\lambda}_{\phi\Delta})\wedge\exists^{\alpha}x\varphi(x)$ is an absurdity which is false under every $\mathscr{L}_T$ model. Hence $W_\xi\cup\{\exists^{\alpha}x\varphi_\xi(x)\}$ is inconsistent. This contradict with the hypothesis. So $W_\xi\cup\{\exists^{\alpha}x\varphi_\xi(x)\}\cup\{d_\eta\neq c^{\beta}_{\phi\Delta}:\beta>\alpha\}$ is consistent. Since $\{d_\eta\neq c^{\beta}_{\phi\Delta}:\beta>\alpha\}\vdash\exists^{\alpha}x\varphi_\xi(x)\rightarrow\varphi_\xi(x/d_\eta)$ is valid $W_\xi\cup\{\exists^{\alpha}x\varphi_\xi(x)\}\cup\{d_\eta\neq c^{\beta}_{\phi\Delta}:\beta>\alpha\}\vdash\exists^{\alpha}x\varphi_\xi(x)\rightarrow\varphi_\xi(x/d_\eta)$ is valid too. And then $W_\xi\cup\{\exists^{\alpha}x\varphi_\xi(x)\}\cup\{d_\eta\neq c^{\beta}_{\phi\Delta}:\beta>\alpha\}\vdash\varphi_\xi(x/d_\eta)$ is valid. Hence $W_\xi\cup\{\exists^{\alpha}x\varphi_\xi(x)\}\cup\{\varphi_\xi(x/d_\eta)\}\cup\{d_\eta\neq c^{\beta}_{\phi\Delta}:\beta>\alpha\}$ is consistent.

(3) For limit λ, if W_α is inconsistent then there is a $\alpha<\lambda$ such that W_α is inconsistent. However, by induction hypothesis, for any $\alpha<\lambda$, W_α is consistent. So W_λ is consistent.

■

Lemma 3.6.3

Lemma

Suppose W is a consistent theory of $\mathscr{L}_T$, D a set of new constants. Then W has a model $\mathfrak{A}$ such that every element of $\mathfrak{A}$ is an explanation of a constant in D∪C i.e. A={$a^{\mathfrak{A}}$: a∈ D∪C } where C is the set of constants in $\mathscr{L}_T$, and $\mathfrak{A} \models W$.

Proof. According to Lindenbaum Theorem W can be extended to a maximal consistent set W′ of sentences of $\mathscr{L}_T$. Then there is a language $\mathscr{L}'_T$ such that $\mathscr{L}'_T = \mathscr{L}_T \cup D$, D $\cap\ \mathscr{L}_T = \varnothing$ and $|D| = |\mathscr{L}_T|$. In line with 3.6.1 W′ can be extended to a set W″ of sentences of $\mathscr{L}_T{}'$ with D as the set of witness. Then we can extended W″ to a maximal consistent set W* of sentences of $\mathscr{L}_T{}'$ with D as the set of witness according to Lindenbaum Theorem.
Defining the binary relation ∼ as follows: for any $a_1, a_2 \in D \cup C$, $a_1 \sim a_2$ iff $T \vdash a_1 = a_2$. For $a \in D \cup C$, $\tilde{a} = \{a' \in D \cup C : a \sim a'\}$ i.e. $\tilde{a}$ is the equivalence class of the constant a. Defining a model $\mathfrak{A} = \langle A, I \rangle$ as follows: $A = \{\tilde{a} : a \in D \cup C\}$, for any $a \in D \cup C$, $I(a) = \tilde{a}$; for any predicate P^n_i and $\widetilde{a_1} \ldots \widetilde{a_n} \in A$

$$W^* \vdash P^n_i(a_1 \ldots a_n) \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1} \ldots \widetilde{a_n}]$$

Next we will prove inductively that for any formula $\varphi(x_1 ... x_n)$ of $\mathscr{L}_T \cup D$ and $\widetilde{a_1} ... \widetilde{a_n} \in A$,

$$W^* \vdash \varphi(a_1 ... a_n) \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1} ... \widetilde{a_n}]$$

(1) If $\varphi(x_1 ... x_n)$ is atomic formula $P_i^n(x_1 ... x_n)$, according to the explanation of the model for predicate we have $a_1 ... a_n \in D \cup C$ such that

$$W^* \vdash P_i^n(a_1 ... a_n \text{ iff } \mathfrak{A} \models P_i^n(x_1 ... x_n)[\widetilde{a_1} ... \widetilde{a_n}] \text{ iff } \mathfrak{A} \models \varphi[\widetilde{a_1} ... \widetilde{a_n}]$$

(2) It is easy to prove that the equivalence holds when $\varphi(x_1 ... x_n)$ is a negative formula or a disjunctive formula.

(3) Suppose $\varphi(x_1...x_n)$ is a formula with the form $\exists^{\alpha} y\psi(x_1...x_n\ y)$. If $\mathfrak{A} \models \varphi[\widetilde{a_1}...\widetilde{a_n}]$, then there is a $\tilde{d} \in M^{\alpha}$ such that $\mathfrak{A} \models \psi[\widetilde{a_1}...\widetilde{a_n}\ \tilde{d}]$.According to the induction hypothesis we have $W^* \vdash \psi(a_1...a_n\ d)$. Since $\tilde{d} \in M^{\alpha} = \max\{M \subseteq A_T$: for any $\beta > \alpha, \sigma_T(c^{\beta}_{\phi\Delta}) \notin M\}$ we have $\tilde{d} \neq \widetilde{c^{\beta}_{\phi\Delta}}$ for any $\beta > \alpha$. And then $d \neq c^{\beta}_{\phi\Delta}$ for any $\beta > \alpha$. Hence $W^* \vdash \exists^{\alpha} y\psi(a_1...a_n\ y)$

If $W^* \vdash \exists^\alpha y\psi(a_1 ... a_n\ y)$, then in accordance with the definition 3.6.1 there is a $d \in D \cup C$ such that $\{\exists^\alpha y\psi(a_1 ... a_n\ y)\} \cup \{\psi(a_1 ... a_n\ d)\} \cup \{d \neq c^\beta_{\phi\Delta} : \beta > \alpha\} \subseteq W^*$. Then we get $W^* \vdash \psi(a_1 ... a_n\ d)$ and $W^* \vdash d \neq c^\beta_{\phi\Delta}$ for any $\beta > \alpha$. By induction hypothesis $\mathfrak{A} \models \psi(a_1 ... a_n\ d)$ and $\mathfrak{A} \models d \neq c^\beta_{\phi\Delta}$ for any $\beta > \alpha$. Hence $d^{\mathfrak{A}} \in M^\alpha$ and then $\mathfrak{A} \models \exists^\alpha y\psi(a_1 ... a_n y)$. ■

3.6.4 Theorem

Theorem

Suppose Σ is a set of formulas of $\mathscr{L}_T$ and φ a formula of $\mathscr{L}_T$. If $\Sigma \models \varphi$ then $\Sigma \vdash \varphi$. Especially, if $\models \varphi$ then $\vdash \varphi$. And if Σ is consistent then there is a model of Σ.

Proof. Suppose $\Sigma \models \varphi$ but $\Sigma \nvdash \varphi$, then obviously $\Sigma \cup \{\neg \varphi\}$ is consistent. Let E be a set of new constants such that $|E|=|\mathscr{L}_T|$ and $E\cap\mathscr{L}_T=\varnothing$. Then arrange the formulas in $\Sigma \cup \{\neg \varphi\}$ in a linear order. And then substitute the new constants in E for the variables in $\Sigma \cup \{\neg \varphi\}$ in such a way that in the same formula we use the same constant to substitute for the same variable, different constants for different variables, in different formulas we use constants in E that haven't appeared in previous formulas to substitute, for sentences in Σ we reserve their form.

Let $W \cup \{\neg\varphi'\}$ be the set of sentences replacing the formulas of $\Sigma \cup \{\neg\varphi\}$ in the above way, therefore $W \cup \{\neg\varphi'\}$ is a set of sentences of of $E \cup \mathscr{L}_T$. Let D be an arbitrary set of new constants such that $D \cap \mathscr{L}_T = E \cap \mathscr{L}_T = D \cap E = \varnothing$ and $|D| = |\mathscr{L}_T|$. In accordance with lemma 3.6.2 $W \cup \{\neg\varphi'\}$ can be extended to a consistent set W' with D as the set of witness. Then according to lemma 3.6.3 W' has a model $\mathfrak{A}'$. Let $\mathfrak{A}$ be $\mathfrak{A}'|_{\mathscr{L}_T \cup E}$, then $\mathfrak{A} \models W \cup \{\neg\varphi'\}$. And then there is a model $\mathfrak{B}$ such that for variable x_i appears in a formula in $\Sigma \cup \{\neg\varphi\}$, let $(x_i)^{\mathfrak{B}} = (e_i)^{\mathfrak{A}}$ where e_i is the constant replacing x_i in the corresponding formula in $W \cup \{\neg\varphi'\}$. For constant t and predicate Q, let $t^{\mathfrak{B}} = t^{\mathfrak{A}}$ and $Q^{\mathfrak{B}} = Q^{\mathfrak{A}}$. Obviously, for any formula ψ in $\Sigma \cup \{\neg\phi\}$, $\mathfrak{B} \models \psi$. Therefore there is a model $\mathfrak{B}$ such that $\mathfrak{B} \models \Sigma$ and $\mathfrak{B} \models \neg\varphi$ i.e. $\mathfrak{B} \models \Sigma$ and $\mathfrak{B} \not\models \varphi$. However, this contradicts with $\Sigma \models \varphi$. So if $\Sigma \models \varphi$ then $\Sigma \vdash \varphi$.

■

The usual understanding of (T) schema is like this: $T(\overline{\ulcorner\phi\urcorner}) \leftrightarrow \phi$. Here the $\overline{\ulcorner\phi\urcorner}$ refers to the Gödel number of the sentence ϕ.

However, in my proposal, I will change the schema a little. My (T) schema is like this: $T(c_{\phi 1}^{\alpha+1}) \leftrightarrow \phi$ (α is the largest ordinal appears in ϕ). In order to distinguish the two schemas I will call my schema (T^*) schema.

It seems that there is no difference between the two schemas except the change of the symbol. However you will see that the meaning of the change is not the variation of the symbols but that the (T^*) schema will be valid without the trouble of liar paradoxes under certain conditions Obviously, according to the explanation of the predicate T in the semantics, the (T^*) schema isn't valid. For example, let τ be a $\mathscr{L}_T$ model. Then let $\tau(c^{\alpha}_{\phi 1})=\tau(c^{\alpha}_{\neg\phi 1})$. If (T^*) schema hold, we will get $\phi \leftrightarrow \neg \phi$ from the (T^*) schema and the explanation of T. So we need some restrictions on the (T^*) schema.

Definition 3.7

Definition

Let σ_{T^*} be a model class, σ_{T^*} =$\{\sigma_T$: for any constants $c^{\alpha}_{\phi\Delta}$, $c^{\beta}_{\varphi\Delta}$, if σ_T $(c^{\alpha}_{\phi\Delta})$=$\sigma_T(c^{\beta}_{\varphi\Delta})$ then $\phi \leftrightarrow \varphi\}$.

Theorem 3.8

Theorem

(T^*) schema is valid in the model class σ_{T^*}.

Proof. We already have $\phi \rightarrow \mathrm{T}(c^{\alpha+1}_{\phi 1})$, α is the largest superscript appears in ϕ, as our axiom, so we need only prove the right to left side of the equivalence. If this implication is false, then we have a model τ_T belonging to σ_{T^*} such that $\tau_T \models \mathrm{T}(c^{\alpha+1}_{\phi 1})$ but $\tau_T \not\models \phi$. Since $\tau_T \models \mathrm{T}(c^{\alpha+1}_{\phi 1})$, according to the semantics of the language, $\tau_T(c^{\alpha+1}_{\phi 1}) \in \tau_T(\mathrm{T})$. Hence there is a constant $c^{\beta}_{\varphi 1}$ such that $\tau_T(c^{\beta}_{\varphi 1}) \in \tau_T(\mathrm{T})$, $\tau_T(\varphi)=1$, and $\tau_T(c^{\beta}_{\varphi 1}) = \tau_T(c^{\alpha+1}_{\phi 1})$. Then, according to the special requirement of the model class σ^*_T, we have $\tau_T(\phi)=\tau_T(\varphi)=1$. Hence $\tau_T \models \phi$. This is contradictory to the hypothesis $\tau_T \not\models \phi$. So $\sigma^*_T \models \phi \leftrightarrow \mathrm{T}(c^{\alpha+1}_{\phi 1})$ (α is the largest superscript appears in ϕ). ■

Theorem 3.9

Theorem

Let $\gamma = \omega$. For any formula ϕ of $\mathscr{L}_T$ and any $\tau \in \sigma_{T^*}$ there is a translation $Tran(\phi)$ from mathcallL_T to $\mathscr{L}_{Tarski}$ such that

$$\text{if } \tau \models \phi, \text{ then } \tau \models Tran(\phi)$$

Proof. First let us define a translation from $\mathscr{L}_T$ to $\mathscr{L}_{Tarski}$.

(1)Tran($\bot$)=$t_1 \neq t_1$

(2)For predicate $P_n \neq T$, Tran($P_n x_i$)= $P_n x_i$

(3)For predicate $P_n \neq T$, Tran($P_n(c^{\alpha}_{\phi\Delta})$)=P(Tran($\overline{\ulcorner \phi \urcorner}$))

(4)For predicate T, if x_i is not bounded by quantifier, Tran(T(x_i))= $T^{\alpha}(x_i)$, if there is a β such that $\tau(c^{\beta}_{\phi\Delta})=\tau(x_i)$ and α=min{β: $\tau(c^{\beta}_{\phi\Delta})=\tau(x_i)$}; if there is not a β such that $\tau(c^{\beta}_{\phi\Delta})=\tau(x_i)$, Tran(T($x_i$))=$t_2 \neq t_2$

(5)For predicate T, if x_i is bounded by quantifier $\forall^{\beta} x_i$ then Tran(T(x_i))=$T^{\beta}(x_i)$

(6)For predicate T, Tran(T($c^{\alpha}_{\phi\Delta}$))=$T^{\alpha}(\overline{\ulcorner Tran(\phi) \urcorner})$

(7) Tran($\neg \phi$)=$\neg$Tranϕ

(8) Tran($\phi \vee \varphi$)=Tran(ϕ)$\vee$Tran(φ)

(9) Tran($\forall^n x\phi$)=$\forall x_i \in Q_n$Tran(ϕ)

Theorem 3.9

Obviously, if ϕ is a formula of $\mathscr{L}_n$ in the construction of the language $\mathscr{L}_T$, then Tran(ϕ) is a formula of $\mathscr{L}_m$ in the construction of $\mathscr{L}_{Tarski}$ where m<n.

Then we can prove the theorem by induction.

Let $\tau(Q_n)$=max{$Q \subseteq A_T$: for any $\beta > \alpha, \tau(c^{\beta}_{\phi\Delta}) \notin Q$}. Let

$\tau(c^{\alpha}_{\phi\Delta}) = \tau(\overline{\ulcorner Tran(\phi) \urcorner})$

(1) Consider atomic formula,

For predicate $P_n \neq T$, if $\tau \models P_n(x_i)$, because Tran($P_n x_i$)= $P_n x_i$, $\tau \models$Tran ($P_n x_i$)

For predicate $P_n \neq T$, if $\tau \models P_n(c^{\alpha}_{\phi\Delta})$, then $\tau(c^{\alpha}_{\phi\Delta}) \in \tau(P_n)$, since $\tau(c^{\alpha}_{\phi\Delta}) = \tau(\overline{\ulcorner Tran(\phi) \urcorner})$, $\tau \models P_n(\overline{\ulcorner Tran(\phi) \urcorner})$.

Theorem 3.9

For predicate T and variable x_i which isn't bounded by quantifier, if $\tau \models T(x_i)$ then according to the definition of T there is a least β such that $\tau(x_i)=\tau(c^{\beta}_{\phi 1})$, then according to the definition of the translation $Tran(T(x_i))= T^{\beta}(x_i)$. According to the construction of the two languages, there is a ϕ in $\mathscr{L}_{\beta-1}$ such that $\tau \models \phi$. Through induction hypothesis $\tau \models Tran(\phi)$. Then according to the definition of T^{β}, $\tau \models T^{\beta}(\overline{\ulcorner Tran(\phi)\urcorner})$. Since $\tau(x_i)=\tau(c^{\beta}_{\phi 1})$ and $\tau(c^{\alpha}_{\phi\Delta})=\tau(\overline{\ulcorner Tran(\phi)\urcorner})$, $\tau \models T^{\beta}(x_i)$.

For predicate T and constant $c^{\alpha}_{\phi\Delta}$, if $\tau \models T(c^{\alpha}_{\phi\Delta})$ then according to the requirement of the model class σ^{*}_{T}, $\tau \models \phi$. Through induction hypothesis, $\tau \models Tran(\phi)$. Because of the construction of the two languages ϕ belongs to $L_{\alpha-1}$. Hence $\tau(\overline{\ulcorner Tran(\phi)\urcorner}) \in \tau(T^{\alpha})$. So $\tau \models Tran(\phi)$

Theorem 3.9

(2) The Boolean cases when $\phi = \neg\varphi$ and $\phi = \varphi \vee \chi$ are easy to prove by induction hypothesis.

(3) $\phi = \forall^{\alpha} x \varphi$. Suppose $\tau \models \forall^{\alpha} x \varphi$. According to the semantics of the L_T, for any d$\in M^n$, $\sigma_T \models \phi(x/d)$, where $M^n = \max\{M \subseteq A_T$: for any $\beta > \alpha$, $\sigma_T(c^{\beta}_{\phi\Delta k_i}) \notin M\}$. By induction hypothesis $\tau \models Tran(\phi)$, so $\tau \models \forall x_i \in Q_n Tran(\phi)$ ■

Theorem 3.9

This theorem assert that *in some sense*, when $\mathscr{L}_T$ can say (T*) schema, $\mathscr{L}_T$ is a fragment of $\mathscr{L}_{Tarski}$, its expressiveness is no more than that of $\mathscr{L}_{Tarski}$. Actually in this sense the expressiveness of $\mathscr{L}_T$ is less than that of $\mathscr{L}_{Tarski}$'s because there are some sentences of $\mathscr{L}_{Tarski}$ cannot be translate into sentences of $\mathscr{L}_T$, for example $\exists x \phi(x)$ cannot be translated into $\mathscr{L}_T$ sentence.

4 Examination

About example (5)

First, I want to say something about the example (5). I don't think that contingent paradox is a real problem. I think that depending on environments is just like depending on models, the same sentence is or isn't a paradox because of different environments is just like the same sentence has different value under different models. In strict artificial language the "environment" is fixed, so whether a sentence is a paradox is also fixed. So it's not our work to make sure whether a sentence is a paradox or not. What we need to do is to treat it when we know a sentence is a paradox. Suppose (1) and (2) in this example are both paradoxes. Then this example is like the example (2). We leave the solution to this example until we handle example (2)

About example (1)

Let's look at the example (1)

$$(1)\text{: } (1) \text{ is not true}$$

or using my language $\mathscr{L}_T$:

$$c^{\alpha}_{\phi\Delta}\text{: } \neg T\,(c^{\alpha}_{\phi\Delta})$$

Remember the reasoning in example(1). Suppose $c^{\alpha}_{\phi\Delta}$ is not true. Then we have $\neg T\,(c^{\alpha}_{\phi\Delta})$. If we want to get the contradiction we must use the intersubstitutivity and the (T) schema.

About example (1)

However, the (T) schema here is (T*) schema i.e. T($c^{\alpha}_{\phi 1}$)$\leftrightarrow \phi$. And this schema is not valid generally but valid only in a model class. What is valid generally here is $\phi \rightarrow$ T($c^{\alpha}_{\phi 1}$). Through it, what we get is $\neg\phi$. In order to get the contradiction we have to suppose ϕ is $\neg$T ($c^{\alpha}_{\phi\Delta}$). This is impossible according to the construction of the language $\mathscr{L}_T$. The process of the construction of the language shows that we first have the sentence ϕ in $\mathscr{L}_\alpha$ then we have the constant $c^{\alpha+1}_{\phi 1}$ in the $\mathscr{L}_{\alpha+1}$. So ϕ cannot be a sentence containing itself as a part in it. Suppose $c^{\alpha}_{\phi\Delta}$ is true. In order to get the contradiction we need T(a)$\rightarrow \phi$ to be valid where a is the name of ϕ. However this is not valid here. (Even if we talk about this example under the model class σ_{T*}, what we get is T($c^{\alpha}_{\phi\Delta}$)$\rightarrow \phi$. However, in this environment we cannot conclude the contradiction because of the construction of the language $\mathscr{L}_T$)

About example (2)

Example (2):

$$c^{\alpha}_{\phi 1}\colon c^{\beta}_{\varphi 1} \text{ is not true}$$
$$c^{\beta}_{\varphi 1}\colon c^{\alpha}_{\phi 1} \text{ is true}$$

Let's start from $c^{\beta}_{\varphi 1}$ *is not true* i.e. $\neg T(c^{\beta}_{\varphi 1})$. Then if we want to get $\neg(c^{\alpha}_{\phi 1}$ is true) we have to use (T) schema. However what is valid is $\phi \rightarrow T(c^{\alpha}_{\phi 1})$ i.e. $\neg T(c^{\alpha}_{\phi 1}) \rightarrow \neg\, \phi$. So we can only get $\neg\, \varphi$ from $\neg T(c^{\beta}_{\varphi 1})$. So in order to construct the paradox, φ must be the sentence $c^{\alpha}_{\phi 1}$ *is true*. So according to the construction of the language $\mathscr{L}_T$ we know $\beta > \alpha$. And then in order to get contradiction we must go through from $\neg(c^{\alpha}_{\phi 1}$ is true) to $\neg$ $(c^{\beta}_{\varphi 1}$ is not true) i.e. we have to use $T(a) \rightarrow \phi$ where a is the name of ϕ. But this is not valid in our system. (Even under model class σ_{T*}, we cannot get paradox because of the construction of the language.)

About example (3)

Example (3), Curry paradox. Let's first translate the Curry sentence into our language. Let ϕ be equivalent to the next sentence

$$T(c^{\alpha}_{\phi 1}) \rightarrow \perp$$

This is not possible according to the construction of the language $\mathscr{L}_T$. Because $c^{\alpha}_{\phi 1}$ has already appeared in the sentence, if we let ϕ be the sentence $T(c^{\alpha}_{\phi 1}) \rightarrow \perp$, that means we have $c^{\alpha+1}_{\phi 1}$ but we already have $c^{\alpha}_{\phi 1}$. According to the construction of the language this is not possible. So we have to let φ be $T(c^{\alpha}_{\phi 1}) \rightarrow \perp$. If we want to infer as we do in the example we have to use (T) schema in the 2nd step of the deduction. But the implication $T(a) \rightarrow \phi$ is not valid in my proposal. So the inference of this example is not true.

About example (4)

Example (4), Yablo's paradox. First Yablo's paradox cannot appear in my language because we don't have the general rule $\forall^-$. This is one of the limit of $\mathscr{L}_T$'s expressive power. However, even if we suppose that we have that rule, this paradox can't emerge. The key step in the inference is the one from S_k *is untrue* to $\neg S_k$. This step is legal in my proposal. According to the construction of the language $\mathscr{L}_T$, for any (S_α) and (S_β), if $\alpha > \beta$ then S_α appears in a lower language $\mathscr{L}_\alpha$ than S_β which appears in a higher language $\mathscr{L}_\beta$. If, because of the construction of the language $\mathscr{L}_T$, the sequence is infinite then we will have a infinite descend chain which contradicts with the well-ordering theorem. And hence it contradict with the construction of the language $\mathscr{L}_T$.

About example (6)

Example (6), the Tarski's undefinability theorem. The key of the proof of this theorem is the construction of the Gödel sentence. First we need an open sentence with one variable like this

$$\exists x(\ x \text{ is the self-application of } v \wedge P(x))$$

where P(x) is any open sentence with x as the only free variable, and the "self-application" means substituting the name (or the Gödel number) of the open sentence for all free occurrence of v in it. Then let D(v) be the open sntence and $<D(v)>$ be the name of it. Let S be the self-application of D(v), i.e. the sentence

$$\exists x(x \text{ is the self-application of } <D(x)> \wedge P(x))$$

But $<S>$, the name of S, is the unique self-application of $<D(x)>$. Then we can get

$$S \leftrightarrow \exists x(x=<S> \wedge P(x))$$

And hence

$$S \leftrightarrow P(<S>)$$

If we change the predicate P() into $\neg$T() and together with the usual (T) schema, we can get $T(<S>) \leftrightarrow \neg T(<S>)$. This is a contradiction.

About example (6)

However, this construction doesn't hold in my proposal. Because we have many existential quantifiers $\exists^{\alpha}$s rather than $\exists$, the well-formed formula is

$$\exists^{\alpha}x(x \text{ is the self-application of } v \wedge P(x))$$

so after substituting $<D(v)>$ (or $c_{\phi 1}^{\alpha+1}$) for v we get

$$\exists^{\alpha}x(x \text{ is the self-application of } <D(v)> \wedge P(x))$$

In order to use the theorem related to T predicate, we need the constant $c_{\phi 1}^{\alpha+1}$ where ϕ is the sentence $\exists^{\alpha}x$(x is the self-application of v $\wedge P(x)$). Then we have

$$\phi \leftrightarrow \exists^{\alpha}x(x=c_{\phi 1}^{\alpha+1} \wedge P(x))$$

The sentence $\exists^{\alpha}x(x=c_{\phi 1}^{\alpha+1} \wedge P(x))$ is always false since there is no x in the field $\max\{M \subseteq A_T\text{: for any } \beta>\alpha, \sigma_T(c_{\phi\Delta k_i}^{\beta}) \notin M\}$ that is $\sigma_T(c_{\phi 1}^{\alpha+1})$ according to our semantic explanation of existential sentence. So we cannot use $P(c_{\phi 1}^{\alpha+1})$ as the abbreviation of $\exists^{\alpha}x(x=c_{\phi 1}^{\alpha+1} \wedge P(x))$. Hence, after substituting $\neg T()$ for $P()$, we can't get the contradiction $\phi \leftrightarrow \neg T(c_{\phi 1}^{\alpha+1})$. But we can get

$$\exists^{\alpha}x(x=c_{\phi 1}^{\alpha+1} \wedge \neg T(x)) \leftrightarrow \exists^{\alpha}x(x=c_{\phi 1}^{\alpha+1} \wedge T(x))$$

since they are both false.

5 Conclusion

Conclusion

In conclusion, the (T) schema is not valid generally, but valid in a model class. I haven't denied direct or indirect self-reference. What I do is that the theorem related to T predicate i.e. $\phi \rightarrow \mathrm{T}(\mathrm{c}^{\alpha}_{\phi 1})$ cannot be used in the self-referential sentence. We can construct self-referential sentence and use the theorem separately. But, according to the form of the theorem, we cannot use it in self-referential sentence. Besides, the other side of the (T) schema i.e. $\mathrm{T}(\mathrm{c}^{\alpha}_{\phi 1}) \rightarrow \phi$ doesn't hold in my proposal. So the problems of the liar paradoxes and the liar-like paradox i.e. Yablo's paradox which lies in the conjunction of self-reference and (T) schema is solved. And the Tarski's undefinability theorem which also lie in the construction of the sentences and (T) schema is avoided. Besides we have only one truth predicate rather than many true_n and our semantics is bivalent.

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Thank you for your attention!