# Natural Logic and Its Applications

Chao Xu

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PEKING UNIVERSITY

# Outline

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# Motivation

On the way to ordinary reasoning, I ran into natural logic.

# What is Natural Logic?

Natural Logic:

1. George Lakoff, 1970.
2. Johan van Benthem, 1983, 1986, 1987, 2008; Jan van Eijck, 2007; 刘新文, 2009; 张立英, 2009, 2012.
3. Larry Moss, 2015. (2008, 2010, 2011, 2012, 2014)
4. 王雨田, 1987; 夏年喜, 2004, 2006;

The Logic of Natural Language:

1. Fred Sommers, 1982.
2. J. Anthony Blair and Ralph H. Johnson.

# George Lakoff, 1970

Lakoff call all any logic meeting the goals of (i)-(v) a 'natural logic':

1. We want to understand the relationship between grammar and reasoning.
2. We require that significant generalizations, especially linguistic ones, be stated.
3. On the basis of (i) and (ii), we have been led tentatively to the generative semantics hypothesis. We assume that hypothesis to see where it leads.
4. We want a logic in which all the concepts expressible in natural language can be expressed unambiguously, that is, in which all non synonymous sentences (at least, all sentences with different truth conditions) have different logical forms.
5. We want a logic which is capable of accounting for all correct inferences made in natural language and which rules out incorrect ones (cf. Linguistics and Natural Logic)

- The construction of a full, nonfragmental natural logic is not an immediate practical goal. In fact, it may not even be a possible goal.
- Assumptions (ii) and (iii) require that a full, descriptively adequate grammar of English is required for there to be a natural logic. That is, all relevant generalizations concerning the relation between logical forms and surface forms must be known.
- No one is about to successfully construct a full natural logic.

# Larry Moss's Review

*Lakoff (1970) is the most prominent antecedent in the linguistic literature. Despite its title, "Linguistics and Natural Logic", the paper is unfortunately not connected to the topic of this chapter. It is a defence of generative semantics; its main points are about the relation of the syntactic and semantic enterprises in generative grammar. In addition, it contains many interesting examples and insightful discussions. However, it is not directly concerned with the issues we raise here. (cf. Moss, 2015)*

# Fred Sommers's Review

*.....The naturalist believes with Aristotle and Leibniz that logical syntax is implicit in the grammar of natural language and that the structure attributed by grammarians to sentences of natural language is in close correspondence to their logical form....... ...... If natural language can lay claim to a syntax that is adequate for logical purpose then this can be taken to mean that the syntax currently attributed to it by most grammarians is faulty and superficial...... (Sommers, 1982)*

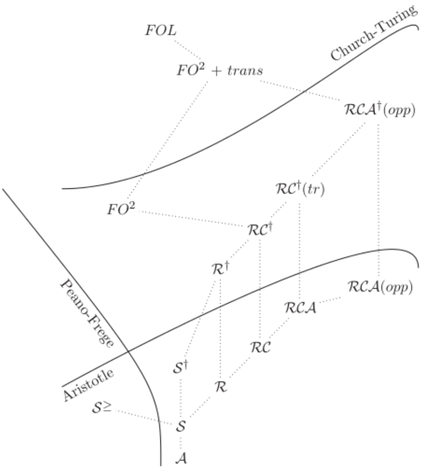
# Johan van Benthem, 1983, 1986,1987, 2008

- 'Natural Logic' is a somewhat loose, but popular and suggestive term for recurrent attempts over the last decades at describing basic patterns of human reasoning directly in natural language without the intermediate of some formal system. (cf. Benthem, 2008)
- Natural logic is in closer contact with natural language surface forms than its modern Fregean successor. (cf. Benthem, 1986)
- 近十几年来，热门尝试不以形式语言为中介，而是重新直接用自然语言来研究人类推理的基本模式，这些尝试被称为 "自然逻辑"。(cf. 张立英, 2009)
- 自然语言不仅仅是说话和交流的媒介，它自身还有一个 "自然逻辑"，人们重新用这个简单的、基于 "表层语法" 推理的演算的想法，尝试直接用自然语言为中介研究人类推理的基本模式。(cf. 刘新文, 2009)

# Larry Moss, 2015

- There is not exactly a well established field of natural logic.
- The subject of natural logic might be defined as "logic for natural language, logic in natural language". By this, we aim to find logical systems that deal with inference in natural language, or something close to it.
- As a methodological point, he use the modern logic research methods and prefer the logical systems that are decidable.

# The Spectrum of Natural Logic




first-order logic

$FO^2$ + "$R$ is trans"

FO2 = 2 variable FOL

† adds full $N$-negation

$\mathcal{RC}(tr)$ + opposites
$\mathcal{RC}$ + (transitive) comparative adjs
$\mathcal{RC} = \mathcal{R}$ + relative clauses
$\mathcal{S}$ + full $N$-negation
$\mathcal{R}$ = relational syllogistic
$\mathcal{S}^\geq$ adds $|p| \geq |q|$
$\mathcal{S}$: all/some/no p are q

$\mathcal{A}$: all p are q

# 王雨田, 1987; 夏年喜, 2004, 2006

- 自然语言逻辑 (Logic of natural logic) 亦称自然逻辑 (Natural logic)。在我国，人们更乐于称它为语言逻辑。(cf. 王雨田, 1987, p431)
- 自然语言逻辑，又叫语言逻辑、自然逻辑或日常语言逻辑。(cf. 夏年喜, 2004, 2006)

# Fred Sommers, 1982

Michael Dummett's study of Frege:

> *Modern logic stand in contrast to all the great logical systems of the past—of classical antiquity, of medieval Europe, and of India—in being able to give an account which depends on the mechanism of quantifiers and bound variables. (Dummett, 1967)*

# Two Perspectives

- Before Frege:
  - (1) Socrates is mortal. = Some Socrates is mortal. = Every Socrates is mortal.
  - (2) Every man is mortal.
  - The difference between (1) and (2) is the difference between 'Socrates' and 'man'.
- Frege:
  - Atomicity thesis: there is a class of of propositions whose subjects are simple names or other singular expressions devoid of any sign of quantity.
  - Socrates is mortal.
  - Every (man is mortal).

# Sommers's Proposal

- ‘some S’ is represented as $\llcorner S \lrcorner$,
- ‘every S’ as $[S]$
- singular subject as $\ulcorner S \urcorner$
- ‘is P’ as $\langle P \rangle$

## Example 1

- A sailor is giving a toy to each of the children.
- Some sailor is giving every child some toy.
- $\llcorner S \lrcorner \langle G^3 [C] \llcorner T \lrcorner \rangle$
- Subject-Predicate Normal Form: $\llcorner S \lrcorner \langle [C] \langle \llcorner T \lrcorner \langle G^3 \rangle \rangle \rangle$

# Correspondence between TFL and MPL

The basic mappings between TFL(Traditional Formal Logic) and MPL(Modern Predicate Logic) are given by three rules.

- R1: $\ulcorner\alpha\urcorner\langle P\rangle = P\alpha$
- R2: $\llcorner S\lrcorner\langle P\rangle = \exists x(Sx \wedge Px)$
- R3: $[S]\langle P\rangle = \forall x(Sx \rightarrow Px)$

**Example 2 ($\llcorner S\lrcorner\langle[C]\langle\llcorner T\lrcorner\langle G^3\rangle\rangle\rangle$)**

- R2: $\exists x(Sx \wedge [C]\langle\llcorner T\lrcorner\langle G^3\rangle\rangle x)$
- Importation*2: $\exists x(Sx \wedge [C]\langle\llcorner T\lrcorner\langle G^3 x\rangle\rangle)$
- R3: $\exists x(Sx \wedge \forall y(Cy \rightarrow \llcorner T\lrcorner\langle G^3 x\rangle y))$
- Importation: $\exists x(Sx \wedge \forall y(Cy \rightarrow \llcorner T\lrcorner\langle G^3 xy\rangle))$
- R2: $\exists x(Sx \wedge \forall y(Cy \rightarrow \exists z(Tz \wedge Gxyz)))$

# Johan van Benthem's Review, 1983

Benthem attacked Sommers's two arguments in favour of traditional logic.

- ☛ ARG1 Sommers proposes a further development of traditional logic whose inference ability is comparable with Fregean predicate logic.
  - ATTACK: The methods he used are beyond the scope of traditional logic.
- ☛ ARG2 Sommers believes that the classical ideal of having a psychologically realistic theory of reasoning based upon ordinary grammatical analysis.
  - ATTACK: This argument lead us to accept the assumption that the more close to natural language, the more psychological reality the logic has.

On the other hand, Benthem think that Sommers's enterprise is useful and stimulating. An analysis of inference directly on grammatical tree structures would be highly desirable. $\Longrightarrow$ Natural Logic.

# J. Anthony Blair and Ralph H. Johnson,

- The logic of natural language, then, is the logic of the language(s) that anyone grew up speaking.
- So the next question is, what is the "logic" of such a language? Any answer to this question steps off the firm ground of established fact and onto the unsettled footing of contested theory, so the reader needs to be aware that other writers might well put matters differently.
- In one sense of 'logic,' its subject matter is the norms for systems of necessary connections. A necessary connection is one that cannot be otherwise. Here is an example. If a flower is yellow, then it is coloured. Indeed, if anything is yellow, then it is coloured. Being yellow, it must be coloured; it cannot be without color. There is, then, a necessary connection between being yellow and being coloured.

# Outline

# De Morgan's Famous Inference

- All horses are animals. So, all horse tails are animal tails.
- $\forall x(Hx \to Ax) \models \forall x((Tx \wedge \exists y(Hy \wedge Rxy)) \to (Tx \wedge \exists y(Ay \wedge Rxy)))$
- This is supposed to show the inadequacy of the traditional logic of 'monadic predicates', because binary relations are essential to understand the validity of the inference.

# Lyndon's Theorem

$$\forall x(Hx \rightarrow Ax) \models \forall x((Tx \wedge \exists y(Hy \wedge Rxy)) \rightarrow (Tx \wedge \exists y(Ay \wedge Rxy)))$$

## Definition 3 (Semantics)

A formula $\phi(X)$ is upward monotone with respect to the predicate $X$ if for all models $\mathcal{M}$, if $\mathcal{M}, P, s \models \phi(X)$ [here we interpret the syntactic predicate $X$ as the set $P$, while $s$ is a tuple of objects whose length equals the arity of $X$], and $P \subseteq Q$, then $\mathcal{M}, Q, s \models \phi(X)$.

## Definition 4 (Syntactic Counterpart)

Let us call an occurrence of $X$ in $\phi(X)$ positive if it lies in the scope of an even number of negations, or stated differently, if the formula $\phi(X)$ is created using only the following inductive syntax rules:
$H$-free formulas$| \wedge | \vee | \forall | \exists$

### Theorem 5 (Lyndon's Theorem)

*A first-order formula $\phi(X)$ is semantically monotone in $X$ iff $\phi(X)$ is equivalent to a formula whose only occurrences of $X$ are positive.*

### Example 6

- The occurrence of '$H$' in $((Tx \wedge \exists y(Hy \wedge Rxy))$ is positive.
- Then, we could use '$A$' to replace '$H$' according to the monotonicity.

# Johan van Benthem's Point

*De Morgan story is misleading and historically false. Inferences like the one with the horse tail were well within the scope of traditional logic, which was much subtler than many modern critics acknowledge. They blame it for defects it never had. (cf. Johan van Benthem, 2008)*

# Doctrine of Medieval Scholastics

- ☛ Dictum de Omni et Nullo
  Whatever is affirmed or denied of a whole kind K may be affirmed or denied (respectively) of any subkind of K.
- ☛ corresponded to admissible inferences of two kinds: (van Eijck 1982, van Benthem 1986, Sanchez Valencia 1991, Hodges 1998)
  - Downward monotonic (substituting stronger predicates for weaker ones)
  - Upward monotonic (substituting weaker predicates for stronger ones)

It means that if every $X$ is $Y$, and '$X$' occurs in upward monotone position in some statement ...$X$... , then that same statement holds for $Y$ : ...$Y$.....

$\Longrightarrow$ Horse tail problem can be subsumed under it.

$\Longrightarrow$ Next, how to mark the phrase structure in a systematic method?

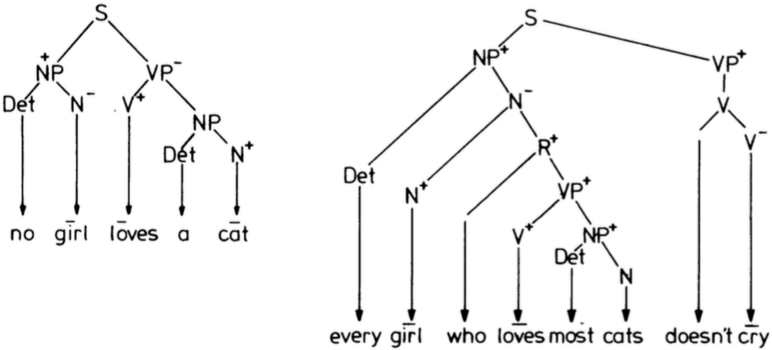
# Some Intuitive Rules

- $S \Longrightarrow \overset{+}{NP}\ VP$
- $NP \Longrightarrow \overset{+}{Det}\ N$
- $NP \Longrightarrow \overset{+}{PN}$(proper names)
- $VP \Longrightarrow \overset{+}{V}$ (intransitive verbs)
- $VP \Longrightarrow V\ \overset{+}{NP}$ (transitive verbs) e.g. love a dog
- $N \Longrightarrow \overset{+}{Adj}\ \overset{+}{N}$ (intersective adjectives)
- $N \Longrightarrow \overset{+}{N}\ \overset{+}{R}$ (relative clauses)
- $V \Longrightarrow \overset{+}{V}\ and\ \overset{+}{V}$
- $V \Longrightarrow not\ \overset{-}{V}$
- $S \Longrightarrow if\ \overset{-}{S},\ \overset{+}{S}$

# Rule of Overall Calculation for Nesting

- $+\ + = +$
- $+\ - = -$
- $-\ + = -$
- $-\ - = +$

# CFG




Actually, it is not enough to just rely on the CFG with tags.

# Another Method: Categorial Grammar

## Definition 7 (Type)

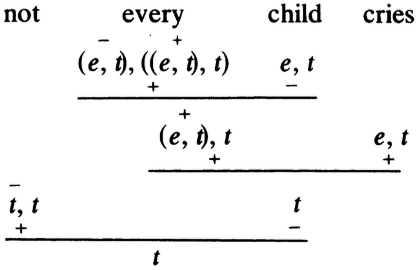
- e ('entity') and t ('truth value') are two basic types.
- If a, b are types, then (a, b) is also a type.

## Example 8

- t(sentences), e (proper names), (t, t) (unary sentential operators),
- (e, t) (intransitive verbs, common nouns),
- (t, (t, t)) (binary sentential connectives),
- (e, (e, t)) (transitive verbs),
- ((e, t), t) (noun phrases),
- ((e, t), ((e, t), t)) (determiners),
- ((e, t), (e, t)) (adverbs, adjectives).

# The Marked Category of Quantifiers

- no $(\overset{-}{e}, t), ((\overset{-}{e}, t), t)$ e.g. No apple is animal.
- every $(\overset{-}{e}, t), ((\overset{+}{e}, t), t)$ e.g. Every apple is a fruit.
- most $(e, t), ((\overset{+}{e}, t), t)$ e.g. Most Sichuan people like spicy food.



# Jan van Eijck's Approach

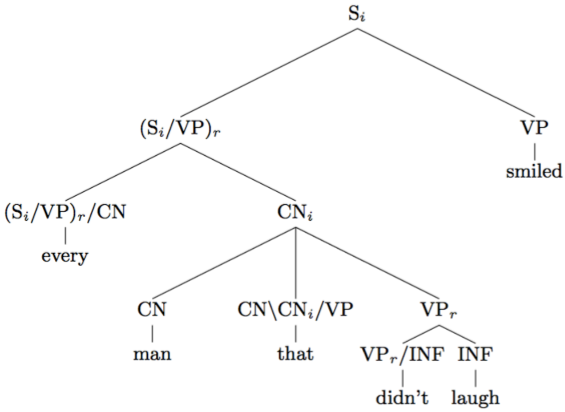
$$
\begin{array}{llll}
C(\text{every}) = (\mathrm{S}_i/\mathrm{VP})_r/\mathrm{CN} & & C(\text{did}) & = \mathrm{VP}_i/\mathrm{INF} \\
C(\text{some}) = (\mathrm{S}_i/\mathrm{VP})_i/\mathrm{CN} & & C(\text{didn't}) & = \mathrm{VP}_r/\mathrm{INF} \\
C(\text{no}) = (\mathrm{S}_r/\mathrm{VP})_r/\mathrm{CN} & & C(\text{man}) & = \mathrm{CN} \\
C(\text{any}) = (\mathrm{S}_i/\mathrm{VP})_i/\mathrm{CN} & & C(\text{that}) & = \mathrm{CN}\backslash\mathrm{CN}_i/\mathrm{VP} \\
C(\text{the}) = (\mathrm{S}_i/\mathrm{VP})_b/\mathrm{CN} & & C(\text{laugh}) & = \mathrm{INF} \\
C(\text{most}) = (\mathrm{S}_i/\mathrm{VP})_b/\mathrm{CN} & & C(\text{laughed}) & = \mathrm{VP} \\
C(\text{Ann}) = \mathrm{S}_i/\mathrm{VP} & & C(\text{kissed}) & = \mathrm{VP}_i/(\mathrm{S}/\mathrm{VP})
\end{array}
$$

# Polarity Marking Algorithm

- Root Marking The main structure C to be marked has positive polarity, so it is marked with $+$.
- Component Marking If a structure C has polarity marking k, then:
  - Leaf Marking If C is a leaf, then done.
  - Composite Marking If C consists of a function (C/A) and argument A (or an argument A and a function A\ C, or an argument A, a function A\ C/B and an argument B), then the function gets polarity marking $k$, and the argument(s) get polarity marking $f(k)$, where f is the polarity marking map at node C.

# An Example

Every man that didn't laugh smiled.



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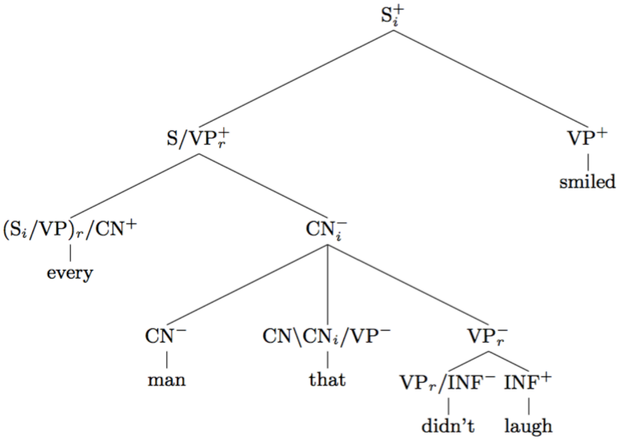


# Conservativity

CONS $QAB \iff QA(B \cap A)$ (Q stands for quantifier.)

## Example 9

- Only willows weep.
- Only willows are weeping willows.

- The first argument set the scene for the second. One can think of this as a sort of domain or role restriction imposed by the initial predicate A on the predicate B.
- Conservativity seems to hold in all human languages.

# Other Properties of Determiners

- QUANT
  - For all sets $E, E'$, all bijections $\pi$ from $E$ to $E'$, and all $A, B \subseteq E$, $Q_E AB \iff Q_{E'} \pi[A] \pi[B]$
- EXT
  - For all $E, E'$, and $A, B \subseteq E \subseteq E'$, $Q_E AB \iff Q_{E'} AB$
- VAR
  - For all non-empty $A \subseteq E$, there exist $B, B' \subseteq E$ such that $Q_E AB$, not $Q_E AB'$

# Other Properties of Determiners

- CONT
  - $QAB, QAC, B \subseteq D \subseteq C$ imply $QAD$.
  - not $QAB$,not $QAC$, $B \subseteq D \subseteq C$ imply not $QAD$.
- PLUS
  - If $(a, b) \in Q$, then $(a + 1, b) \in Q$ or $(a, b + 1) \in Q$,
  - If $(a, b) \notin Q$, then $(a + 1, b) \notin Q$ or $(a, b + 1) \notin Q$.
- UNIF
  - For each truth value, the addition experiment has the same triangle of outcomes everywhere.

# An Important Theorem

**Theorem 10**

*On the finite sets, the only generalized quantifiers satisfying QUANT, CONS, EXT, VAR as well as CONT, PLUS, UNIF are all, some, no and not all.*

**Proof.**

Please refer to chapter 2.4 (van Benthem, 1986) for more details. □

# Outline

# Motivating Examples

1. P. A Revenue Cutter, the ship was named for Harriet Lane, niece of President James Buchanan, who served as Buchanan's White House hostess.
   H. Harriet Lane worked at the White House. yes
2. P. Two Turkish engineers and an Afghan translator kidnapped in July were freed Friday.
   H. translator kidnapped in Iraq no
3. P. The memorandum noted the United Nations estimated that 2.5 million to 3.5 million people died of AIDS last year.
   H. Over 2 million people died of AIDS last year. yes
4. P. Mitsubishi Motors Corp.'s new vehicle sales in the US fell 46 percent in June.
   H. Mitsubishi sales rose 46 percent. no

(cf. MacCartney, 2015)

# Natural Language Inference

Does premise P justify an inference to hypothesis H?

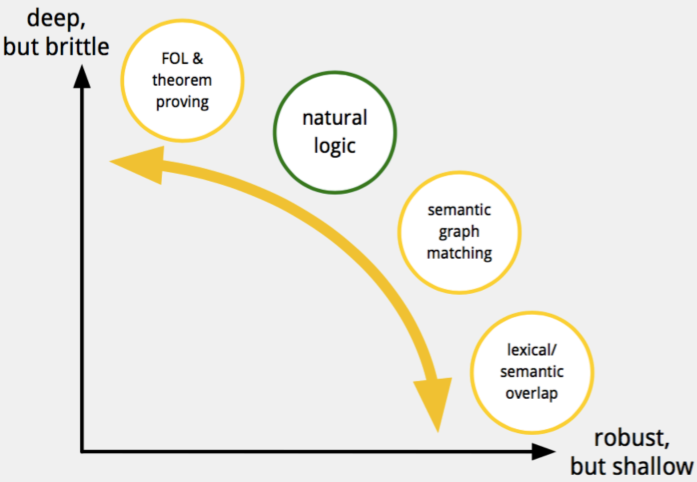
- ☛ An informal, intuitive notion of inference: not strict logic
- ☛ Focus on local inference steps, not long chains of deduction
- ☛ Emphasis on variability of linguistic expression

Relationship between NLI and Natural Language Understanding

- ☛ If you can't recognize that P implies H, then you haven't really understood P (or H).
- ☛ Thus, a capacity for natural language inference is a necessary (though probably not sufficient) condition for real NLU.

(cf. MacCartney, 2015)

# Approaches to NLI




(cf. MacCartney, 2015)

# Upward Monotonicity

- My cat ate a rat $\Longrightarrow$ My cat ate a rodent.
- My cat ate a rat $\Longrightarrow$ My cat consumed a rat.
- My cat ate a rat this morning $\Longrightarrow$ My cat ate a rat today.
- My cat ate a fat rat $\Longrightarrow$ My cat ate a rat.

# Downward Monotonicity

- No cats ate rats $\Longleftarrow$ No cats ate rodents
- Every rat fears my cat $\Longleftarrow$ Every rodent fears my cat
- My cat ate at most three rats $\Longleftarrow$ My cat ate at most three rodents
- If my cat eats a rat, he'll puke $\Longleftarrow$ If my cat eats a rodent, he'll puke
- My cat avoids eating rats $\Longleftarrow$ My cat avoids eating rodents
- My cat denies eating a rat $\Longleftarrow$ My cat denies eating a rodent
- My cat rarely eats rats $\Longleftarrow$ My cat rarely eats rodents

# Non-monotonicity

- Most rats like cheese # Most rodents like cheese
- My cat ate exactly three rats # My cat ate exactly three rodents
- I climbed the tallest building in Asia # I climbed the tallest building
- He is our first black president # He is our first president

# Semantic Containment

There are many different ways to broaden meaning!

- Deleting modifiers, qualifiers, adjuncts, appositives, etc.:
  tall girl standing by the pool $\sqsubset$ tall girl $\sqsubset$ girl
- Generalizing instances or classes into superclasses:
  Einstein $\sqsubset$ a physicist $\sqsubset$ a scientist
- Spatial & temporal broadening:
  in Palo Alto $\sqsubset$ in California, this month $\sqsubset$ this year
- Relaxing modals:
  must $\sqsubset$ could, definitely $\sqsubset$ probably $\sqsubset$ maybe
- Relaxing quantifiers:
  six $\sqsubset$ several $\sqsubset$ some
- Dropping conjuncts, adding disjuncts:
  danced and sang $\sqsubset$ sang $\sqsubset$ hummed or sang

# Semantic Exclusion

**Example 11 (negation (exhaustive exclusive))**

- slept ^ didn't sleep     able ^ unable
- living ^ nonliving     sometimes ^ never

**Example 12 (alternation (non-exhaustive exclusive))**

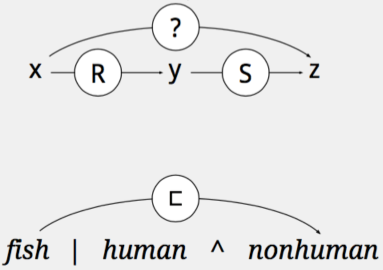
cat | dog     red | blue     all | none
male | female     hot | cold     here | there
teacup | toothbrush     French | German     today | tomorrow

# 7 Basic Semantic Relations

| | | | |
|---|---|---|---|
|  | $x \equiv y$ | equivalence | couch ≡ sofa |
|  | $x \sqsubset y$ | forward entailment (strict) | crow ⊏ bird |
|  | $x \sqsupset y$ | reverse entailment (strict) | European ⊐ French |
|  | $x \wedge y$ | negation (exhaustive exclusion) | human ^ nonhuman |
|  | $x \mid y$ | alternation (non-exhaustive exclusion) | cat \| dog |
|  | $x \smile y$ | cover (exhaustive non-exclusion) | animal ‿ nonhuman |
|  | $x \# y$ | independence | hungry # hippo |

# What is $|\bowtie|$?

$$R \bowtie S \stackrel{\text{def}}{=} \{\langle x, z\rangle : \exists y\, (\langle x, y\rangle \in R \wedge \langle y, z\rangle \in S)\}$$




| | | | | |
|---|---|---|---|---|
| $\equiv$ | $\bowtie$ | $\equiv$ | $\Rightarrow$ | $\equiv$ |
| $\sqsubset$ | $\bowtie$ | $\sqsubset$ | $\Rightarrow$ | $\sqsubset$ |
| $\sqsupset$ | $\bowtie$ | $\sqsupset$ | $\Rightarrow$ | $\sqsupset$ |
| $\wedge$ | $\bowtie$ | $\wedge$ | $\Rightarrow$ | $\equiv$ |
| R | $\bowtie$ | $\equiv$ | $\Rightarrow$ | R |
| $\equiv$ | $\bowtie$ | R | $\Rightarrow$ | R |

# Joining Semantic Relations

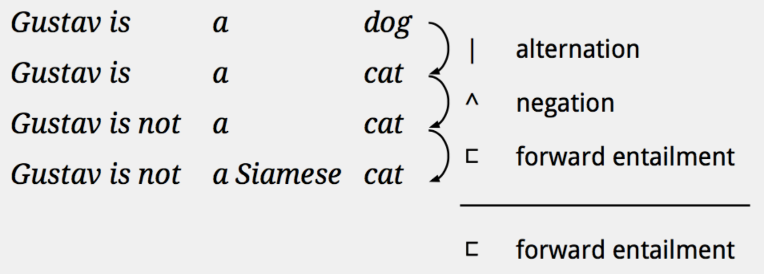
| $x \mid y$ | $y \mid z$ | $x\ ?\ z$ |
|---|---|---|
| *couch* \| *table* | *table* \| *sofa* | *couch* ≡ *sofa* |
| *pistol* \| *knife* | *knife* \| *gun* | *pistol* ⊏ *gun* |
| *dog* \| *cat* | *cat* \| *terrier* | *dog* ⊐ *terrier* |
| *rose* \| *orchid* | *orchid* \| *daisy* | *rose* \| *daisy* |
| *woman* \| *frog* | *frog* \| *Eskimo* | *woman* # *Eskimo* |

$$| \bowtie | = \{\equiv, \sqsubset, \sqsupset, |, \#\}$$

# The Complete Join Table

| $\bowtie$ | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\mid$ | $\smile$ | $\#$ |
|---|---|---|---|---|---|---|---|
| $\equiv$ | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\mid$ | $\smile$ | $\#$ |
| $\sqsubset$ | $\sqsubset$ | $\sqsubset$ | $\equiv\sqsubset\sqsupset\mid\#$ | $\mid$ | $\mid$ | $\sqsubset\wedge\mid\smile\#$ | $\sqsubset\mid\#$ |
| $\sqsupset$ | $\sqsupset$ | $\equiv\sqsubset\sqsupset\smile\#$ | $\sqsupset$ | $\smile$ | $\sqsupset\wedge\mid\smile\#$ | $\smile$ | $\sqsupset\smile\#$ |
| $\wedge$ | $\wedge$ | $\smile$ | $\mid$ | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\#$ |
| $\mid$ | $\mid$ | $\sqsubset\wedge\mid\smile\#$ | $\mid$ | $\sqsubset$ | $\equiv\sqsubset\sqsupset\mid\#$ | $\sqsubset$ | $\sqsubset\mid\#$ |
| $\smile$ | $\smile$ | $\smile$ | $\sqsupset\wedge\mid\smile\#$ | $\sqsupset$ | $\sqsupset$ | $\equiv\sqsubset\sqsupset\smile\#$ | $\sqsupset\smile\#$ |
| $\#$ | $\#$ | $\sqsubset\smile\#$ | $\sqsupset\mid\#$ | $\#$ | $\sqsupset\mid\#$ | $\sqsubset\smile\#$ | $\equiv\sqsubset\sqsupset\wedge\mid\smile\#$ |

# Joining Chains



# Projectivity

How do the entailments of a compound expression depend on the entailments of its parts?

# Example: Projectivity of Not

| | projection | | | example |
|---|---|---|---|---|
| | $\equiv$ | $\rightarrow$ | $\equiv$ | *not happy* $\equiv$ *not glad* |
| downward monotonicity | $\sqsubset$ | $\rightarrow$ | $\sqsupset$ | *didn't kiss* $\sqsupset$ *didn't touch* |
| | $\sqsupset$ | $\rightarrow$ | $\sqsubset$ | *isn't European* $\sqsubset$ *isn't French* |
| | # | $\rightarrow$ | # | *isn't swimming* # *isn't hungry* |
| | $\wedge$ | $\rightarrow$ | $\wedge$ | *not human* $\wedge$ *not nonhuman* |
| swaps these too | $\mid$ | $\rightarrow$ | $\smile$ | *not French* $\smile$ *not German* |
| | $\smile$ | $\rightarrow$ | $\mid$ | *not more than 4* $\mid$ *not less than 6* |

# Example: Projectivity of Refuse

| | projection | example |
|---|---|---|
| | $\equiv \;\rightarrow\; \equiv$ | |
| downward monotonicity | $\sqsubset \;\rightarrow\; \sqsupset$ | *refuse to tango* $\sqsupset$ *refuse to dance* |
| | $\sqsupset \;\rightarrow\; \sqsubset$ | |
| | $\# \;\rightarrow\; \#$ | |
| switch | $\wedge \;\rightarrow\; \mid$ | *refuse to stay* $\mid$ *refuse to go* |
| blocks, not swaps | $\mid \;\rightarrow\; \#$ | *refuse to tango* # *refuse to waltz* |
| | $\smile \;\rightarrow\; \#$ | |

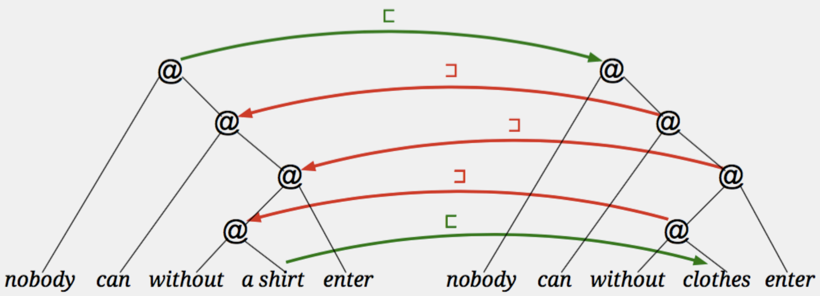
# The Projectivity of Connectives

| | projectivity | | | | | | |
|---|---|---|---|---|---|---|---|
| connective | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\mid$ | $\smile$ | # |
| negation (*not*) | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\wedge$ | $\smile$ | $\mid$ | # |
| conjunction (*and*) / intersection | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\mid$ | $\mid$ | # | # |
| disjunction (*or*) | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\smile$ | # | $\smile$ | # |
| conditional (*if*) (antecedent) | $\equiv$ | $\sqsupset$ | $\sqsubset$ | # | # | # | # |
| conditional (*if*) (consequent) | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\mid$ | $\mid$ | # | # |
| biconditional (*if and only if*) | $\equiv$ | # | # | $\wedge$ | # | # | # |

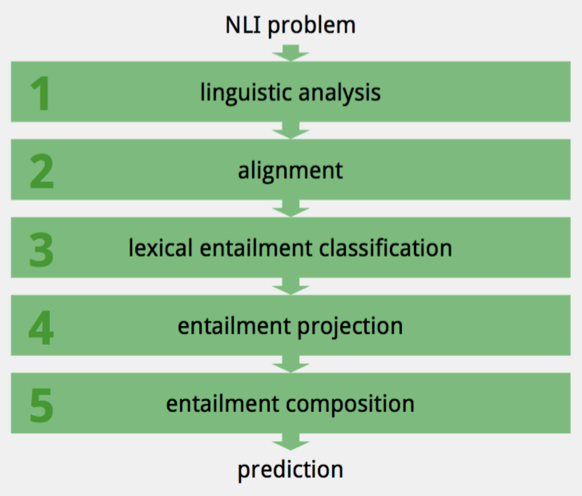
# The Projectivity of Quantifiers

| | projectivity for 1st argument | | | | | | | projectivity for 2nd argument | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| quantifier | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\mid$ | $\smile$ | # | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\mid$ | $\smile$ | # |
| *some* | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\smile^{\dagger}$ | # | $\smile^{\dagger}$ | # | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\smile^{\dagger}$ | # | $\smile^{\dagger}$ | # |
| *no* | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\mid^{\dagger}$ | # | $\mid^{\dagger}$ | # | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\mid^{\dagger}$ | # | $\mid^{\dagger}$ | # |
| *every* | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\mid^{\ddagger}$ | # | $\mid^{\ddagger}$ | # | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\mid^{\dagger}$ | $\mid^{\dagger}$ | # | # |
| *not every* | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\smile^{\ddagger}$ | # | $\smile^{\ddagger}$ | # | $\equiv$ | $\sqsupset$ | $\sqsubset$ | $\smile^{\dagger}$ | $\smile^{\dagger}$ | # | # |
| *at least two* | $\equiv$ | $\sqsubset$ | $\sqsupset$ | # | # | # | # | $\equiv$ | $\sqsubset$ | $\sqsupset$ | # | # | # | # |
| *most* | $\equiv$ | # | # | # | # | # | # | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\mid$ | $\mid$ | # | # |
| *exactly one* | $\equiv$ | # | # | # | # | # | # | $\equiv$ | # | # | # | # | # | # |
| *all but one* | $\equiv$ | # | # | # | # | # | # | $\equiv$ | # | # | # | # | # | # |

# A Simple Example



# The NatLog System



# What Natural Logic Can Not Do?

Many types of inference not amenable to natural logic.

- ☛ Paraphrase: Eve was let go $\equiv$ Eve lost her job
- ☛ Verb/frame alternation: he drained the oil $\sqsubset$ the oil drained
- ☛ Relation extraction: Aho, a trader at UBS... $\sqsubset$ Aho works for UBS
- ☛ Common-sense reasoning: the sink overflowed $\sqsubset$ the floor got wet

# Outline

# Summary

- An overview of natural logic
- Natural logic
- Natural language inference

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# THANKS!