Modal AGM model of preference changes

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2012.12.18

Preference and preference change

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- Dynamic turn and preference dynamics.

AGM framework

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- Sentential repersentations, input-assimilation and minimal change
- Sven Ove Hansson introduced a AGM-style framework to deal with preference changes.

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- However, we notice that *DEL* framework is flexible in the sense that new modalities can be added to the language as long as we need.
- It is interesting to find whether there is a *DEL*-like modal AGM framework so that original change operators can be represented faithfully by some modalities in it.

Preliminary

Definition

$\mathcal{L}$ is a minimal set satisfying the following rules:

(1) If $x, y \in \mathcal{U}$, then $x \leqslant y \in \mathcal{L}$,

(2) if $\alpha, \beta \in \mathcal{L}$, then $\neg\alpha \in \mathcal{L}$ and $\alpha \wedge \beta \in \mathcal{L}$.

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Definition

$|\Sigma|$, the set of alternatives mentioned by a set of sentences, is defined according to the following rules:

(1) $|\{x \leqslant y\}| = |x \leqslant y| = \{x, y\}$,

(2) $|\{\neg\alpha\}| = |\neg\alpha| = |\alpha|$,

(3) $|\{\alpha \wedge \beta\}| = |\alpha \wedge \beta| = |\alpha| \cup |\beta|$,

(4) $|\Sigma| = \bigcup\{|\{\alpha\}| \mid \alpha \in \Sigma\}$.

Preliminary, cont.

Definition

Let $\mathcal{A} \subseteq \mathcal{U}$ and $\Sigma \subseteq \mathcal{L}$, then:

(1) $\Sigma \uparrow \mathcal{A} = \{\alpha \in \Sigma \mid |\alpha| \subseteq \mathcal{A}\}$,

(2) $\Sigma \downarrow \mathcal{A} = \{\alpha \in \Sigma \mid |\alpha| \cap \mathcal{A} = \emptyset\}$.

Preliminary, cont.

Definition

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Definition

Let $\Phi \subseteq \mathcal{L}$. $\mathbf{sub}(\Phi)$ is the set of substitution-instances of elements of Φ. Furthermore, Cn_0 is the classical truth-functional consequence operator. Let Cn_Φ be the operator on subset of $\mathcal{L}$ such that for any $\Sigma \subseteq \mathcal{L}$, $Cn_\Phi(\Sigma) = Cn_0(\mathbf{sub}(\Phi) \cup \Sigma)$.

Preference set

Definition (preference set)

Let $\Phi = \{x \leqslant x, x \leqslant y \wedge y \leqslant z \rightarrow x \leqslant z\}$. A set $\Sigma \subseteq \mathcal{L}$ is a preference set if and only if:

(1) $\Sigma = (Cn_{\Phi}(\Sigma)) \uparrow |\Sigma|$ and

(2) for all $\alpha \in \Sigma$, $\neg\alpha \notin \Sigma$.

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- Φ means *rationality constraints* which rational preference state should obey.
- We assume that agents are perfect reasoners.

Preference model

Definition (preference model)

A preference model for $\mathcal{L}$ is a multi-tuple $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$ where $\mathcal{U}$ is the fixed domain, $\mathcal{A} \subseteq \mathcal{U}$ and for any $x \in \mathcal{U}$, $\sigma(x) = x$. Furthermore, for any $i \leqslant n$, R_i is a reflexive and transitive binary relation on $\mathcal{A}$. The satisfaction relation $\vDash$ is defined as follows:

(1) for any $x, y \in \mathcal{U}$, $\mathbb{R} \vDash x \leqslant y$ iff for any $i \leqslant n$, $(\sigma(x), \sigma(y)) \in R_i$,

(2) $\mathbb{R} \vDash \neg\alpha$ iff $(\mathcal{U}, \mathcal{A}, R_i, \sigma) \nvDash \alpha$ for any $i \leqslant n$,

(3) $\mathbb{R} \vDash \alpha \wedge \beta$ iff $\mathbb{R} \vDash \alpha$ and $\mathbb{R} \vDash \beta$.

Sentential repersentation and relational representation

- Let $[\![\mathbb{R}]\!] = \{\alpha \in \mathcal{L} \mid \mathbb{R} \vDash \alpha\}$ and $[\mathbb{R}] = [\![\mathbb{R}]\!] \uparrow \mathcal{A}$, we can get the following theorem.

Theorem

Let $\Sigma \subseteq \mathcal{L}$, Σ is a preference set if and only if there is a model $\mathbb{R}$ for $\mathcal{L}$ such that $\Sigma = [\mathbb{R}]$.

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- Relational repersentation allows for simple and natural definitions of operators of change.

Basic types of preference changes: $\oplus$, $\ominus$

Definition (Subtraction $\ominus$)

If preference model $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$ and $x \in \mathcal{U}$, then $\mathbb{R} \ominus x = (\mathcal{U}, \mathcal{A}', R_1', \cdots, R_n', \sigma)$ where

(1) $\mathcal{A}' = \mathcal{A} \setminus \{x\}$ and

(2) for any $i \leqslant n$, $R_i' = R_i \cap (\mathcal{A} \times \mathcal{A})$

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Definition (Addition $\oplus$)

If preference model $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$ and $x \in \mathcal{U}$, then $\mathbb{R} \oplus x = (\mathcal{U}, \mathcal{A}', R_1', \cdots, R_n', \sigma)$ where

(1) $\mathcal{A}' = \mathcal{A} \cup \{x\}$ and

(2) for any $i \leqslant n$, $R_i' = R_i \cup \{(x, x)\}$

Similarity relation

- In AGM framework, every basic change should be minimal. So in order to define $\otimes$ and $\odot$, we need a tool to measure the similarity between two preference models.

Similarity relation

- In AGM framework, every basic change should be minimal. So in order to define $\otimes$ and $\odot$, we need a tool to measure the similarity between two preference models.
- For all sets X and Y, let the symmetrical difference $X \Delta Y$ between X and Y be equal to $(X \setminus Y) \cup (Y \setminus X)$.

Similarity relation, cont.

Definition

For any finite set X, $\#(X)$ is the number of elements of X. Let $\mathcal{B} \subseteq \mathcal{U}$ and R_1, R_2 are binary relations on $\mathcal{A}$. The similarity relation between R_1 and R_2 is defined as follows:

(1) $\delta(R_1, R_2)_{\mathcal{B}} = \langle \#((R_1 \Delta R_2) \cap ((\mathcal{U} \setminus \mathcal{B})) \times (\mathcal{U} \setminus \mathcal{B}))), \#(R_1 \Delta R_2) \rangle$,

(2) $\langle a, b \rangle \sqsubseteq \langle c, d \rangle$ iff $a < c$ or $a < c \wedge b \leqslant d$,

(3) $\langle a, b \rangle \sqsubset \langle c, d \rangle$ iff $\langle a, b \rangle \sqsubseteq \langle c, d \rangle$ and $\neg \langle c, d \rangle \sqsubseteq \langle a, b \rangle$.

Similarity relation, cont.

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Definition

If $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$, $\mathcal{B} \subseteq \mathcal{U}$ and R is a binary relation on $\mathcal{U}$, then $R \in \mathbb{R}$ if and only if there is some $i \leqslant n$ such that $R_i = R$. Furthermore, $\delta_{\mathcal{B}}(R, \mathbb{R}) = \mathbf{min}\{\delta_{\mathcal{B}}(R, R') \mid R' \in \mathbb{R}\}$

Basic types of preference changes: $\otimes$

Definition (Revision $\otimes$)

Let preference model $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$. If α is a Φ-consistent sentence in $\mathcal{L} \uparrow \mathcal{A}$, then

$\mathbb{R}_{\mathcal{B}} \otimes \alpha = (\mathcal{U}, \mathcal{A}, R'_1, \cdots, R'_m, \sigma)$ where for any $i \leqslant m$,

(1) $(\mathcal{U}, \mathcal{A}, R'_i, \sigma) \vDash \alpha \cup Cn_0(\mathbf{Sub}(\Phi))$ and

(2) there is no R' satisfying both the above condition and that $\delta_{\mathcal{B}}(R', \mathbb{R}) \sqsubset \delta_{\mathcal{B}}(R, \mathbb{R})$.

Otherwise, $\mathbb{R}_{\mathcal{B}} \otimes \alpha = \mathbb{R}$.

Basic types of preference changes: $\odot$

- If $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$ and $\mathbb{R}' = (\mathcal{U}, \mathcal{A}', R'_1, \cdots, R'_m, \sigma)$, then $\mathbb{R} \cup \mathbb{R}' = (\mathcal{U}, \mathcal{A} \cup \mathcal{A}', R_1, \cdots, R_n, R'_1, \cdots, R'_m, \sigma)$.

Basic types of preference changes: $\odot$

- If $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$ and $\mathbb{R}' = (\mathcal{U}, \mathcal{A}', R'_1, \cdots, R'_m, \sigma)$, then $\mathbb{R} \cup \mathbb{R}' = (\mathcal{U}, \mathcal{A} \cup \mathcal{A}', R_1, \cdots, R_n, R'_1, \cdots, R'_m, \sigma)$.

Definition (Contraction)

Let $\mathcal{B} \subseteq \mathcal{U}$. If $\alpha \in \mathcal{L} \uparrow |\mathbb{R}|$ and $\alpha \notin Cn_{\Phi}(\emptyset)$, then $\mathbb{R}_{\mathcal{B}} \odot \alpha = \mathbb{R} \cup (\mathbb{R}_{\mathcal{B}} \otimes \neg\alpha)$. Otherwise, $\mathbb{R} \odot \alpha = \mathbb{R}$

Modal preference logic: syntax

Definition

Let $\mathbf{P}$ be a finite set of atomic propositions and $\#(\mathbf{P}) \geqslant \log_2^{\#(\mathcal{U})}$.

$\mathcal{L}^*$ is defined as follows:

(1) If $p \in \mathbf{P}$, then $p \in \mathcal{L}^*$,

(2) If $\phi, \psi \in \mathcal{L}^*$, then $\neg\phi \in \mathcal{L}^*$ and $\phi \wedge \psi \in \mathcal{L}^*$,

(1) If $\phi \in \mathcal{L}^*$, then $E\phi \in \mathcal{L}^*$ and $\langle\leqslant\rangle\phi \in \mathcal{L}^*$.

Modal preference logic: semantics

Definition (Modal preference model)

A modal preference model for $\mathcal{L}^*$ is $\mathcal{M} = (\mathcal{U}, \mathcal{A}, \leqslant_1, \cdots, \leqslant_n, V)$ where $\mathcal{U}$ is the fixed domain, $\mathcal{A} \subseteq \mathcal{U}$ and for any $i \leqslant n$, $\leqslant_i$ is a reflexive and transitive relation on $\mathcal{A}$. V is a fixed function mapping every $x \in \mathcal{U}$ to $V_x : \mathbf{P} \to \{\top, \bot\}$. If $x, y \in \mathcal{U}$ and $x \neq y$, then $V_x \neq V_y$. The satisfaction relation $\Vdash$ is defined as follows:

(1) $\mathcal{M}, x \Vdash p$ iff $V_x(p) = \top$,

(2) $\mathcal{M}, x \Vdash \neg\phi$ iff $(\mathcal{U}, \mathcal{A}, \leqslant_i, V), x \nVdash \phi$, for any $i \leqslant n$,

(3) $\mathcal{M}, x \Vdash \phi \wedge \psi$ iff $\mathcal{M}, x \Vdash \phi$ and $\mathcal{M}, x \Vdash \psi$,

(4) $\mathcal{M}, x \Vdash \langle\leqslant\rangle\phi$ iff for any $i \leqslant n$, there exists some y such that $x \leqslant_i y$ and $\mathcal{M}, x \Vdash \phi$,

(5) $\mathcal{M}, x \Vdash E\phi$ iff there exists some $y \in \mathcal{U}$, $\mathcal{M}, y \Vdash \phi$.

Translation τ

- Let $N = \{\phi \in \mathcal{L}^* \mid \phi = (\neg)p_1 \wedge \cdots \wedge (\neg)p_i\}$ where $i = \#(\mathbf{P})$, $\{p_1, \cdots, p_i\} = \mathbf{P}$ and $(\neg)p_j$ is p_j or $\neg p_j$.

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- Let $N^{\mathcal{U}} = \{\phi \in N \mid \mathcal{M}, x \Vdash \phi \text{ for some } x \in \mathcal{U}\}$.

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- Let $N^{\mathcal{U}} = \{\phi \in N \mid \mathcal{M}, x \Vdash \phi$ for some $x \in \mathcal{U}\}$.
- The bijective map $f\colon N^{\mathcal{U}} \to \mathcal{U}$ can be defined as $f(x) = \phi$ if and only if $\mathcal{M}, x \Vdash \phi$. If $\mathcal{A} \subseteq \mathcal{U}$, then $f(\mathcal{A}) = \{f(x) \mid x \in \mathcal{A}\}$.

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- Let $N^{\mathcal{U}} = \{\phi \in N \mid \mathcal{M}, x \Vdash \phi$ for some $x \in \mathcal{U}\}$.
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Definition

(1) For any $x, y \in \mathcal{U}$, $\tau(x \leqslant y) = E(\phi_x \wedge \langle\leqslant\rangle\phi_y)$ where $f(x) = \phi_x$ and $f(y) = \phi_y$,

(2) If $\alpha = \neg\beta$, then $\tau(\alpha) = \neg\tau(\beta)$,

(3) If $\alpha = \beta \wedge \lambda$, then $\tau(\alpha) = \tau(\beta) \wedge \tau(\lambda)$,

Let $\tau(\Sigma) = \bigcup\{\tau(\alpha) \mid \alpha \in \Sigma\}$ when $\Sigma \subseteq \mathcal{L}$.

Translation τ^*

Definition

For any preference model $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$, there is a modal preference model $\tau^*(\mathbb{R}) = \mathcal{M} = (\mathcal{U}, \mathcal{A}, \leqslant_1, \cdots, \leqslant_n, V)$ where for any $i \leqslant n$, $\leqslant_i = R_i$, and vice versa.

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- The relation $\vDash$ is preserved under the translations τ and τ^*.

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- The relation $\vDash$ is preserved under the translations τ and τ^*.

Theorem

If $\alpha \in \mathcal{L}$, $\phi = \tau(\alpha)$ and $\mathcal{M} = \tau^*(\mathbb{R})$, then for any $x \in \mathcal{U}$

$$\mathbb{R} \vDash \alpha \text{ iff } \mathcal{M}, x \Vdash \phi\,.$$

Translation τ^*

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For any preference model $\mathbb{R} = (\mathcal{U}, \mathcal{A}, R_1, \cdots, R_n, \sigma)$, there is a modal preference model $\tau^*(\mathbb{R}) = \mathcal{M} = (\mathcal{U}, \mathcal{A}, \leqslant_1, \cdots, \leqslant_n, V)$ where for any $i \leqslant n$, $\leqslant_i = R_i$, and vice versa.

- The relation $\vDash$ is preserved under the translations τ and τ^*.

Theorem

If $\alpha \in \mathcal{L}$, $\phi = \tau(\alpha)$ *and* $\mathcal{M} = \tau^*(\mathbb{R})$,*then for any* $x \in \mathcal{U}$

$$\mathbb{R} \vDash \alpha \text{ iff } \mathcal{M}, x \Vdash \phi\,.$$

- Thus, modal preference model can also be seen as a semantic counterpart of preference set.

Dynamic modal preferecne logic: syntax

- We extend the static $\mathcal{L}^*$ to dynamic language $\mathcal{L}^{*+}$.

Definition

$\mathcal{L}^{*+}$ is a minimal set satisfying the following rules:

(1) if $\phi \in \mathcal{L}^*$, then $\phi \in \mathcal{L}^{*+}$

(2) if $\phi \in \mathcal{L}^{*+}$, then $[\oplus\psi]\phi$, $[\ominus\psi]\phi \in \mathcal{L}^{*+}$ for any $\psi \in N^{\mathcal{U}}$

(3) if $\phi \in \mathcal{L}^{*+}$, then $[\otimes\psi|\chi]\phi$, $[\odot\psi|\chi] \in \mathcal{L}^{*+}$ for any $\psi \in \tau(\mathcal{L})$ and $\chi = \bigvee \Phi$ where $\Phi \subseteq N^{\mathcal{U}}$.

Dynamic modal preferecne logic: semantics

Definition

Let τ^{*-} denote the inverse function of τ^*.

(1) $\mathcal{M}, x \Vdash [\oplus\phi]\psi$ iff $\tau^*(\tau^{*-}(\mathcal{M}) \oplus y), x \Vdash \psi$ where $\mathcal{M}, y \Vdash \phi$,

(2) $\mathcal{M}, x \Vdash [\ominus\phi]\psi$ iff $\tau^*(\tau^{*-}(\mathcal{M}) \ominus y), x \Vdash \psi$ where $\mathcal{M}, y \Vdash \phi$,

(3) $\mathcal{M}, x \Vdash [\otimes\phi|\chi]\psi$ iff $\tau^*((\tau^{*-}(\mathcal{M}))_{\mathcal{B}} \otimes \alpha), x \Vdash \psi$ where $\mathcal{B} = \{x \mid \mathcal{M}, x \Vdash \chi\}$ and $\tau(\alpha) = \phi$,

(4) $\mathcal{M}, x \Vdash [\odot\phi|\chi]\psi$ iff $\mathcal{M}, x \Vdash \psi \wedge [\otimes\neg\phi|\chi]\psi$.

Change operators and action modalities

Theorem

For any $\alpha \in \mathcal{L}$ *and preference model* $\mathbb{R}$*, if* $\mathcal{M} = \tau^*(\mathbb{R})$*, then for any* $y \in \mathcal{U}$*,*

(1) $\mathbb{R} \oplus x \vDash \alpha$ *iff* $\mathcal{M}, y \Vdash [\oplus\phi]\tau(\alpha)$ *where* $f(\phi) = x$*,*

(2) $\mathbb{R} \ominus x \vDash \alpha$ *iff* $\mathcal{M}, y \Vdash [\ominus\phi]\tau(\alpha)$ *where* $f(\phi) = x$*,*

(3) $\mathbb{R}_{\mathcal{B}} \otimes \beta \vDash \alpha$ *iff* $\mathcal{M}, y \Vdash [\otimes\tau(\beta) | \bigvee \Phi]\tau(\alpha)$*,*

(4) $\mathbb{R}_{\mathcal{B}} \odot \beta \vDash \alpha$ *iff* $\mathcal{M}, y \Vdash \tau(\alpha) \wedge [\otimes\neg\tau(\beta) | \bigvee \Phi]\tau(\alpha)$*,*

where $\Phi = \{\alpha \mid \exists z(z \in \mathcal{B} \wedge z = f(\alpha))\}$.

Observations on $\otimes$

Theorem

The following formulas or rules are valid on class of modal preference models.

(1) $[\otimes\phi|\chi](((\langle\leqslant\rangle\psi \rightarrow \langle\leqslant\rangle\langle\leqslant\rangle\psi) \wedge (\psi \rightarrow \langle\leqslant\rangle\psi))$ (closure)

(2) $\phi^* \rightarrow [\otimes\phi|\chi]\phi$ (success)

(3) $\phi \rightarrow (\psi \leftrightarrow [\otimes\phi|\chi]\psi)$ (vacuity)

(4) $\Vdash \phi \leftrightarrow \psi$, *then* $\Vdash [\otimes\phi|\chi]\lambda \leftrightarrow [\otimes\psi|\chi]\lambda$ (extensionality)

(5) $\psi^* \wedge \neg[\otimes\phi|\chi]\neg\psi \rightarrow ([\otimes\phi|\chi][\otimes\psi|\chi]\lambda \leftrightarrow [\otimes(\phi\wedge\psi)|\chi]\lambda)$ (conjunction)

(6) $([\otimes(\phi\vee\psi)|\chi]\lambda \leftrightarrow [\otimes\phi|\chi]\lambda) \vee ([\otimes(\phi\vee\psi)|\chi]\lambda \leftrightarrow [\otimes\psi|\chi]\lambda) \vee [\otimes(\phi\vee\psi)|\chi]\lambda \leftrightarrow [\otimes\phi|\chi]\lambda \wedge [\otimes\psi|\chi]\lambda)$ (factoring)

where $\phi^* = \bigwedge\{E(\psi \wedge \langle\leqslant\rangle\psi) \mid \psi \in f(|\tau^-(\phi)|)\}$, *if* $\phi \in \tau(\mathcal{L})$.

Observations on $\odot$

Theorem

The following formulas or rules are valid on class of modal preference models.

(1) $[\odot\phi|\chi]((\langle\leqslant\rangle\psi \to \langle\leqslant\rangle\langle\leqslant\rangle\psi) \wedge (\psi \to \langle\leqslant\rangle\psi))$ (closure)

(2) $[\odot\phi|\chi]\psi \to \psi$ (inclusion)

(3) $\neg\phi \to ([\odot\phi|\chi]\psi \leftrightarrow \psi)$ (vacuity)

(4) $\phi^* \to [\odot|\chi]\psi$, *for any invalid* ϕ (success)

(5) $\Vdash \phi \leftrightarrow \psi$, *then* $\Vdash [\odot\phi|\chi]\lambda \leftrightarrow [\odot\psi|\chi]\lambda$ (extensionality)

(6) $([\odot(\phi\wedge\psi)|\chi]\lambda \leftrightarrow [\odot\phi|\chi]\lambda) \vee ([\odot(\phi\wedge\psi)|\chi]\lambda \leftrightarrow [\odot\psi|\chi]\lambda) \vee [\odot(\phi\wedge\psi)|\chi]\lambda \leftrightarrow [\odot\phi|\chi]\lambda \wedge [\odot\psi|\chi]\lambda)$ (factoring)

Observations on relation between $\otimes$ and $\odot$

Theorem

The following formulas are valid on class of modal preference models.

(1) $\phi \wedge \phi^* \rightarrow (\psi \leftrightarrow [\odot\phi|\chi][\otimes\phi|\chi]\psi)$ (recovery)

(2) $[\otimes\phi|\chi]\psi \leftrightarrow [\odot(\neg\phi)|\chi][\otimes\phi|\chi]\psi$ (Levi identity)

(3) $[\odot\phi|\chi]\psi \leftrightarrow \psi \wedge [\otimes(\neg\phi)|\chi]\psi$ (Harpern identity)

Observations on $\oplus$ and $\ominus$

Theorem

The following formulas are valid on class of modal preference models.

(1) $\neg E(\phi \wedge \langle\leqslant\rangle\phi) \to ([\ominus\phi]\psi \leftrightarrow \psi)$ (vacuity)

(2) $[\ominus\psi]E(\phi \wedge \langle\leqslant\rangle\phi) \leftrightarrow E(\phi \wedge \langle\leqslant\rangle\phi) \wedge \neg(\phi \leftrightarrow \psi)$, *for any* $\phi \in f(\mathcal{U})$ (success)

(3) $[\ominus\phi][\ominus\psi]\lambda \leftrightarrow [\ominus\psi][\ominus\phi]\lambda$ (commutativity)

(4) $[\ominus\phi]((\langle\leqslant\rangle\psi \to \langle\leqslant\rangle\langle\leqslant\rangle\psi) \wedge (\psi \to \langle\leqslant\rangle\psi))$ (closure)

Observations on $\oplus$ and $\ominus$, cont.

Theorem (Cont.)

(5) $E(\phi \wedge \langle\leqslant\rangle\phi) \rightarrow ([\oplus\phi]\psi \leftrightarrow \psi)$ (vacuity)

(6) $[\oplus\psi]E(\phi \wedge \langle\leqslant\rangle\phi) \leftrightarrow E(\phi \wedge \langle\leqslant\rangle\phi) \vee (\phi \leftrightarrow \psi)$, *for any* $\phi \in f(\mathcal{U})$ (success)

(7) $[\oplus\phi][\oplus\psi]\lambda \leftrightarrow [\oplus\psi][\oplus\phi]\lambda$ (commutativity)

(8) $[\oplus\phi]((\langle\leqslant\rangle\psi \rightarrow \langle\leqslant\rangle\langle\leqslant\rangle\psi) \wedge (\psi \rightarrow \langle\leqslant\rangle\psi))$ (closure)

(9) $\neg E(\phi \wedge \langle\leqslant\rangle\phi) \rightarrow (\psi \leftrightarrow [\oplus\phi][\ominus\phi]\psi)$ (subtractive recovery)

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- With further studies, it is possible to find more differences and relations of interest between these two framework.

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Thank you very much for your attention!