Multiple-Path vs. Single-Path Solutions to Skepticism

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External World Skepticism

- Skepticism about the external wold: we have no empirical, contingent knowledge about the external world.
- The main argument (P. Unger, 1975):
  1. $\neg K \neg biv$ Premise
  2. $(Kh \wedge K(h \rightarrow \neg biv)) \rightarrow K \neg biv)$ Premise (ECP)
  3. $K(h \rightarrow \neg biv)$ Premise (Let us make it true now!)
  4. $Kh \rightarrow K \neg biv$ by 2 and 3
  5. $\neg Kh$. by 1 and 4

Relevant Alternative Theories

- In order to attain knowledge p about the external world, the epistemic subject does not have to be able to evidentially exclude every ¬p-possibility (or every 'alternative'), all s/he needs is to be able to evidentially exclude every relevant ¬p-possibility (or every relevant alternative).
- It is possible that an alternative *w* is relevant to ψ but not relevant to ϕ even though (one knows that) ϕ entails ψ. Hence, it is also possible that one knows ϕ but does not know ψ, even if (one knows that) ϕ entails ψ. Thus, ECP is not valid.

Holliday's Formalization

A relevant alternative model (RA model) is a tuple $\mathfrak{M} = <\mathrm{W}_{\mathfrak{M}}$, $\Rightarrow_{\mathfrak{M}}$, $\leq_{\mathfrak{M}}$, $\mathrm{V}_{\mathfrak{M}}>$:

- $\mathrm{W}_{\mathfrak{M}}$ is a non-empty set;
- $\Rightarrow_{\mathfrak{M}}$ is a reflexive binary relation on $\mathrm{W}_{\mathfrak{M}}$;
- $\leq_{\mathfrak{M}}$ assigns to each $w \in \mathrm{W}$ a binary relation $\leq^{w}_{\mathfrak{M}}$ on some $\mathrm{W}_{w} \subseteq \mathrm{W}$:
  3.1 $\leq^{w}_{\mathfrak{M}}$ is reflexive and transitive in W_{w} (preorder);
  3.2 $w \in \mathrm{W}_{w}$, and for all $v \in \mathrm{W}_{w}$, $w \leq^{w}_{\mathfrak{M}} v$ (weak centering)
- $\mathrm{V}_{\mathfrak{M}}$: $\mathrm{At} \to \mathcal{P}(\mathrm{W})$

Truth in an RA Model

- Given an RA model $\mathfrak{M} = <W_{\mathfrak{M}}, \Rightarrow_{\mathfrak{M}}, \leq_{\mathfrak{M}}, V_{\mathfrak{M}}>$, a world $w \in W$, a formula ϕ in the epistemic language, we define $\mathfrak{M}, w \models \phi$ as follows (call this D-semantics):
  $\mathfrak{M}, w \models \neg\phi$ iff not $\mathfrak{M}, w \models \phi$;
  $\mathfrak{M}, w \models \phi \wedge \psi$ iff $\mathfrak{M}, w \models \phi$ and $\mathfrak{M}, w \models \psi$;
  $\mathfrak{M}, w \models K\phi$ iff $\forall v \in \mathrm{Min}_{\leq^w_{\mathfrak{M}}}[\neg\phi]_{\mathfrak{M}}$: not $w \Rightarrow v$;
- $[\neg\phi]_{\mathfrak{M}} = \{v \in W \mid \mathfrak{M}, v \models \neg\phi\}$ and $\mathrm{Min}_{\leq^w_{\mathfrak{M}}}[\neg\phi]_{\mathfrak{M}} = \{v \in [\neg\phi]_{\mathfrak{M}} \cap W_w \mid \neg\exists u(u \in [\neg\phi]_{\mathfrak{M}} \wedge u \leq^w_{\mathfrak{M}} v \wedge \neg v \leq^w_{\mathfrak{M}} u\}$.
- D-validity is defined in the usual way.

Some Easily Provable Results

- '$K\phi \wedge K(\phi \rightarrow \psi) \rightarrow K\psi$' is not D-valid. In the terminology of Dretske, knowledge operator is not fully penetrating; so ECP fails in D-semantics.
- However, '$K(\phi \wedge \psi) \rightarrow K\phi$' and '$K\phi \rightarrow K(\phi \vee \psi)$' are also not D-valid. This is surprising, for it shows that 'K' may not even be semi-penetrating.
- These results point to a dilemma: skepticism or the problem of containment.

Basic Ideas of Heller (1999)

- Relevance (realistic) order just is similarity order. The worlds are ordered identically for SC (subjunctive conditionals) and for RA (relevant alternatives). (Heller, 1989, p. 25)
- Some possibilities are realistic (close) enough while others are not. Those that are not are too remote (too irrealistic) to be eliminated by your evidence.
- Some sentences are, while others are not, such that those possibilities that falsify them are all too remote.
- ERA (Heller 1999, p. 201): S knows p only if S does not believe p in any of the closest not-p world or any more distant not-p worlds that are still close enough.

Holliday's Formalization

A counterfactual belief model (CB model) is a tuple $\mathfrak{M} = <\mathrm{W}_{\mathfrak{M}}, \mathrm{D}_{\mathfrak{M}}, \leq_{\mathfrak{M}}, \mathrm{V}_{\mathfrak{M}}>$:

- $\mathrm{W}_{\mathfrak{M}}$ is a non-empty set;
- $\mathrm{D}_{\mathfrak{M}}$ is a serial binary relation on $\mathrm{W}_{\mathfrak{M}}$;
- $\leq_{\mathfrak{M}}$ assigns to each $w \in \mathrm{W}$ a binary relation $\leq^{w}_{\mathfrak{M}}$ on some $\mathrm{W}_w \subseteq \mathrm{W}$:
  3.1 $\leq^{w}_{\mathfrak{M}}$ is reflexive and transitive in W_w (preorder);
  3.2 $w \in \mathrm{W}_w$, and for all $v \in \mathrm{W}_w$, $w \leq^{w}_{\mathfrak{M}} v$ (weak centering)
- $\mathrm{V}_{\mathfrak{M}}$: $\mathrm{At} \to \mathcal{P}(\mathrm{W})$

Truth in a CB Model

- Given a CB model $\mathfrak{M} = <W_{\mathfrak{M}}, D_{\mathfrak{M}}, \leq_{\mathfrak{M}}, V_{\mathfrak{M}}>$, a world $w \in W$, a formula ϕ in the epistemic language, we define $\mathfrak{M}, w \models \phi$ as follows (call this H-semantics):
  $\mathfrak{M}, w \models B\phi$ iff $\forall v \in W$: if $wD_{\mathfrak{M}}v$ then $\mathfrak{M}, v \models \phi$;
  $\mathfrak{M}, w \models K\phi$ iff $\mathfrak{M}, w \models B\phi$ and
  (sensitivity) $\forall v \in \mathrm{Min}_{\leq^{w}_{\mathfrak{M}}}[\neg\phi]_{\mathfrak{M}}$: not $\mathfrak{M}, w \models B\phi$.
- H-semantics avoid skepticism by invalidating ECP, but all of the closure principles shown to fail for D-semantics also fail for H-semantics, so they all face the problem of containment. H-semantics may also avoid the problem of vacuous knowledge.

The Impossibility Result, Holliday (2014)

For any scenario w, context C, and area Σ (if $\phi \in \Sigma$ and ψ is a TF-consequence of ϕ, then $\psi \in \Sigma$), the following principles are jointly inconsistent in the standard alternative picture (where r is a selection function that select, for any sentence ϕ and any world w, a set of relevant alternatives):

- contrast/enough$_\Sigma$ – $\forall\phi \in \Sigma$: $r_C(\phi, w) \subseteq (W - [\phi]_C)$;
- e-fallibilism$_\Sigma$ – $\exists\phi \in \Sigma\ \exists\psi \in \Sigma$: $r_C(\phi, w) \subseteq [\psi]_C$ and it is not the case that $(W_w - [\phi]_C) \subseteq [\psi]_C$;
- noVK$_\Sigma$ – $\forall\phi \in \Sigma$: $(W_w \cap [\phi]_C) \neq W_w$ implies $r_C(\phi, w) \neq \emptyset$;
- TF-cover$_\Sigma$ – $\forall\phi \in \Sigma\ \forall\psi \in \Sigma$: if ψ is a TF-consequence of ϕ, then $r_C(\psi, w) \subseteq r_C(\phi, w)$.

Multiple Paths (Holliday, 2014)

- In some cases, there are multiple sets of scenarios such that, if one is to know ϕ, one must exclude *all of the scenarios in at least one of those sets*.
- In some cases, it is sufficient for an agent to know ϕ that s/he *only eliminates non-contrasting scenarios in which ϕ is true*.
- Consider a disjunction 'p $\vee$ q' for an example. There seem to be at least three paths to know it: one could start by eliminating relevant $\neg$p-alternatives, or by eliminating relevant $\neg$q-alternatives, or by eliminating relevant $\neg$(p $\vee$ q)-alternatives. These three sets may not be the same. Further, all of the $\neg$p-alternatives may also be q-scenarios, therefore be (p $\vee$ q)-scenarios.

Symbols and Terminology

- CCNF: A canonical conjunctive normal form of a sentence ϕ ('CCNF(ϕ)' in symbols) is a conjunction ϕ' of nontrivial (i.e., does not include both 'p' and '$\neg$p') clauses (i.e., disjunctions of TF-basic sentences) such that for each p $\in$ at(ϕ'), each clause in ϕ' contains either 'p' or '$\neg$p'. Each sentence ϕ that is not a tautology is TF-equivalent to a ϕ' in CCNF with at(ϕ) = at(ϕ') that is unique up to reordering of the conjuncts and disjuncts.
- If ϕ is in CCNF, $\mathfrak{c}(\phi)$ is the set of all subclauses C of conjuncts in ϕ such that every nontrivial superclause C' of C with at(C') = at(ϕ) is a conjunct of ϕ. It turns out that $\mathfrak{c}(\phi)$ is the set of all nontrivial clauses C with at(C) $\subseteq$ at(ϕ) that are TF-consequences of ϕ.

From Single Path to Multiple Paths

Given an RA model $\mathfrak{M}$ and the standard alternatives function $\mathrm{r}_{\mathfrak{M}}$, we define a multipath alternatives function $\mathbf{r}^r_{\mathfrak{M}}$ as follows:
for any clause C,

- $\mathbf{r}^r_{\mathfrak{M}}(\mathrm{C}, w) = \{\mathrm{r}_{\mathfrak{M}}(\mathrm{C'}, w) |\ \mathrm{C'} \text{ is a subclause of } \mathrm{C}\}$;

for any CCNF conjunction $\mathrm{C}_1 \wedge \ldots \wedge \mathrm{C}_n$ of clauses with $\mathfrak{c}(\mathrm{C}_1 \wedge \ldots \wedge \mathrm{C}_n) = \{\psi_1, \ldots, \psi_m\}$,

- $\mathbf{r}^r_{\mathfrak{M}}(\mathrm{C}_1 \wedge \ldots \wedge \mathrm{C}_n, w) = \{\mathrm{A} \subseteq \mathrm{W} |\ \exists \mathrm{A}_1 \in \mathbf{r}^r_{\mathfrak{M}}(\psi_1, w) \ldots \exists \mathrm{A}_m \in \mathbf{r}^r_{\mathfrak{M}}(\psi_m, w)\text{: } \mathrm{A} = \cup_{1 \leq i \leq m} \mathrm{A}_i\}$;

If ϕ is not in CCNF, we define:

- $\mathbf{r}^r_{\mathfrak{M}}(\phi, w) = \mathbf{r}^r_{\mathfrak{M}}(\mathrm{CCNF}(\phi), w)$.

$\mathfrak{M}, w \models K\phi$ iff $\exists \mathrm{A} \in \mathbf{r}^r_{\mathfrak{M}}$: $\mathrm{A} \cap \{v \mid w \Rightarrow_{\mathfrak{M}} v\} = \emptyset$.

The Impossibility Result Again, Holliday (2014)

- contrast/enough$_{\Sigma}$ – $\forall\phi \in \Sigma$: $\forall$A(A$\in$ $\mathbf{r_C}(\phi, w) \rightarrow$ A $\subseteq$ (W – $[\phi]_C$)); ✗
- e-fallibilism$_{\Sigma}$ – $\exists\phi \in \Sigma$ $\exists\psi \in \Sigma$: $\exists$A(A$\in$ $\mathbf{r_C}(\phi, w) \wedge$ A$\subseteq$ $[\psi]_C$) and it is not the case that $(W_w - [\phi]_C) \subseteq [\psi]_C$; ✔
- noVK$_{\Sigma}$ – $\forall\phi \in \Sigma$: $(W_w \cap [\phi]_C) \neq W_w$ implies that $\emptyset \notin \mathbf{r_C}(\phi, w)$; ✔
- TF-cover$_{\Sigma}$ – $\forall\phi \in \Sigma$ $\forall\psi \in \Sigma$: if ψ is a TF-consequence of ϕ, then $\forall$A(A$\in$ $\mathbf{r_C}(\phi, w) \rightarrow \exists$B(B$\in$ $r_C(\psi, w)$ $\wedge$B$\subseteq$A)). ✔

Advantages and Disadvantages

- Holliday's semantics invalidates ECP, at the same time avoids the problem of containment and the problem of vacuous knowledge.
- It leads to path chaos. For example, even a sentence as simple as 'p $\vee$ (q $\wedge$ r)' could have $7^3 \times 3^2 = 3087$ paths to know it. There is also a smell of *ad hocness* here.
- Worse, it is hard to avoid skepticism: if we allow that there are multiple paths of knowing p each of which is via a single sentence, what can stop us from saying that there are also multiple paths of knowing ψ each of which via several sentences together, say, via knowing both 'ϕ' and '$\phi \rightarrow \psi$'?
- So the problem remains: can we have a single-path relevant alternative theory that avoids skepticism, the problem of containment, and the problem of vacuous knowledge?

Basic Idea of Heller (1999) Again

- ERA (Heller 1999, p. 201): S knows p only if S does not believe p in any of the closest not-p world *or* any more distant not-p worlds that are still close enough. (Notice that the 'or' in ERA gives it a smell of *ad hocness*, but I will utilize it.)
- Heller cashes out 'S can rule out not-p' in terms of 'S does not believe p in any of the relevant not-p world', so ERA can also be understood as:
  ERA*: S knows p (if and) only if S can rule out both the closest not-p world and all not-p worlds that are close enough.

My Formalization of Heller (1999)

An RA* model $\mathfrak{M}$ is a tuple $<W_{\mathfrak{M}}, \$_{\mathfrak{M}}, \Rightarrow_{\mathfrak{M}}, V_{\mathfrak{M}}>$ that satisfies the following conditions:

- $W_{\mathfrak{M}}$ is a non-empty set.
- $\$_{\mathfrak{M}}$ is a function from $W_{\mathfrak{M}}$ to $\mathcal{P}(\mathcal{P}(W_{\mathfrak{M}}))$ that is weakly centered, nested, closed under unions and nonempty intersection, and satisfies the Limit Assumption.
- $\Rightarrow_{\mathfrak{M}}$ is a reflexive binary relation on $W_{\mathfrak{M}}$ and contains every pair $<w, v>$, where $v \in W_{\mathfrak{M}} - \cup\$_{\mathfrak{M}}(w)$. (Think of those worlds in $W_{\mathfrak{M}} - \cup\$_{\mathfrak{M}}$ as unelimin*able*.)
- $V_{\mathfrak{M}}$: At $\rightarrow \mathcal{P}(W_{\mathfrak{M}})$.

Truth Condition and Validity

- Given an RA* model $<W_{\mathfrak{M}}, \$_{\mathfrak{M}}, \Rightarrow_{\mathfrak{M}}, V_{\mathfrak{M}}>$, a world $w \in W$, a formula ϕ in the epistemic language, we define $\mathfrak{M}, w \models \phi$ as follows (call this H*-semantics):
  $\mathfrak{M}, w \models \neg\phi$ iff not $\mathfrak{M}, w \models \phi$;
  $\mathfrak{M}, w \models \phi \wedge \psi$ iff $\mathfrak{M}, w \models \phi$ and $\mathfrak{M}, w \models \psi$;
  $\mathfrak{M}, w \models K\phi$ iff $r_{\mathfrak{M}}(\phi, w) \cap \{v \mid w \Rightarrow_{\mathfrak{M}} v\} = \emptyset$.
- We define $Min_{\leq^w_{\mathfrak{M}}}[\phi]_{\mathfrak{M}}$ to be the intersection of $[\phi]_{\mathfrak{M}}$ and the smallest sphere S if there is such an S, of $\$_{\mathfrak{M}}(w)$ such that $[\phi]_{\mathfrak{M}} \cap S$ is not empty and we define it to be $[\phi]_{\mathfrak{M}}$, if otherwise.
- H*-validity is defined in the usual way.

The Single-Path Function r

Given $\mathfrak{M}$, a tautology ϕ and a world w, we define $r_{\mathfrak{M}}(\phi, w) = r_{\mathfrak{M}}(p \vee \neg p, w)$. Given $\mathfrak{M}$, a non-tautology ϕ, and a world w, we define $r_{\mathfrak{M}}(\phi, w) = r_{\mathfrak{M}}(CCNF(\phi), w)$. For any CCNF, we define its relevant set inductively as follows:

- $r_{\mathfrak{M}}(p, w) = Min_{\leq^w_{\mathfrak{M}}}[\neg p]_{\mathfrak{M}} \cup (\cup \$_{\mathfrak{M}}(w) \cap [\neg p]_{\mathfrak{M}})$ if p is TF-basic.
- If $[p_1 \vee \ldots \vee p_n]_{\mathfrak{M}} = W_{\mathfrak{M}}$, then $r_{\mathfrak{M}}(p_1 \vee \ldots \vee p_n, w) = \emptyset$.
- If $[p_1 \vee \ldots \vee p_n]_{\mathfrak{M}} \neq W_{\mathfrak{M}}$, then (i) if there is no p_i (where $1 \leq i \leq n$) such that $(\cup \$_{\mathfrak{M}}(w) \cap [\neg p_i]_{\mathfrak{M}}) \neq \emptyset$, $r_{\mathfrak{M}}(p_1 \vee \ldots \vee p_n, w) = \emptyset$; otherwise, (ii) $r_{\mathfrak{M}}(p_1 \vee \ldots \vee p_n, w) = \cup \$_{\mathfrak{M}}(w) \cap ([\neg p_{i_1}]_{\mathfrak{M}} \cap \ldots \cap [\neg p_{i_m}]_{\mathfrak{M}})$, where $1 \leq i_j \leq n$ for each j between 1 and m and $(\cup \$_{\mathfrak{M}}(w) \cap [\neg p_{i_j}]_{\mathfrak{M}}) \neq \emptyset$ for each i_j;
- $r_{\mathfrak{M}}(C_1 \wedge \ldots \wedge C_n, w) = \cup\{r_{\mathfrak{M}}(A, w) \mid A \in \mathfrak{c}(C_1 \wedge \ldots \wedge C_n)\}$

ECP Is Not Valid in H*-Semantics

Here is a model that invalidate ECP:

$W_{\mathfrak{M}} = \{w_1, w_2, w_3\}$

$\$_{\mathfrak{M}}(w_1) = \{\{w_1\}, \{w_1, w_2\}\}$

$\$_{\mathfrak{M}}(w_2) = \$_{\mathfrak{M}}(w_3) = \{\{w_1, w_2, w_3\}\}$

$\cup\$_{\mathfrak{M}}(w_1) = \{w_1, w_2\}$

$\cup\$_{\mathfrak{M}}(w_2) = C_{\mathfrak{M}}(w_3) = \{w_1, w_2, w_3\}$

$\Rightarrow_{\mathfrak{M}} = \{<w_1, w_1>, <w_1, w_3>, <w_2, w_2>, <w_3, w_3>\}$

$V_{\mathfrak{M}}(p) = \{w_1\}, V_{\mathfrak{M}}(q) = \{w_3\}$

$\mathfrak{M}, w_1 \models Kp$ for $\{w_2\} = Min_{\leq^{w_1}_{\mathfrak{M}}}[\neg p]_{\mathfrak{M}} \cup (\cup\$_{\mathfrak{M}}(w_1) \cap [\neg p]_{\mathfrak{M}})$

$\mathfrak{M}, w_1 \models K(p \rightarrow \neg q)$ for $[p \rightarrow \neg q]_{\mathfrak{M}} = W_{\mathfrak{M}}$

Not $\mathfrak{M}, w_1 \models K\neg q$ for $w_3 \in Min_{\leq^{w_1}_{\mathfrak{M}}}[q]_{\mathfrak{M}} \cup (\cup\$_{\mathfrak{M}}(w_1) \cap [q]_{\mathfrak{M}})$

The Impossibility Result, putting $C = \mathfrak{M}$, $\mathrm{W}_w = \cup\$_{\mathfrak{M}}(w)$

- contrast/enough$_\Sigma$ – $\forall\phi \in \Sigma$: $\mathrm{r}_{\mathfrak{M}}(\phi, w) \subseteq (\mathrm{W}_{\mathfrak{M}} - [\phi]_{\mathfrak{M}})$; ✗
- e-fallibilism$_\Sigma$ – $\exists\phi \in \Sigma\ \exists\psi \in \Sigma$: $\mathrm{r}_{\mathfrak{M}}(\phi, w) \subseteq [\psi]_{\mathfrak{M}}$ and it is not the case that $(\cup\$_{\mathfrak{M}}(w) - [\phi]_{\mathfrak{M}}) \subseteq [\psi]_{\mathfrak{M}}$; ✗
- noVK$_\Sigma$ – $\forall\phi \in \Sigma$: $(\cup\$_{\mathfrak{M}}(w) \cap [\phi]_{\mathfrak{M}}) \neq \cup\$_{\mathfrak{M}}(w)$ implies $\mathrm{r}_{\mathfrak{M}}(\phi, w) \neq \emptyset$; ✔
- TF-cover$_\Sigma$ – $\forall\phi \in \Sigma\ \forall\psi \in \Sigma$: if ψ is a TF-consequence of ϕ, then $\mathrm{r}_{\mathfrak{M}}(\psi, w) \subseteq \mathrm{r}_{\mathfrak{M}}(\phi, w)$. ✔

A Simple Proof of noVK_Σ (part I)

Proof: We prove noVK_Σ by induction. The case for TF-basics is trivial. The case for clauses when $[\mathrm{p}_1 \vee \ldots \vee \mathrm{p}_n]_{\mathfrak{M}} = \mathrm{W}_{\mathfrak{M}}$ is also trivial. Suppose $[\mathrm{p}_1 \vee \ldots \vee \mathrm{p}_n]_{\mathfrak{M}} \neq \mathrm{W}_{\mathfrak{M}}$. If there is no p_i $(1 \leq i \leq n)$ such that $(\cup\$_{\mathfrak{M}}(w) \cap [\neg \mathrm{p}_i]_{\mathfrak{M}}) \neq \emptyset$, then $(\cup\$_{\mathfrak{M}}(w) \cap [\mathrm{p}_1 \vee \ldots \vee \mathrm{p}_n]_{\mathfrak{M}}) = \cup\$_{\mathfrak{M}}(w)$ and the case is trivial again. So assume that there is p_i $(1 \leq i \leq n)$ such that $(\cup\$_{\mathfrak{M}}(w) \cap [\neg \mathrm{p}_i]_{\mathfrak{M}}) \neq \emptyset$ and $\mathrm{r}_{\mathfrak{M}}(\mathrm{p}_1 \vee \ldots \vee \mathrm{p}_n, w) = \cup\$_{\mathfrak{M}}(w) \cap ([\neg \mathrm{p}_{i_1}]_{\mathfrak{M}} \cap \ldots \cap [\neg \mathrm{p}_{i_m}]_{\mathfrak{M}})$, where $1 \leq i_j \leq n$ for each j between 1 and m and $(\cup\$_{\mathfrak{M}}(w) \cap [\neg \mathrm{p}_{i_j}]_{\mathfrak{M}}) \neq \emptyset$ for each i_j: (a) if $\cup\$_{\mathfrak{M}}(w) \cap ([\neg \mathrm{p}_{i_1}]_{\mathfrak{M}} \cap \ldots \cap [\neg \mathrm{p}_{i_m}]_{\mathfrak{M}}) \neq \emptyset$, then noVK_Σ holds; (b) if $\cup\$_{\mathfrak{M}}(w) \cap ([\neg \mathrm{p}_{i_1}]_{\mathfrak{M}} \cap \ldots \cap [\neg \mathrm{p}_{i_m}]_{\mathfrak{M}}) = \emptyset$, then $(\cup\$_{\mathfrak{M}}(w) \cap [\mathrm{p}_1 \vee \ldots \vee \mathrm{p}_n]_{\mathfrak{M}}) = \cup\$_{\mathfrak{M}}(w)$ and the case is trivial again. (To be continued)

A Simple Proof of noVK_Σ (part II)

Finally, suppose that $(\cup\$_{\mathfrak{M}}(w)\cap[C_1 \wedge \ldots \wedge C_n]_{\mathfrak{M}}) \neq \cup\$_{\mathfrak{M}}(w)$. So $\cup\$_{\mathfrak{M}}(w)\cap[\neg C_i]_{\mathfrak{M}} \neq \emptyset$ and $(\cup\$_{\mathfrak{M}}(w)\cap[C_i]_{\mathfrak{M}}) \neq \cup\$_{\mathfrak{M}}(w)$ for some i between 1 and n. It follows from the previous result that $r_{\mathfrak{M}}(C_i, w) \neq \emptyset$. Since $r_{\mathfrak{M}}(C_i, w) \neq \emptyset$, $C_i \in \mathfrak{c}(C_1 \wedge \ldots \wedge C_n)$, and $r_{\mathfrak{M}}(C_1 \wedge \ldots \wedge C_n, w) = \cup\{r_{\mathfrak{M}}(A, w) \mid A \in \mathfrak{c}(C_1 \wedge \ldots \wedge C_n)\}$, it follows that $r_{\mathfrak{M}}(C_1 \wedge \ldots \wedge C_n, w) \neq \emptyset$. Q.E.D.

A Simple Proof of TF-cover$_\Sigma$

Proof: Suppose that ψ is a TF-consequence of ϕ. If ψ is a tautology, then $r_{\mathfrak{M}}(\psi, w)$ is $\emptyset$ for any w and $\mathfrak{M}$. So $r_{\mathfrak{M}}(\psi, w) \subseteq r_{\mathfrak{M}}(\phi, w)$. Suppose that ψ is not a tautology on the other hand, then CCNF(ψ) is still a TF-consequence of CCNF(ϕ) by our initial assumption and the fact that every formula is TF-equivalent to its CCNF. But then, by the definition of $\mathfrak{c}$, $\mathfrak{c}$(CCNF(ψ)) is a subset of $\mathfrak{c}$(CCNF(ϕ)). By the definition of $r_{\mathfrak{M}}$ and the fact that $\mathfrak{c}$(CCNF(ψ)) is a subset of of $\mathfrak{c}$(CCNF(ϕ)), $r_{\mathfrak{M}}$(CCNF(ψ), w) $\subseteq$ $r_{\mathfrak{M}}$(CCNF(ϕ), w) for any w in any model $\mathfrak{M}$. Since $r_{\mathfrak{M}}(\phi, w) = r_{\mathfrak{M}}$(CCNF($\phi$), w) and $r_{\mathfrak{M}}(\psi, w) = r_{\mathfrak{M}}$(CCNF($\psi$), w) for any w and $\mathfrak{M}$, it follows that $r_{\mathfrak{M}}(\psi, w) \subseteq r_{\mathfrak{M}}(\phi, w)$ for any w and $\mathfrak{M}$. Q.E.D.

contrast/enough$_\Sigma$ & e-fallibilism$_\Sigma$

- Like Holliday, we agree that one may know a proposition ϕ by eliminating some ϕ-alternatives. For example, if p is an ordinary empirical proposition while q is a 'heavy-weight' proposition that is impossible to know by empirical method, there is no way to know 'p or q' except by knowing p, i.e., by eliminating all relevant ¬p-possibilities. Since these relevant ¬p-possibilities may also be q-possibilities (and therefore p-or-q-possibilities), contrast/enough$_\Sigma$ is violated.
- Even though e-fallibilism$_\Sigma$ is violated, there is a weaker form of fallibilism that is sustained: $\exists\phi \in \Sigma\ \exists\psi \in \Sigma$: $r_{\mathfrak{M}}(\phi, w) \subseteq [\psi]_{\mathfrak{M}}$ and it is not the case that $(W - [\phi]_{\mathfrak{M}}) \subseteq [\psi]_{\mathfrak{M}}$. There is no reason why this weaker form should not be called 'falliblism'.

That's all, Folks.