LOGIC in the COMMUNITY

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1 Reasoning about Social Relations

Communities consist of individuals bounds together by social relationships and roles. Within communities, individuals reason about each other's beliefs, knowledge and preferences. Knowledge, belief, preferences and even the social relationships are constantly changing, and yet our ability to keep track of these changes is an important part of what it means to belong to a community. In the past 50 years, our patterns of reasoning about knowledge, beliefs and preferences have been extensively studied by logicians (cf. notably, [12], [19], [1], [6], [10], [18], and [17].), but the way in which we are influenced by social relationships has received little attention. The country and culture in which we are born, our families, friends, partners or work colleagues all play a part in the formation, rejection and modification of our attitudes. One might update one's beliefs about the impact of human activities on climate change after reading a scientific report, become vegetarian after moving to California, decide to change one's appearance because of peer pressure, vote for a candidate one doesn't like personally for the sake of one's department, or argue in a court of law for the innocence of someone one believes to be guilty. From the perspective of individual rationality, such changes are difficult to understand, but they are not arbitrary and are governed by norms that we internalise as readily as the rules of logic. It is the logic of these internalised norms of social behaviour, a social conception of rationality, that we intend to investigate from the standpoint of logic.

This paper lays out the problems we wish to address, with a view to promoting the logic of community as an interesting area of research in applied philosophical logic. As a small test case, we will provide some technical details for the first of the following examples. But the main aim of our paper is to describe what we take to be a coherent and fruitful topic for future research, some of which is already under way.

1.1 Facebook Friends

Perhaps the simplest example of a social structure is that of online communities such as Facebook (`www.facebook.com`). Cutting out bells and whistles, the structure is just that of a symmetric relation of 'friendship'. The relation is not necessarily transitive and is arguably irreflexive. Yet even with this simple structure, we get an interesting model of communities. Define an agent's 'community' to be the set of her friends (together with the agent herself, if friendship is not reflexive). In this way, we get a picture of the social world as a collection of overlapping communities. Even this is a little more subtle and flexible than the naive view of communities as isolated groups: nations, schools, families, etc. Moreover, we also get a simple model of the interaction between social relations and propositional attitudes, specifically knowledge. Privacy protocols ensure that information can be restricted to be viewable only to one's friends (in theory at least!). This can be implemented (in logic) with an announcement operator – details will be given in Section 2 below. The social network itself is also subject to change: one can add or remove friends, so altering both social and epistemic relations.

1.2 Distributed knowledge

> I have a friend in Minsk, who has a friend in Pinsk, whose friend in Omsk, has friend in Tomsk, with friend in Akmolinsk. His friend in Alexandrovsk has friend in Petropavlovsk, whose friend somehow is solving now the problem in Dnepropetrovsk. (Tom Lehrer, Lobachevsky)

Within any community, knowledge propagates via social relations. Actual transmission of knowledge depends on communication but as a first approximation, one can reason about who knows what on the basis of social relations. If you tell your colleague about some important secret at work, the chances are that his or her lover will also know. The same is true of academic networks, as immortalised by Tom Lehrer. A network of 'friends' gives rise to degrees of accessibility of information, which can be captured by the sequence of propositions

0. I know p
1. I have a friend who knows p
2. I have a friend who has a friend who knows p

∞ I am connected by friendship to someone who knows that p

The inference from p being accessible to an agent to that agent knowing p is defeasible, but there is no need to build in such defeasibility to a logic of distributed knowledge. Instead, one can reason directly about accessible information, defining an operator 'I have access to the information that p' or even the more fine-grained 'I have access in n steps to the information that p' for proposition n in the above list.

1.3 Social Information flow

The mechanism for distributing knowledge is communication, which is a hugely complex matter in reality. Nonetheless, one can model the transmission of information within a simple social network on the assumption that announcements are made to friends. This gives an analogous sequence of propositions:

0. I tell a friend that p
1. My friend tells her friends that p
2. My friends' friends' tell their friends that p

∞ I told your secret to only one person, but now everyone knows!

By modelling this using announcement operators restricted to friends, we get an elementary logic of gossip.

1.4 Deference to expert opinion

Whereas friendship is a symmetric relation, the way in which our attitudes are shaped by society is typically asymmetric. Different people have different access to information and different capacities to absorb, process and transmit it. Our reasoning about knowledge in the community often takes these asymmetries into account. A clear example is the way in which we defer to expert opinion on matters that require specific training, ability or experience. In a court of law, or in policy making committees, the testimony of experts carries more weight than the opinion of ordinary folk. Even in our daily lives, when we seek council from older or more experienced members of our communities, we do it with an attitude of deference. If the opinion of the expert is in line with my own, then I may feel more confident in my attitudes but there is little practical consequence. The interesting case is when there is some difference.

Suppose I initially believe that $\sim p$ but that I consult Prof. X, who is an expert on matters to do with p. Prof. X is of the opinion that p. In reasoning about community belief, it may be sufficient to leave it at this point. There is a conflict, but Prof. X's opinion is that of an expert. Thus we have

1. I believe $\sim p$
2. Concerning p, I defer to the opinion of Prof X
3. Prof X believes p

This is consistent, but if I do nothing with Prof. X's advice, then I have wasted my time in consulting him, at possibly also the large sum of money paid as his fee. One action I might take is to change my belief to $\sim p$. A great deal has been written on the subject of belief revision and much of it applies to the analysis of this situation. But another possibility is for me to defer to Prof. X in the sense that I act in accordance with his belief, taking it to be a safer guide to action, but retaining a private conviction in $\sim p$. This is consistent but requires further analysis of the relation between belief, desire and action, all of which may also have a social dimension. Further possibilities are opened when considering the behaviour of groups. We, as a society, may agree to consult a panel of scientific experts when formulating policy regarding climate change. Given the expert opinion that significantly unpleasant consequences will result if the rate of carbon emissions is not drastically reduced (p for short), we may revise the group belief accordingly, admitting that some members of the group – perhaps most of them – retain their belief that $\sim p$. For all this to be modelled in a logical system, we must have a mechanism for belief aggregation that is sensitive to social relations within the group.

1.5 Peer pressure

Deference to experts requires the addition of asymmetric social relationships to our model. But even within the symmetric friendship model, there is an asymmetry between myself and others. Suppose that I have a magnificent, well-developed and well-groomed handlebar moustache. I like it very much but most (perhaps all) of my friends think it is ridiculous and are even somewhat embarrassed to be seen with me in public. Our preferences are clear. Other things being equal, I prefer to have the moustache than not to have it, but my friends prefer the opposite. We can then define 'peer pressure' as adopting a deferential attitude to one's friends. Although the friendship relation is symmetric, it is important that it is also irreflexive: I am not my own friend. There are two further differences with the case of deference to expert opinion. First, the attitude involved is preference rather than belief. And second, one is deferential not to an individual but to a group. We therefore have to employ techniques of preference aggregation. Interestingly, the group is indexically determined. When bowing to peer pressure, I am deferential to the aggregated preference of *my* friends.

1.6 Community norms

A somewhat similar scenario occurs whenever individual preferences are contrary to community norms, such as paying one's taxes. Everyone (let's assume) prefers not to pay tax but also prefers that everyone else in the community does pay tax. The Golden Rule of many ethical systems tells us what we ought to do in such situations but duty and preference may diverge. Again we have the logical structure of peer pressure but with an added asymmetry. In this case, everyone's preferences are the same *de re*; they only differ *de se*. Fleshing this out a bit, letting $T(x)$ stand for the predicate 'x pays his/her taxes', we can say that everyone agrees to the following indexical proposition

I prefer $\sim T(I)$ and $T(x)$ for all $x \neq I$

The socially acceptable resolution of this problem is for everyone to adopt a deferential attitude to the group's aggregated preference, which if we assume that a majority of $n-1$ to 1 is sufficient for suitably large n, results in everyone paying their taxes - even if they retain a private preference not to.

1.7 Mutual subordination

Our final example of an interesting puzzle concerning logic in communities commonly arises in more intimate settings. There is a young couple, a boy and a girl, desperately in love and yet lacking a little in self-assurance. They have just moved in together. All is well except for one small problem about their sleeping arrangements. Both are used to sleeping on the right side of the bed and they both prefer this strongly. Yet they also both prefer to sacrifice their personal preferences in favour of the other - such is the power of love. Communication is obviously the answer to this problem but they are faced with the paradox of Mutual Subordination:

1. He prefers to sleep on the right.
2. He knows that she also prefers to sleep on the right.
3. He is deferential to her preferences and so revises his preference to that of sleeping on the left.
4. She does the same.

5. Now they both prefer to sleep on the left.

The last two examples have a game theoretical flavour and similar game theoretical scenarios have been extensively studied.[4] Our analysis, however, stresses the interplay between individuals and their communities, and how each is affected in attitude attribution, something which is hardly captured in a utility oriented calculus. For instance, there is a distinction between *my* preferences, *my friends'* preferences and *our* aggregated preferences. In a lot of cases, *my* preferences may not correspond to those of my friends, nor to *our* aggregated preferences. When *I* am saying that x is preferable, I might be reporting my own preference, that of my friends, or that of our community. I might in the same day say that "x is preferable" and "x is not preferable" without contradicting myself, as I might be reporting preferences in the name of my community on one occasion and my own on another occasion. We think that rationality for community has to accommodate this if it is to make sense at all, and the logic of the next section is devised with this purpose in mind.

2 Facebook Logic

In the remainder of the paper, we develop an epistemic logic of communities. This logic emphasises the multi-faceted attitude analysis of the above examples with a two-dimensional approach, one dimension standing for each agent's epistemic possibilities, the second for each agent's community (one's friends). As a starting point for this new paradigm of research, we sketch an approach to modelling the first of the applications mentioned above: that of Facebook Friends.

Define a **social network** $\langle A, \asymp \rangle$ to consists of a set A of *agents* and a binary relation $\asymp$ of *friendship* between agents that is irreflexive and symmetric. In the simplest case, we will only be interested in one propositional attitude: knowledge. For this, we adopt a minor variant of the standard definition from epistemic logic (e.g. [6]). An **epistemic model** $\langle W, A, \sim, V \rangle$ consists of a set W of *epistemic alternatives*, a set A of *agents*, a partial equivalence relation $\sim_a$ on W for each agent a in A, and a propositional valuation function V, assigning a subset of $W \times A$ to each propositional variable.

There are two main differences from the standard definition. First the relation $\sim_a$, which is interpreted by the relation between epistemic alternatives of being indistinguishable by a, is a partial equivalence relation.

[4] One can even trace back such analyses to traditional community wisdom, for instance in Indian culture, the so-called *tragedy of the commons* and *Birbal story* (cf. [14]).

This means that it is symmetric and transitive but not necessarily reflexive. We do not insist on reflexivity because we allow for the possibility that some epistemic alternatives have been ruled out by some but not all of the agents; this will be important when we consider the dynamics of announcements. Second, propositional variables (and formulas more generally) are interpreted as expressing *indexical* propositions, represented as subsets of $W \times A$ instead of subsets of W.

Now, combining the two ideas we define an **epistemic social network model** M to consists of a social network model $\langle A, \asymp_w \rangle$ for each w in W and an epistemic model $\langle W, A, \sim, V \rangle$. A social network model is linked to each epistemic alternative so that we can represent an agent's ignorance about the structure of the social network. We use indexical modal operators K and F, read as 'I know that' and 'all my friends' with a semantics in which satisfaction is relative to both an epistemic alternative w and an agent a. The salient clauses are:

$$
\begin{array}{lll}
M, w, a \models p & \text{iff} & (w, a) \in V(p) \\
M, w, a \models K\varphi & \text{iff} & M, v, a \models \varphi \text{ for every } v \sim_a w \\
M, w, a \models F\varphi & \text{iff} & M, w, b \models \varphi \text{ for every } b \asymp_w a
\end{array}
$$

A simple example illustrates the difference between the alternations of modalities K and F. Let p be the proposition 'I am in danger'. Then

KFp : I know that all my friends are in danger
FKp : Each of my friends knows that s/he is in danger

We define the existential duals as usual: $\langle K \rangle = {\sim}K{\sim}$, $\langle F \rangle = {\sim}F{\sim}$.

2.1 Distributed Knowledge

The basic scenario of Distributed Knowledge, as discussed above, can be represented as follows:

${\sim}(Kp \vee K{\sim}p)\ \&\ \langle F \rangle (Kp \vee K{\sim}p)$
I don't know whether p, but I have a friend who does.
Kp, $\langle F \rangle Kp$, $\langle F \rangle \langle F \rangle Kp$, etc.
I know p, I have a friend who knows p, I have a friend who has a friend who knows p, etc.
$\langle F^* \rangle Kp$
I am connected by friendship to someone who knows that p

The latter requires a new operator, F^*, which can be introduced (following *PDL* in [11]) as the modality of the transitive closure of the friendship relation.

2.2 Talking about friends

To talk about your friends, you need to give them *names*. We therefore introduce a syntactic category of nominals and extend the valuation function V to apply to nominals as well as propositional variables (for further details of hybrid logic, we refer to [3]). We will assume that names are 'rigid designators' in the epistemic sense, i.e., that every agent knows who is whom. So for each nominal n we insist that

there is an agent $\underline{n} \in A$ such that for all $a \in A$ and $w \in W$, $(w, a) \in V(n)$ iff $a\underline{n}$.

Now we can say

$\langle F \rangle n$: n is my friend

Also borrowed from hybrid logic is an operator $@_n$ for shifting the evaluation to the agent named n. This enables us to say

$@_n Kp$: n knows that p

Finally, another hybrid logic device: a way of *indexically referring* to the current agent. This is provided by the operator $\downarrow x$ which names the current agent 'x'. This enables us to express some nice interactions between friendship and knowledge:

$\downarrow x \langle F \rangle K @_n \langle F \rangle x$: I have a friend who knows that n is friends with me.

To capture the semantics of $\downarrow x$ we need the help of an assignment function g assigning agents to variables. Variables are of the same syntactic category as nominals and so we also write $\underline{x}$ for $g(x)$. With the help of assignment functions, we get the following satisfaction conditions:

$$
\begin{array}{lll}
M, g, w, a \models x & \text{iff} & g(x) = a \\
M, g, w, a \models @_n \varphi & \text{iff} & M, g, w, \underline{n} \models \varphi \\
M, g, w, a \models \downarrow x \varphi & \text{iff} & M, g^x_a, w, a \models \varphi
\end{array}
$$

where, as usual, g^x_a is defined by $g^x_a(y) = a$ if $x = y$ and $g(y)$ otherwise.

2.3 Indexical public announcements

In dynamic epistemic logic, the result of publicly announcing that p is given by eliminating epistemic alternatives in which p is not true.[5]. The operator $[!\varphi]$ for 'after announcing φ' is defined by

5 Public announcement logic was introduced in [15]

$$M, w \models [!\varphi]\psi \quad \text{iff} \quad \text{if } M, w \models \varphi \text{ then } M_\varphi, w \models \psi$$

where M_φ is the result of restricting M to the set of epistemic alternatives v such that $M, v \models \varphi$. The logic is pleasingly simple, thanks to the following (now well-known) reduction axioms:

$$\begin{array}{l}
[!\varphi]p \;\equiv\; (\varphi \supset p) \\
[!\varphi]{\sim}\psi \;\equiv\; (\varphi \supset {\sim}[!\varphi]\psi) \\
[!\varphi](\psi_1 \,\&\, \psi_2) \;\equiv\; ([!\varphi]\psi_1 \,\&\, [!\varphi]\psi_2) \\
[!\varphi]K_a\psi \;\equiv\; (\varphi \supset K_a[!\varphi]\psi)
\end{array}$$

With these axioms, a completeness result for the base epistemic logic can be lifted to its dynamic extension. A crucial feature of the operator is the restriction to announcements that are true. Without this, the model M_φ would not contain w and the satisfaction condition would be rendered meaningless.

To interpret public announcement in indexical epistemic models, we give the obvious definition:

$$M, w, a \models [!\varphi]\psi \quad \text{iff} \quad \text{if } M, w, a \models \varphi \text{ then } M_{a,\varphi}w, a \models \psi$$

where $M_{a,\varphi}$ is the restriction of M to those epistemic alternatives v such that $M, v, a \models \varphi$. But there is a problem: public announcements cannot be reduced when we add the hybrid shifting operator $@_n$. The equivalence

$$[!\varphi]@_n\psi \;\equiv\; (\varphi \supset @_n[!\varphi]\psi)$$

is not in general valid. If φ is a non-indexical proposition then the equivalence holds: in fact we have the simpler equivalence $[!\varphi]@_n\psi \equiv @_n[!\varphi]\psi$. Since the truth of a non-indexical φ does not depend on the agent, it does not matter which agent announces it. But when the truth of φ is indexical, varying by agent, then the equivalence breaks down.

Suppose, for example that a but not $\underline{n}$ is in danger. Then evaluating at a, the following two propositions are not equivalent:

1. $[!p]@_n p$
   After I announce that I am in danger, n is in danger.
2. $(p \supset @_n[!p]p)$
   If I am in danger then after n announces that he is in danger, he is in danger.

Proposition 1 can easily be falsified; there is no implication from a's being in danger to $\underline{n}$'s being in danger. But Proposition 2 is true: if $\underline{n}$ is not in danger then he cannot announce that he is and so the consequent of the conditional is trivially true. Moreover, there is no way of avoiding the problem. To do so, we would need an announcement by $\underline{n}$ that is equivalent to a's announcement, but for indexical announcements this is impossible.

Our solution is to introduce a new operator $[n!\varphi]$ for 'after n announces φ', with satisfaction conditions

$$M, g, w, a \models [n!\varphi]\psi \quad \text{iff} \quad \text{if } M, g, w, \underline{n}, \models \varphi \text{ then } M_{\underline{n},\varphi}, g, w, a \models \psi$$

This has the advantage of admitting reduction equivalences as follows:

$$\begin{array}{l}
[n!\varphi]p \;\equiv\; (@_n\varphi \supset p) \\
[n!\varphi]{\sim}\psi \;\equiv\; (@_n\varphi \supset {\sim}[n!\varphi]\psi) \\
[n!\varphi](\psi_1 \,\&\, \psi_2) \;\equiv\; ([n!\varphi]\psi_1 \,\&\, [n!\varphi]\psi_2) \\
[n!\varphi]@_m\psi \;\equiv\; @_m[n!\varphi]\psi \\
[n!\varphi]\downarrow\! x\psi \equiv\; \downarrow\! x[n!\varphi]\psi \\
[n!\varphi]F\psi \;\equiv\; F[n!\varphi]\psi \\
[n!\varphi]K\psi \;\equiv\; K[n!\varphi]\psi
\end{array}$$

(with a change of bound variables in the line for $\downarrow\! x$, if necessary.)

The new operator also allows us to recover reduction for the indexical notion of public announcement via the equivalence

$$[!\varphi]\psi \equiv\; \downarrow\! x[x!\varphi]\psi$$

in which x is a new variable.

2.4 Talking to friends

Of greater interest to logic in the community is the possibility of making announcements only to one's friends. Here we adopt a simplistic approach, noting some of its limitations and a direction for further research.

We define an operator $[F!\varphi]$ for 'after I announce φ to my friends' by

$$M, g, w, a \models [F!\varphi]\psi \quad \text{iff} \quad \text{if } M, g, w, a \models \varphi \text{ then } M', g, w, a \models \psi$$

where $M' = \langle W, A, F, \sim', V\rangle$ has the same set W of epistemic alternatives as M but has an indistinguishability relation $\sim'$ defined as follows:

if $b \asymp a$ then
$$u \sim'_b v \quad \text{iff} \quad u \sim_b v \text{ and } M, g, u, a \models \varphi \text{ and } M, g, v, a \models \varphi$$
otherwise $\sim'_b = \sim_b$

In other words, the epistemic indistinguishability relation of agents that are not friends with a remains unchanged, but that of a's friends is changed so as to remove links between alternatives that are incompatible with a's announcement.

For example, suppose that $\underline{n}$ is in danger (p) and announces this to her friends. After the announcement, all of $\underline{n}$'s friends will know $@_n p$ that $\underline{n}$ is in danger. So the formula

$$@_n[F!p]FK@_n p$$

is valid.

Scenarios of this kind are somewhat similar to what is called *private announcement* in [4]. Our way of handling the announcement here is to take them to be *soft information* (see detailed discussions on soft information vs. hard information in [16]). We think that the approach of product update with event model in [4] can be adapted to this context, too.

3 Prospects

The sketch of Facebook Logic is only a beginning. Even within this simple model of communities there is much to investigate. We hope that this case study has shown the readers where we are heading: our goal is to use recent developments in dynamic logics of knowledge, belief and preference to model the subtleties of the communication and relationship between agents in communities. Going back to the topics outlined in Section 1, there are immediate directions we would like to explore. Due to limitations of space, we finish with a few preliminary remarks on how to proceed.

1. **Preference and belief** To model preference and belief, we can introduce two orderings to the model, one for preference relation, one for plausibility relation, between alternatives. From there, we can consider changes in preference and beliefs within communities, again extending the existing framework on preference change and belief revision, e.g., [16], [13], [5] and [9].
2. **Dominance** To model *asymmetric* social relations, we can replace friendship relation in the model with a new preorder S (for 'is subordinate to') between agents, and investigate, for instance, what the paradox of mutual subordination in Section 1.7 means to us within communities.
3. **Aggregation** As agents are modelled explicitly, we can easily add groups of agents by imposing an algebra on the set of A, such as a

semilattice $\sqcup$ whose atoms are interpreted as individuals. Different aggregation procedures can be defined in terms of the structure of the social network, see studies in this line [2], [7] and [8].

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