Descriptor Revision, Epistemic Proximity Ordering and Believability Relation

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  - $Cn(X) = Cn(Cn(X))$ (iteration)
  - If $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$ (Monotony)

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- If $\phi \in Cn(X)$, then $\phi \in Cn(X')$ for some finite subset $X' \subseteq X$. (compactness)

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Notations:

- $\phi \vdash \psi$: $\psi \in Cn(\{\phi\})$.
- K: agent's current belief state.

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- Contraction $\div$: a specified sentence has to be removed from the epistemic agent's set of beliefs
- Revision $\star$: a specified sentence has instead to be consistently added.

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- To employ a choice function to select a number of belief sets from a **remainder set** each of which satisfies the **success condition** for the operation (namely that of not containing the specified sentence).

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Revision

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- Then take their intersection as outcome.

Limitations and problems of AGM

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- We need a “select-direct” approach.

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- $\{\mathfrak{B}\mathfrak{B}\phi\}$ and $\{\phi \wedge \mathfrak{B}\phi\}$ are not descriptors.

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- *Descriptor revision* $\circ$ is an operation that can be applied to any success condition expressed by descriptor: $K \circ \Phi$ yields a belief set for all Φ.
- How to specify $K \circ \Phi$?

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- $\mathbb{X}$ is a *Outcome set* with respect to K iff it is a set of belief sets with $K \in \mathbb{X}$.
- f is a *monoselective choice function* for $\mathbb{X}$ iff it holds that if $\varnothing \neq \mathbb{Y} \subseteq \mathbb{X}$ then $f(\mathbb{Y}) \in \mathbb{Y}$, and otherwise $f(\mathbb{Y})$ is undefined.

Semantics 2

- Let $\mathbb{X}$ be a outcome set (with respect to K), f a monoselective choice function for $\mathbb{X}$ and $\mathbb{X}^{\Phi}$ denote $\{X \in \mathbb{X} \mid X \Vdash \Phi\}$.

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- We say a descriptor revision $\circ$ is *based on* ordered pair $(\mathbb{X}, f)$ iff

$$K \circ \Phi = \begin{cases} f(\mathbb{X}^{\Phi}) & \text{if } \mathbb{X}^{\Phi} \neq \varnothing, \\ K & \text{otherwise.} \end{cases}$$

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- We call the ordered pair $(\mathbb{X}, f)$ *monoselective model* (for descriptor revision) and $\circ$ based on this kind of model *monoselective (descriptor revision)* operator.

Axiomatic characterization of monoselective $\circ$

$\circ$ is a monoselective operator iff it satisfies:

- $Cn(K \circ \Phi) = K \circ \Phi$ (closure)
- $K \circ \Phi \Vdash \Phi$ or $K \circ \Phi = K$ (relative success)
- $K \circ \Phi \Vdash \Psi$, then $K \circ \Psi \Vdash \Psi$ (regularity)
- If $\Phi \dashv\Vdash \Psi$, then $K \circ \Phi = K \circ \Psi$ (extensionality)

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- $\Phi \Vdash \Psi$ iff for any belief set X, if $X \Vdash \Phi$ then $X \Vdash \Psi$
- $\Phi \dashv\Vdash \Psi$ iff $\Phi \Vdash \Psi$ and $\Psi \Vdash \Phi$

Linear descriptor revision

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- One plausible way to construct the monoselective choice function f is to let $K \circ \Phi$ be closest element of the outcome set that satisfies Φ. This requires an ordering or distance relation on outcome set.

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Observation

If $(\mathbb{X}, \leqq)$ is a relational model for descriptor revision, then $\leqq$ is linear (transitive, anti-symmetric and complete)

A descriptor revision operator $\circ$ is based on relational model $(\mathbb{X}, \leqq)$ iff

$$K \circ \Phi = \begin{cases} \mathbb{X}^{\Phi}_{<} & \text{if } \mathbb{X}^{\Phi} \neq \varnothing, \\ K & \text{otherwise.} \end{cases}$$

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We call this kind of $\circ$ *linear (descriptor revision) operator*.

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Observation

If $\circ$ satisfies relative success, regularity and reciprocity, then it satisfies:
If $\Phi \dashv\Vdash \Psi$, then $K \circ \Phi = K \circ \Psi$ (extensionality)

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- It is assumed that there is **a set $\mathbb{X}$ of potential outcomes** of belief change, and the mechanism of change is a **direct choice** among these potential outcomes.
- In the linearly ordered form of descriptor revision, this choice is based on **a linear relation** on $\mathbb{X}$ that can be interpreted as **representing distance** from K.

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  - If $\psi \leq \phi$ for all ψ, then $\vdash \phi$. (maximality)

An alternative model

We say a contraction operator $\div$ is *based on* an epistemic entrenchment ordering $\leqslant$ iff

- $\phi \in K \div \psi$ if and only if $\phi \in K$ and either $\psi \lessdot \phi \vee \psi$ or $\psi \in Cn(\varnothing)$.

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Observation

$\div$ is a transitively partial meet contraction operator iff it is based on a standard epistemic entrenchment ordering.

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- Intuitively: $\Phi \leq \Psi$ (Φ is at least as epistemically proximate as Ψ) if and only if the change in the belief system required to satisfy Φ is not larger (more radical) than that required to satisfy Ψ.
- From the point of view of relational model: $\Phi \leq \Psi$ if and only if the distance from K to the closest Φ-satisfying potential outcome is not longer than that to the closest Ψ-satisfying potential outcome.

Formal definition

A binary relation $\preceq$ (with strict part $\prec$ and symmetric part $\simeq$) on descriptors is a *epistemic proximity ordering* iff there exists a relational model $(\mathbb{X}, \leqq)$ such that

> $\Phi \preceq \Psi$ if and only if either (i) there exists $\mathbb{X}^{\Phi}_{<}$ and $\mathbb{X}^{\Psi}_{<}$ with $\mathbb{X}^{\Phi}_{<} \leqq \mathbb{X}^{\Psi}_{<}$ or (ii) Ψ is not satisfiable in $\mathbb{X}$.

Representation theorem for $\preceq$

Let $\bot = \{\mathfrak{B}\phi \wedge \neg\mathfrak{B}\phi\}$. $\preceq$ is a epistemic proximity ordering iff it satisfies

- If $\Phi \preceq \Psi$ and $\Psi \preceq \Xi$, then $\Phi \preceq \Xi$. (transitivity)
- If $\Phi \Vdash \Psi$, then $\Psi \preceq \Phi$. (counter-dominance)
- If $\Phi \simeq \Psi$, then $\Phi \simeq \Phi \cup \Psi$.(coupling)
- Either $\Phi \cup \{\mathfrak{B}\phi\} \simeq \Phi$ or $\Phi \cup \{\neg\mathfrak{B}\phi\} \simeq \Phi$ (amplification)
- $\Phi \prec \bot$ for some Φ. (absurdity avoidance)

Alternative model

Let $\preceq$ be some epistemic proximity ordering and
$[\Phi]_{\preceq} = \{\phi \in \mathcal{L} \mid \Phi \simeq \Phi \cup \{\mathfrak{B}\phi\}\}$.
We say a descriptor revision operator $\circ$ is *based on* $\preceq$ iff

$$K \circ \Phi = \begin{cases} [\Phi]_{\preceq} & \text{if } \Phi \prec \perp, \\ K & \text{if } \Phi \simeq \perp. \end{cases}$$

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Observation

A descriptor revision operator $\circ$ is based on some relational model iff it is based on the epistemic proximity ordering derived from the relational model.

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- $K \star \phi = K \circ \mathfrak{B}\phi$.
- Restricted operators, relational models and epistemic entrenchment orderings.

Restricted relational model

- Let $\mathfrak{D}$ be the set of all descriptors.

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  - For any descriptor $\Phi \in \mathfrak{S}$, if $\mathbb{X}^{\Phi} \neq \varnothing$, then there exists a unique $\leqq$-minimal element (denoted by $\mathbb{X}^{\Phi}_{<}$) in it.
- It does not in general hold that $\leqq$ is linear if $(\mathbb{X}, \leqq)$ is a restricted relational model.

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- $\circ$ is a *relational* restricted descriptor revision operator with respect to $\mathfrak{S}$ iff it is based on some relational model restricted to $\mathfrak{S}$, i.e. for any $\Phi \in \mathfrak{S}$,

$$K \circ \Phi = \begin{cases} \mathbb{X}^{\Phi}_{<} & \text{if } \mathbb{X}^{\Phi} \neq \varnothing, \\ K & \text{otherwise.} \end{cases}$$

Sentential descriptor revision

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- $\circ$ is a *relational* sentential descriptor revision operator iff $\circ$ is sentential descriptor revision operator based on some relational model restricted to $\mathfrak{A}$.
- $K \star \phi = K \circ \mathfrak{B}\phi$: (relational) sentential operator derived from (relational) sentential descriptor revision operator.

Epistemic proximity ordering restricted to $\mathfrak{A}$

- $\leq$ on $\mathfrak{A}$ is an epistemic proximity ordering restricted to $\mathfrak{A}$ iff there exists a relational model restricted to $\mathfrak{A}$ such that $\Phi \leq \Psi$ if and only if $\mathbb{X}^{\Phi}_{<}$ and $\mathbb{X}^{\Psi}_{<}$ exist and $\Phi \leq \Psi$.

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- It is not same as the "either...or..." definition of general epistemic proximity ordering.
- Let $\leq$ be an epistemic proximity ordering restricted to $\mathfrak{A}$ and $Ref(\leq)$ denote the domain of $\leq$. It is possible that $Ref(\leq) \neq \mathfrak{A}$.
- Intuitively: $\Phi \notin Ref(\leq)$ means Φ is not under consideration.

Why a difference

- $Cn(\phi) < Cn(\perp)$ and $Cn(\phi)$: Two different relational sentential descriptor revision operator are derived from them.

Why a difference

- $Cn(\phi) < Cn(\bot)$ and $Cn(\phi)$: Two different relational sentential descriptor revision operator are derived from them.
- Either...or...: The same restricted epistemic proximity ordering is derived from them.

Believability relation

- A binary relation $\trianglelefteq$ (with strict part $\triangleleft$ and symmetric part $\bowtie$) on $\mathcal{L}$ is a *believability relation* iff there exists an epistemic proximity ordering restricted to $\mathfrak{A}$ satisfying the following condition:

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  - $\phi \trianglelefteq \psi$ iff $\mathfrak{B}\phi \preceq \mathfrak{B}\psi$
- Intuitively: $\phi \trianglelefteq \psi$ means belief in ϕ is at least as easily acquired by the agent as belief in ψ.

Axiomatic characterization of believability relation

$\trianglelefteq$ is a believability relation iff it satisfies

- Given $\phi \bowtie \phi \wedge \psi$, (i) $\phi \trianglelefteq \lambda$ if and only if $\phi \wedge \psi \trianglelefteq \lambda$, and (ii) $\lambda \trianglelefteq \phi$ if and only if $\lambda \trianglelefteq \phi \wedge \psi$. (weak transitivity)
- If $\phi \in Ref(\trianglelefteq)$ and $\phi \vdash \psi$, then $\psi \trianglelefteq \phi$. (relative counter-dominance)
- If $\phi \bowtie \phi \wedge \psi$ and $\phi \bowtie \phi \wedge \lambda$, then $\phi \bowtie \phi \wedge (\psi \wedge \lambda)$. (weak coupling)
- $\phi \in K$ if and only if $\phi \in \mathrm{Ref}(\trianglelefteq)$ and $\phi \trianglelefteq \psi$ for all $\psi \in \mathrm{Ref}(\trianglelefteq)$. (relative minimality)

An alternative model for sentential revision

We say an sentential operator $\star$ is based on some believability relation $\trianglelefteq$ if and only if it satisfies:

$$K \star \phi = \begin{cases} \{\psi \mid \phi \bowtie \phi \wedge \psi\} & \text{if } \phi \in \mathrm{Ref}(\trianglelefteq), \\ K & \text{otherwise.} \end{cases}$$

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Observation

Let $(\mathbb{X}, \leqq)$ be some relational model restricted to $\mathfrak{A}$ and $\circ$ the relational sentential descriptor revision operator based on $(\mathbb{X}, \leqq)$. Then, the following two conditions are equivalent:

(1) *$\star$ is the sentential operator derived from $\circ$.*

(2) *$\star$ is based on the believability relation derived from $(\mathbb{X}, \leqq)$.*

Axiomatic characterization

$\star$ is a sentential operator derived from some relational sentential descriptor revision operator (namely based on some believability relation) iff it satisfies following postulates:

- $Cn(K \star \phi) = K \star \phi$ (closure)
- If $K \star \phi \neq K$, then $\phi \in K \star \phi$ (relative success)
- If $\phi \in K$, then $K \star \phi = K$ (confirmation)
- If $\psi \in K \star \phi$, then $\psi \in K \star \psi$ (regularity)
- If $\psi \in K \star \phi$ and $\phi \in K \star \psi$, then $K \star \phi = K \star \psi$ (reciprocity)

More properties on...

...believability relation:

- transitivity: If $\phi \trianglelefteq \psi$ and $\psi \trianglelefteq \lambda$, then $\phi \trianglelefteq \lambda$.

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- maximality: If $\psi \trianglelefteq \phi$ for all ψ, then $\phi \equiv \bot$.
- coupling: If $\phi \bowtie \psi$ then $\phi \bowtie \phi \wedge \psi$.
- complete: Either $\phi \trianglelefteq \psi$ or $\psi \trianglelefteq \phi$.

More properties on...

...relational model restricted to $\mathfrak{A}$

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More properties on...

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- $\leqq$ is linear.
- $Cn(\{\bot\}) \in \mathbb{X}$.
- For any $\phi \not\equiv \bot$, $\mathbb{X}_{<}^{\mathfrak{B}\phi} < Cn(\{\bot\})$.

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- success: $\phi \in K \star \phi$ for all $\phi \in \mathcal{L}$.
- consistency: If $\neg\phi \notin Cn(\varnothing)$, then $K \star \phi$ is consistent.
- strong reciprocity: Given $n \in \mathbb{N}$, if for every $0 \leqslant i < n$, $\phi_i \in K \star \phi_{i+1}$ and $\phi_n \in K \star \phi_0$, then $K \star \phi_0 = K \star \phi_2 = \cdots = K \star \phi_n$.)

Correspondences among these properties

Omit...

Relationships with AGM revision and entrenchment relation

- Given K is consistent, AGM revision operator on K can be exactly characterized by a subset of believability relations which can be specified in an explicit way.

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- Assume elements in $\mathbb{X}$ are all maximal consistent sets, the derived believability relations and related entrenchment relations are inter-definable.

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Thanks for your attention!