Losing Connection: the Modal Logic of Definable Link Deletion

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Outline

Sabotage Game and Sabotage Modal Logic

This work is inspired by the work on Sabotage Game (SG) [10] and Sabotage Modal Logic (SML) [5].

A sabotage game is played on a graph.

- Players: Traveler vs. Demon.
- Action: In each round, Traveler moves along a link to arrive at a goal node, while Demon deletes one link to prevent Traveler.
- Winning Condition: Traveler wins iff he arrives at some goal node.

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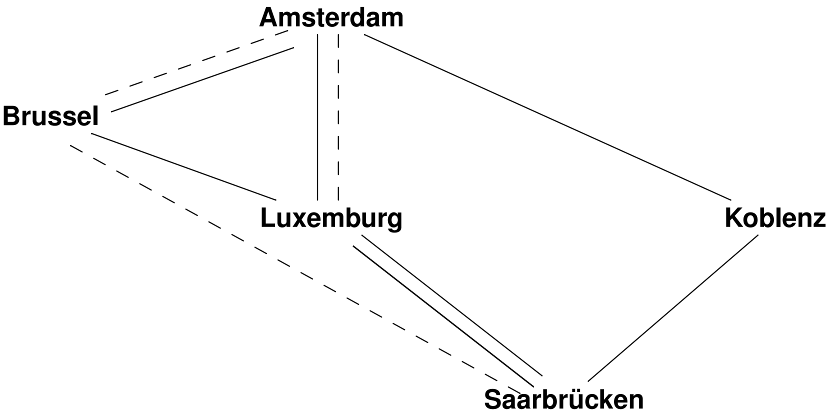
Example (Sabotage Game)



Sabotage Modal Logic

Definition (Language $\mathcal{L}_S$)

Let $\mathbf{P}$ be a countable set of propositional atoms. Formulas of $\mathcal{L}_S$ are defined as follows:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond\varphi \mid \blacklozenge\varphi$$

where $p \in \mathbf{P}$.

Definition (Semantics)

Let $\langle W, R, V\rangle$ be a standard relational model. The truth condition for $\blacklozenge$ is as follows:

$$\langle W, R, V\rangle, w \vDash \blacklozenge\varphi$$

$$\Updownarrow$$

$$\text{there is } \langle u, v\rangle \in R \text{ such that } \langle W, R\backslash\{\langle u, v\rangle\}, V\rangle, w \vDash \varphi$$

The features of the link deletion:

- Global
- Arbitrary
- Stepwise

While, there are also some other cases in every day life. Here we study the update of the relationship in the social network. The link deletion for this case has the following features:

- Local
- Definable
- Uniform

Definable Sabotage Modal Logic (S_dML)

- Language: $\mathcal{L}_d \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box\varphi \mid [-\varphi]\varphi$
  where $p \in \mathbf{P}$ and $\mathbf{P}$ is a countable set of propositional atoms.

- Semantics: We just show the true condition for $[-\ ]$:

$$\mathcal{M}, w \vDash [-\varphi]\psi \iff \mathcal{M}|_{\langle w,\varphi\rangle}, w \vDash \psi$$

  where

  - $\mathcal{M}|_{\langle w,\varphi\rangle} := \langle W, R \setminus (\{w\} \times V(\varphi) \cap R(w)), V\rangle$
  - $R_w := \{w' \in W \mid \langle w, w'\rangle \in R\}$

  Intuitively, $\mathcal{M}, w \vDash [-\varphi]\psi$ means that ψ *is true at w after deleting all links from w to the nodes that are φ*.

Some logical validities:

$$[-\varphi](\varphi_1 \to \varphi_2) \to ([-\varphi]\varphi_1 \to [-\varphi]\varphi_2) \quad (1)$$

$$[-\varphi]\psi \leftrightarrow \langle -\varphi \rangle \psi \quad (2)$$

$$[-\varphi]p \leftrightarrow p \quad (3)$$

$$[-p]\Diamond q \leftrightarrow \Diamond(\neg p \land q) \quad (4)$$

$$[-p][-q]\varphi \leftrightarrow [-q][-p]\varphi \quad (5)$$

Example

Consider the general schema $[-\varphi_1][-\varphi_2]\varphi \leftrightarrow [-\varphi_2][-\varphi_1]\varphi$ for (5). Let $\varphi_1 := p$, $\varphi_2 := \Diamond\Diamond p$, and $\varphi := \Diamond q$. Define a model $\mathcal{M}$ as follows:

$\mathcal{M}, w \models [-p][-\Diamond\Diamond p]\Diamond q$ and $\mathcal{M}, w \not\models [-\Diamond\Diamond p][-p]\Diamond q$.

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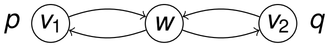

$\mathcal{M}, w \models [-p][-\Diamond\Diamond p]\Diamond q$ and $\mathcal{M}, w \not\models [-\Diamond\Diamond p][-p]\Diamond q$.

First-Order Translation

To translate the SML into first-order logic, we need a finite set of ordered pairs of nodes in the translation. Intuitively, the ordered pairs are links that have already been deleted. Can this method work here?

1. The links deleted by one $[-]$ operator in a formula may be infinite.
2. We should take care of the order of different $[-]$ operators in a formula.

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1. The links deleted by one $[-]$ operator in a formula may be infinite.
2. We should take care of the order of different $[-]$ operators in a formula.

Our method: use a sequence of ordered pairs of the form $\langle x, \varphi \rangle$.

Definition (Standard Translation for S_dML)

Let x be a designated variable, and O be a finite sequence $\langle v_0, \psi_0 \rangle; ...; \langle v_i, \psi_i \rangle; ...; \langle v_n, \psi_n \rangle (0 \leqslant i \leqslant n)$, where $\psi_{0 \leqslant i \leqslant n} \in \mathcal{L}_d$ and $v_{0 \leqslant i \leqslant n}$ is a variable. The main clauses of the translation $ST_x^O : \mathcal{L}_d \to \mathcal{L}_1$ is as follows:

$$ST_x^O(\Diamond\varphi) = \exists y(Rxy \wedge \neg(x \equiv v_0 \wedge ST_y^{\langle x, \perp \rangle}(\psi_0)) \wedge \bigwedge_{0 \leq i \leq n-1} \neg(x \equiv v_{i+1} \wedge ST_y^{\langle v_0, \psi_0 \rangle; \ldots; \langle v_i, \psi_i \rangle}(\psi_{i+1})) \wedge ST_y^O(\varphi))$$

$$ST_x^O([-\varphi_1]\varphi_2) = ST_x^{O; \langle x, \varphi_1 \rangle}(\varphi_2)$$

Bisimulation

Fact

The logic $S_d ML$ is not invariant under the standard bisimulation.

Consider the following two models:

We have $\mathcal{M}_1, w_1 \models [-q]\Diamond\Diamond q$ and $\mathcal{M}_2, v_1 \not\models [-q]\Diamond\Diamond q$.

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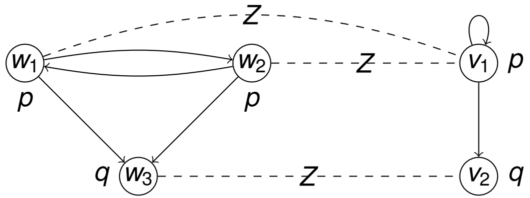

We have $\mathcal{M}_1, w_1 \vDash [-q]\Diamond\Diamond q$ and $\mathcal{M}_2, v_1 \nvDash [-q]\Diamond\Diamond q$.

We introduce a new notion of bisimulation for S_dML, and call it d-bisimulation.

Formally, it extends the standard bisimulation with the following two clauses:

$\mathbf{Zig}_{[-]}$: If $\langle \mathcal{M}_1, w \rangle Z_d \langle \mathcal{M}_2, v \rangle$ and U is definable relative to $R_1(w)$ in $\mathcal{M}_1$, then it holds that $\langle \mathcal{M}_1|_{\langle w,U \rangle}, w \rangle Z_d \langle \mathcal{M}_2|_{\langle v, Z_d(U) \rangle}, v \rangle$.

$\mathbf{Zag}_{[-]}$: If $\langle \mathcal{M}_1, w \rangle Z_d \langle \mathcal{M}_2, v \rangle$ and U' is definable relative to $R_2(v)$ in $\mathcal{M}_2$, then it holds that $\langle \mathcal{M}_1|_{\langle w, Z_d^{-1}(U') \rangle}, w \rangle Z_d \langle \mathcal{M}_2|_{\langle v, U' \rangle}, v \rangle$.

where $Z_d(U) = \{v' \in R_2(v) \mid \langle \mathcal{M}_1, w' \rangle Z_d \langle \mathcal{M}_2, v' \rangle$ for some $w' \in U\}$ and $Z_d^{-1}(U') = \{w' \in R_1(w) \mid \langle \mathcal{M}_1, w' \rangle Z_d \langle \mathcal{M}_2, v' \rangle$ *for some* $v' \in U'\}$. We write $\langle \mathcal{M}_1, w \rangle \underline{\leftrightarrow}_d \langle \mathcal{M}_2, v \rangle$ if there exists a d-bisimulation Z_d s.t. $\langle \mathcal{M}_1, w \rangle Z_d \langle \mathcal{M}_2, v \rangle$.

Theorem ($\underline{\leftrightarrow}_d \subseteq \leftrightsquigarrow_d$)

For any pointed models $\langle \mathcal{M}_1, w \rangle$ and $\langle \mathcal{M}_2, v \rangle$, if $\langle \mathcal{M}_1, w \rangle \underline{\leftrightarrow}_d \langle \mathcal{M}_2, v \rangle$, then $\langle \mathcal{M}_1, w \rangle \leftrightsquigarrow_d \langle \mathcal{M}_2, v \rangle$.

Theorem ($\leftrightsquigarrow_d \subseteq \underline{\leftrightarrow}_d$)

For any ω-saturated $\langle \mathcal{M}_1, w \rangle$ and $\langle \mathcal{M}_2, v \rangle$, if $\langle \mathcal{M}_1, w \rangle \leftrightsquigarrow_d \langle \mathcal{M}_2, v \rangle$, then $\langle \mathcal{M}_1, w \rangle \underline{\leftrightarrow}_d \langle \mathcal{M}_2, v \rangle$.

Theorem (Characterization of S_dML by d-bisimulation Invariance)

An $\mathcal{L}_1$-formula is equivalent to the translation of an $\mathcal{L}_d$-formula iff it is invariant for d-bisimulation.

S_dML and Hybrid Logics

The recursion axioms for Boolean cases are as usual. Namely,

$$[-\varphi]p \leftrightarrow p$$
$$[-\varphi]\neg\psi \leftrightarrow \neg[-\varphi]\psi$$
$$[-\varphi](\psi_1 \wedge \psi_2) \leftrightarrow [-\varphi]\psi_1 \wedge [-\varphi]\psi_2$$

But what should the principle for $[-\varphi]\Box\psi$ be like?

$$[-\varphi]\Box\psi \leftrightarrow ?$$

We focus on the hybrid logic with *nominals*, *at operator* @ and *down-arrow* $\downarrow$, which is denoted by $\mathcal{H}(\downarrow)$.

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Definition (The Hybrid Translation for S_dML)

Let O be a finite sequence of pairs of variables of nominals and properties, denoted with $\langle x_0, \psi_0\rangle; \ldots; \langle x_i, \psi_i\rangle; \ldots; \langle x_n, \psi_n\rangle (0 \leq i \leq n)$. The translation $T^O : \mathcal{L}_d \to \mathcal{H}(\downarrow)$ is recursively defined in the following way:

$$\begin{array}{rcl}
T^O(p) & = & p \\
T^O(\neg\varphi) & = & \neg T^O(\varphi) \\
T^O(\varphi_1 \wedge \varphi_2) & = & T^O(\varphi_1) \wedge T^O(\varphi_2) \\
T^O(\Diamond\varphi) & = & \downarrow x \Diamond(\neg(@_x x_0 \wedge T^{\langle x_0, \bot\rangle}(\psi_0)) \wedge \\
 & & \bigwedge_{0 \leqslant i \leqslant n-1} \neg(@_x x_{i+1} \wedge T^{\langle x_0, \psi_0\rangle;\ldots;\langle x_i, \psi_i\rangle}(\psi_{i+1})) \wedge T^O(\varphi)) \\
T^O([-\psi]\varphi) & = & \downarrow x T^{O;\langle x, \psi\rangle}(\varphi)
\end{array}$$

Fact

$\mathcal{H}(\downarrow)$ is more expressive than S_dML over models.

We first show the axiom for formula $[-\varphi_1][-\varphi_2]\psi$:

$$[-\varphi_1][-\varphi_2]\psi \leftrightarrow \downarrow x[- \downarrow y(\varphi_1 \vee @_x[-\varphi_1]@_y\varphi_2)]\psi$$

For formula $[-\varphi]\Box\psi$, we have the following equivalence:

$$[-\varphi]\Box\psi \leftrightarrow \downarrow x\Box \downarrow y(\neg\varphi \to @_x[-\varphi]@_y\psi)$$

But it is not the solution!

$$[-\varphi]@_x p \leftrightarrow @_x p$$
$$[-\varphi]@_x\neg\psi \leftrightarrow \neg[-\varphi]@_x\psi$$
$$[-\varphi]@_x(\psi \wedge \chi) \leftrightarrow [-\varphi]@_x\psi \wedge [-\varphi]@_x\chi$$
$$[-\varphi]@_x\Box\psi \leftrightarrow \downarrow y@_x\Box \downarrow z(\neg(\varphi \wedge @_x y) \to @_y[-\varphi]@_z\psi)$$

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Undecidability of S_dML

Theorem

The logic S_dML does not have the tree model property.

$$(R_1) \qquad p \wedge \Diamond p \wedge \Diamond \neg p$$
$$(R_2) \qquad \Box(p \rightarrow \Diamond p \wedge \Diamond \neg p)$$
$$(R_3) \qquad [-\neg p]\Box\Box p$$

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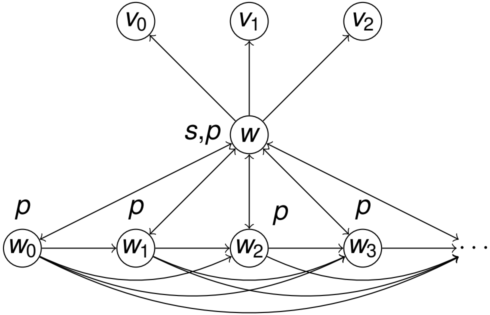
Theorem

The logic $S_d ML$ does not have the finite model property.

(F_1)	$s \wedge p \wedge \Box\neg s \wedge \Diamond p \wedge \Diamond\neg p \wedge \Box(\neg p \to \Box\bot)$
(F_2)	$\Box(p \to \Diamond s \wedge \Diamond\neg s \wedge \Box p)$
(F_3)	$\Box(p \to \Box(s \to \Box\neg s \wedge \Diamond\neg p))$
(F_4)	$[-\neg p]\Box\Box(s \to \neg\Diamond\neg p)$
(F_5)	$\Box(p \to \Box(\neg s \to \Diamond s \wedge \Diamond\neg s \wedge \Box p))$
(F_6)	$\Box(p \to \Box(\neg s \to \Box(s \to \Box\neg s \wedge \Diamond\neg p)))$
(F_7)	$[-\neg p]\Box\Box(\neg s \to \Box(s \to \neg\Diamond\neg p))$
(Spy)	$\Box(p \to \Box(\neg s \to [-\neg s]\Box\Diamond(p \wedge \Box s)))$
(Irr)	$\Box(p \to [-s]\Box\Diamond s)$
$(No\text{-}3cyc)$	$\neg\Diamond(p \wedge [-s]\Diamond[-s]\Diamond\Diamond(\neg s \wedge \Box\neg s))$
$(Trans)$	$\Box(p \to [-s]\Box\Box(\neg s \to [-\neg s]\Box\Diamond(\Box\neg s \wedge \Diamond\Box s)))$


v_0
v_1
v_2
s,p
w
p
p
p
p
w_0
w_1
w_2
w_3
$\cdots$

Theorem

The satisfiability problem for S_dML is undecidable.

$$
\begin{array}{rll}
(M_1) & s \wedge p \wedge \Box_s \neg s \wedge \Diamond_s p \wedge \Diamond_s \neg p \wedge \Box_s(\neg p \rightarrow \Box_s \bot) & \\
(M_2) & \Box_s(p \rightarrow \Diamond_s \top \wedge \Box_s(s \wedge \Diamond_s \neg p)) & \\
(M_3) & [-\neg p]\Box_s\Box_s(s \wedge \neg\Diamond_s \neg p) & \\
(M_4) & \Box_s(p \rightarrow \Diamond_\dagger \top \wedge \Box_\dagger(\neg s \wedge p \wedge \Diamond_s \top \wedge \Box_s(s \wedge \Diamond_s \neg p))) & \dagger \in \{u, r\} \\
(M_5) & [-\neg p]\Box_s\Box_\dagger\Box_s \neg\Diamond_s \neg p & \dagger \in \{u, r\} \\
(M_6) & \Box_s(p \rightarrow \Box_\dagger(\Diamond_u \top \wedge \Diamond_r \top \wedge \Box_u(\neg s \wedge p) \wedge \Box_r(\neg s \wedge p))) & \dagger \in \{u, r\} \\
(M_7) & \Box_s(p \rightarrow [-s]\Box_\dagger(\Diamond_s s \wedge \neg\Diamond_\dagger \neg\Diamond_s s)) & \dagger \in \{u, r\} \\
(Spy) & \Box_s(p \rightarrow \Box_\dagger[-\neg s]\Box_s\Diamond_s(p \wedge \Box_u \bot \wedge \Box_r \bot)) & \dagger \in \{u, r\} \\
(Func) & \Box_s(p \rightarrow [-s]\Box_\dagger[-\neg s]\Diamond_s\Diamond_s(p \wedge \neg\Diamond_s s \wedge \Diamond_\dagger \top \wedge & \\
 & \Box_\dagger(\Box_u \bot \wedge \Box_r \bot)) & \dagger \in \{u, r\}
\end{array}
$$

$$(\textit{No-UR}) \quad \Box_s(p \to [-s]\Box_u\Box_r\Diamond_s s \wedge [-s]\Box_r\Box_u\Diamond_s s)$$

$$(\textit{No-URU}) \quad \Box_s(p \to [-s]\Box_u\Box_r\Box_u\Diamond_s s)$$

$$(\textit{Conv}) \quad \Box_s(p \to [-s]\Diamond_u[-s]\Diamond_r[-\neg s]\Diamond_s\Diamond_s(p \wedge \neg\Diamond_s s \wedge \Box_r(\Diamond_u\top \wedge \Diamond_r\top) \wedge \Diamond_u\neg\Diamond_s s \wedge \Diamond_r\Diamond_u(\Box_u\bot \wedge \Box_r\bot)))$$

$$(\textit{Unique}) \quad \Box_s(p \to \bigvee_{1\leq i\leq n} t_i \wedge \bigwedge_{1\leq i<j\leq n} (t_i \to \neg t_j))$$

$$(\textit{Vert}) \quad \Box_s(p \to \bigwedge_{1\leq i\leq n} (t_i \to \Diamond_u \bigvee_{1\leq j\leq n,\ u(T_i)=d(T_j)} t_j))$$

$$(\textit{Horiz}) \quad \Box_s(p \to \bigwedge_{1\leq i\leq n} (t_i \to \Diamond_r \bigvee_{1\leq j\leq n,\ r(T_i)=l(T_j)} t_j))$$

What is the source of the high complexity of S_dML?

The features of our link deletion:

- Uniform (SML[5] (undecidable))
- Definable (dynamic-epistemic logics of link deletion [7] (decidable))
- Local

If we define $[-\ ]$ in a global way, i.e.,

$$\langle W, R, V\rangle, w \vDash [-\varphi]\psi \ \ \textit{iff} \ \ \langle W, R \setminus \{\langle s, t\rangle \in R \mid \mathcal{M}, t \vDash \varphi\}, V\rangle, w \vDash \psi$$

We can easily show the reduction axiom for $\Box$, and it is

$$[-\varphi]\Box\psi \leftrightarrow \Box(\neg\varphi \rightarrow [-\varphi]\psi).$$

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We can easily show the reduction axiom for $\Box$, and it is

$$[-\varphi]\Box\psi \leftrightarrow \Box(\neg\varphi \rightarrow [-\varphi]\psi).$$

Open Problem: Does making update operations local (world-relative) generate undecidability in general for decidable dynamic-epistemic logics?

Thanks for Your Attention!

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