A Logic of Knowing How with Skippable Plans

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Language [Wang 2018]

Definition (Language)

Given a set of propositional letters $\mathbf{P}$, the language $\mathbf{L_{Kh}}$ is defined by the following BNF where $p \in \mathbf{P}$:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}h(\varphi, \varphi).$$

$\mathcal{K}\mathrm{h}(\psi, \varphi)$ expresses that the agent knows how to achieve φ given ψ.

Know-how expressions often come with implicit preconditions. This language makes such preconditions explicit by introducing the binary modality $\mathcal{K}\mathrm{h}$.

$\mathcal{U}\varphi$ is defined as $\mathcal{K}\mathrm{h}(\neg\varphi, \bot)$. $\mathcal{U}\varphi$ is a universal modality as it will become more clear after defining the semantics.

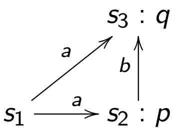
Model [Wang 2018]

Definition (Model)

A model is a labelled transition system $(\mathcal{S}, \Sigma, \mathcal{R}, \mathcal{V})$ where:

- $\mathcal{S}$ is a non-empty set of states;
- Σ is a non-empty set of actions;
- $\mathcal{R} : \Sigma \to 2^{\mathcal{S} \times \mathcal{S}}$ is a collection of transitions labelled by Σ;
- $\mathcal{V} : \mathcal{S} \to 2^{P}$ is a valuation function.

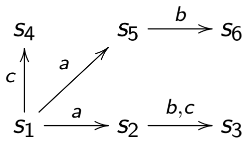
We write $s \xrightarrow{a} t$ and say t is an a-successor of s, if $(s, t) \in \mathcal{R}(a)$.



Strongly Executable Plans [Wang 2018]

$$\mathcal{M}, s \models \mathcal{K}h(\psi, \varphi) \quad \Leftrightarrow \quad \begin{array}{l} \text{there exists } \sigma \in \Sigma^* \text{ s.t. for all } \mathcal{M}, s' \models \psi : \\ (1)\ \sigma \text{ is strongly executable at } s', \text{ and} \\ (2)\ \text{for all } t, \text{ if } s' \xrightarrow{\sigma} t \text{ then } \mathcal{M}, t \models \varphi \end{array}$$

where we say $\sigma = a_1 \cdots a_n$ is strongly executable at s' if: s' has an a_1-successor and for any $1 \leq k < n$ and any r, $s' \xrightarrow{\sigma_k} r$ implies that r has at least one a_{k+1}-successor.



ab is strongly executable at s_1.
b is not strongly executable at s_1.
ac is not strongly executable at s_1.

Strongly Executable Plans [Wang 2018]

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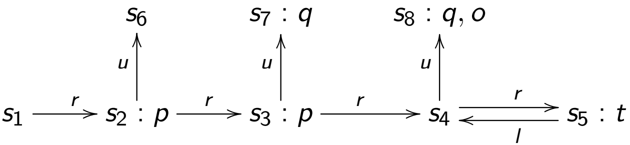
$$\text{(1) } \sigma \text{ is strongly executable at } s', \text{ and}$$
$$\text{(2) for all } t, \text{ if } s' \xrightarrow{\sigma} t \text{ then } \mathcal{M}, t \models \varphi$$



$\mathcal{M}, s_1 \vDash \mathcal{K}h(p, q)$

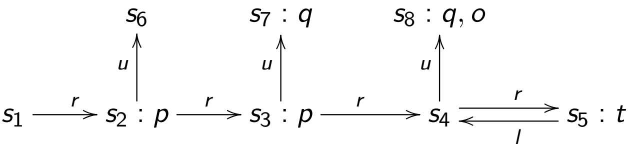
Note that the semantics of $\mathcal{K}h$-formulas ignores the current state s. The formula of the form $\mathcal{K}h(\psi, \varphi)$ is globally true or false.

$\mathcal{M} \vDash \mathcal{K}h(p, q)$, $\mathcal{M} \vDash \neg\mathcal{K}h(p, t)$

Stopping means achieving [Li 2016]

$\mathcal{M}, s \models \mathcal{K}hw(\psi, \varphi) \quad \Leftrightarrow \quad$ there exists $\sigma \in \Sigma^*$ s.t. for all $\mathcal{M}, s' \models \psi$: for all t, if $s' \xrightarrow{\sigma}_w t$ then $\mathcal{M}, t \models \varphi$

where $s' \xrightarrow{\sigma}_w t$ means that t is a state at which executing σ on s' might terminate.



$\mathcal{K}hw(p, t)$, $\mathcal{K}hw(t, o)$, $\neg\mathcal{K}hw(p, o)$

The Composition Axiom

COMPKh: $\mathcal{K}h(p, r) \wedge \mathcal{K}h(r, q) \rightarrow \mathcal{K}h(p, q)$

$\mathcal{K}hw$-interpretation of knowing-how results in a wearker logic where the composition axiom in [Wang, 2018] no longer holds.

Language and Model

Definition (Language)

Given a set of propositional letters $\mathbf{P}$, the language $\mathbf{L_{Khs}}$ is defined by the following BNF where $p \in \mathbf{P}$:

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Skippable Plans

Given a model $(\mathcal{S}, \Sigma, \mathcal{R}, \mathcal{V})$, a state $w \in \mathcal{S}$ and an action sequence $\sigma = a_1 \cdots a_n \in \Sigma^*$, $\text{ArrSta}(w, \sigma)$ is the set of states at which executing σ on w might arrive.

Definition (Arrival States)

$$\text{ArrSta}(w, a) = \begin{cases} \{w\}, & \text{if } w \text{ has no } a\text{-successor} \\ \{t \in \mathcal{S} \mid w \xrightarrow{a} t\}, & \text{otherwise} \end{cases}$$

We write $w \xrightarrow{a}_s t$ if $t \in \text{ArrSta}(w, a)$.

$$\text{ArrSta}(w, \sigma) = \{t \mid \exists t_1 \cdots t_{n-1} : w \xrightarrow{a_1}_s t_1 \xrightarrow{a_2}_s \cdots t_{n-1} \xrightarrow{a_n}_s t\}.$$

We write $w \xrightarrow{\sigma}_s t$ if $t \in \text{ArrSta}(w, \sigma)$.

In particular, σ can be the empty sequence ϵ. We set that $w \xrightarrow{\epsilon}_s w$.

Semantics

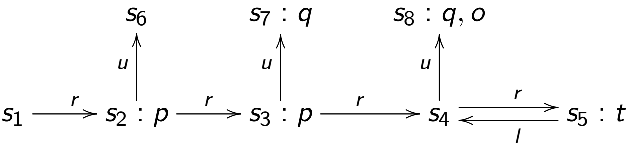
Definition (Semantics)

The satisfaction relation $\vDash$ is defined as follows:

$$
\begin{array}{lcl}
\mathcal{M}, s \vDash \top & & \text{always} \\
\mathcal{M}, s \vDash p & \Longleftrightarrow & p \in \mathcal{V}(s) \\
\mathcal{M}, s \vDash \neg\varphi & \Longleftrightarrow & \mathcal{M}, s \nvDash \varphi \\
\mathcal{M}, s \vDash \varphi \wedge \psi & \Longleftrightarrow & \mathcal{M}, s \vDash \varphi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \vDash \mathcal{K}hs(\psi, \varphi) & \Longleftrightarrow & \text{there exists } \sigma \in \Sigma^* \text{ such that for each} \\
& & w \in [\![\psi]\!]^{\mathcal{M}} \text{ and each } t \in \text{ArrSta}(w, \sigma) \\
& & \text{we have } \mathcal{M}, t \vDash \varphi
\end{array}
$$

where $[\![\psi]\!]^{\mathcal{M}} = \{s \mid \mathcal{M}, s \vDash \psi\}$.

The formula of the form $\mathcal{K}hs(\psi, \varphi)$ is globally true or false.



$$\mathcal{K}hs(p, o) : rrrlu$$

The Operator $\mathcal{U}$

$\mathcal{U}$ is a universal modality:

$$\mathcal{M}, s \vDash \mathcal{U}\varphi \iff \mathcal{M}, w \vDash \varphi \text{ for all } w \in \mathcal{S}$$

To see this, check the following:

$$\begin{array}{lcl}
\mathcal{M}, s \vDash \mathcal{U}\varphi & \iff & \mathcal{M}, s \vDash \mathcal{K}hs(\neg\varphi, \bot) \\
& \iff & \text{there is } \sigma \in \Sigma^* \text{ such that for each } w \in [\![\neg\varphi]\!]^{\mathcal{M}} \\
& & \text{and each } t \in \text{ArrSta}(w, \sigma)\text{: } \mathcal{M}, t \models \bot \\
& \iff & \text{there is } \sigma \in \Sigma^* \text{ such that for each } w \in [\![\neg\varphi]\!]^{\mathcal{M}}\text{:} \\
& & \text{there is no } t \text{ such that } t \in \text{ArrSta}(w, \sigma) \\
& \iff & \text{there is } \sigma \in \Sigma^* \text{ such that there is no } w \text{ such} \\
& & \text{that } \mathcal{M}, w \vDash \psi \\
& \iff & \mathcal{M}, w \vDash \varphi \text{ for all } w \in \mathcal{S}
\end{array}$$

Proof System $\mathbb{SKHS}$

Axioms

TAUT	all axioms of propositional logic
DISTU	$\mathcal{U}p \land \mathcal{U}(p \to q) \to \mathcal{U}q$
COMPKh	$\mathcal{K}hs(p, r) \land \mathcal{K}hs(r, q) \to \mathcal{K}hs(p, q)$
EMP	$\mathcal{U}(p \to q) \to \mathcal{K}hs(p, q)$
TU	$\mathcal{U}p \to p$
4KU	$\mathcal{K}hs(p, q) \to \mathcal{U}\mathcal{K}hs(p, q)$
5KU	$\neg\mathcal{K}hs(p, q) \to \mathcal{U}\neg\mathcal{K}hs(p, q)$

Rules

MP	$\dfrac{\varphi,\ \varphi\to\psi}{\psi}$
NECU	$\dfrac{\varphi}{\mathcal{U}\varphi}$
SUB	$\dfrac{\varphi(p)}{\varphi[(\psi/p)]}$

Soundness

Theorem (Soundness)

$\mathbb{SKHS}$ *is sound w.r.t. the class of all models.*

Proof.

The only non-trivial case is COMPKh. Note that if there is an action sequence σ_1 leading you from any p-state to some r-state, and there is a sequence σ_2 from any r-state to some q-state, then $\sigma_1\sigma_2$ will make sure that you end up with q-states from any p-state. □

Canonical Model

Since $\mathcal{K}hs$-formulars are globally true or false, it is not possible to satisfy all $\mathcal{K}hs$-formulas simultaneously in a single model. We built a separate canonical model for each maximal consistent set.

Given a set of $\mathbf{L_{Khs}}$ formulas Δ, let $\Delta|_{\mathcal{K}hs}$ and $\Delta|_{\neg\mathcal{K}hs}$ be the collections of its positive and negative $\mathcal{K}hs$ formulas:

$$\Delta|_{\mathcal{K}hs} = \{\theta \mid \theta = \mathcal{K}hs(\psi, \varphi) \in \Delta\},$$

$$\Delta|_{\neg\mathcal{K}hs} = \{\theta \mid \theta = \neg\mathcal{K}hs(\psi, \varphi) \in \Delta\}.$$

Definition (Canonical Models)

Given a maximal consistent set Γ w.r.t. $\mathbb{SKHS}$, the canonical model for Γ is $\mathcal{M}^c_\Gamma = \langle \mathcal{S}^c_\Gamma, \Sigma_\Gamma, \mathcal{R}^c, \mathcal{V}^c \rangle$ where:

- $\mathcal{S}^c_\Gamma = \{\Delta \mid \Delta$ is a MCS w.r.t. $\mathbb{SKHS}$ and $\Gamma|_{\mathcal{K}hs} = \Delta|_{\mathcal{K}hs}\}$,
- $\Sigma_\Gamma = \{\langle \psi, \varphi \rangle \mid \mathcal{K}hs(\psi, \varphi) \in \Gamma\}$,
- $\Delta \xrightarrow{\langle \psi, \varphi \rangle}_c \Theta$ iff 1. $\mathcal{K}hs(\psi, \varphi) \in \Gamma$, $\psi \in \Delta$, $\varphi \in \Theta$, or
  2. $\mathcal{K}hs(\psi, \varphi) \in \Gamma$, $\neg\psi \in \Delta$, $\Delta = \Theta$, or
  3. $\mathcal{K}hs(\psi, \varphi) \in \Gamma$, $\neg\psi \in \Delta$, $\psi \in \Theta$,
- $p \in \mathcal{V}^c(\Delta)$ iff $p \in \Delta$.

We say that $\Delta \in \mathcal{S}^c_\Gamma$ is a φ-state if $\varphi \in \Delta$.

Completeness

Proposition

If $\varphi \in \Delta$ *for all* $\Delta \in \mathcal{S}^c_\Gamma$*, then* $\mathcal{U}\varphi \in \Delta$ *for all* $\Delta \in \mathcal{S}^c_\Gamma$*.*

Proof.

Suppose $\varphi \in \Delta$ for all $\Delta \in \mathcal{S}^c_\Gamma$. Then $\neg\varphi$ is not consistent with $\Gamma|_{\mathcal{K}hs} \cup \Gamma|_{\neg\mathcal{K}hs}$.

$$\vdash \bigwedge_{1\leq i\leq k} \mathcal{K}hs(\psi_i, \varphi_i) \wedge \bigwedge_{1\leq j\leq l} \neg\mathcal{K}hs(\psi'_j, \varphi'_j) \to \varphi$$

$$\vdash \mathcal{U}(\bigwedge_{1\leq i\leq k} \mathcal{K}hs(\psi_i, \varphi_i) \wedge \bigwedge_{1\leq j\leq l} \neg\mathcal{K}hs(\psi'_j, \varphi'_j)) \to \mathcal{U}\varphi$$

We have that $\mathcal{U}(\bigwedge \mathcal{K}hs(\psi_i, \varphi_i) \wedge \bigwedge \neg\mathcal{K}hs(\psi'_j, \varphi'_j)) \in \Gamma$.
Thus $\mathcal{U}\varphi \in \Gamma$. □

Completeness

Corollary

If $\psi \to \varphi \in \Delta$ *for all* $\Delta \in \mathcal{S}^c_\Gamma$*, then* $\mathcal{K}hs(\psi, \varphi) \in \Gamma$.

Proof.

If $\psi \to \varphi \in \Delta$ for all $\Delta \in \mathcal{S}^c_\Gamma$, then $\mathcal{U}(\psi \to \varphi) \in \Gamma$, then by EMP $\mathcal{K}hs(\psi, \varphi) \in \Gamma$. □

Completeness

Proposition

For any $\mathcal{K}hs(\psi, \varphi) \in \Gamma$, any $\Delta \in \mathcal{S}^c_\Gamma$, if $\psi \in \Delta$ then there exists $\Delta' \in \mathcal{S}^c_\Gamma$ such that $\varphi \in \Delta'$.

Proposition

For any $\langle\psi, \varphi\rangle \in \Sigma_\Gamma$ and any $\Delta \in \mathcal{S}^c_\Gamma$, Δ has a $\langle\psi, \varphi\rangle$-successor. Moreover, if $\psi \in \Delta$ then $\mathrm{ArrSta}(\Delta, \langle\psi, \varphi\rangle) = \{\Pi \in \mathcal{S}^c_\Gamma \mid \varphi \in \Pi\} \neq \emptyset$.

The first proposition reflects our intuition that if we know how to achieve φ from ψ and we are at a ψ-state, then there must be a φ-state where we could arrive. Moreover, the second one reflects the intuition that the states where we arrive after executing the plan for achieving φ must be φ-states.

Completeness

Lemma (Truth Lemma)

For any formula φ, $\mathcal{M}^c_\Gamma, \Delta \models \varphi$ *iff* $\varphi \in \Delta$.

Proof.

The proof is by structural inducition on $\mathbf{L_{Khs}}$-formulas. We only focus on the case of $\mathcal{K}hs(\psi, \varphi)$.

($\Leftarrow$) ψ-states $\xrightarrow{\langle\psi,\varphi\rangle}_s$ φ-states.

($\Rightarrow$) If there exists a plan for achieving φ from ψ, then there exists a one-step plan for achieving φ from ψ. □

Theorem (Completeness)

$\mathbb{SKHS}$ *is strongly complete w.r.t. the class of all models.*

Decidability

Theorem (Decidability)

If φ is satisfiable then it is satisfiable on a finite model. Indeed, it is satisfiable on a finite model containing at most 2^k states, where k is the number of subformulas of φ. It follows that $\mathbb{SKHS}$ is decidable.

Proof.

Note that given a satisfiable formula φ, only the proposition letters that occur in φ matter. Thus we can consider a fragment of $\mathbf{L_{Khs}}$ based on the finite set of proposition letters in φ. Clearly, if φ is satisfiable in some model w.r.t. the full set of proposition letters P then it is satisfiable in a model w.r.t. the restricted set of proposition letters: we can simply forget the valuation of other propositions.

Decidablity

cont.

Note that $\mathcal{K}hs$-formulas hold globally in the canonical model. It follows that in the canonical model construction for the restricted language, the maximal consistent sets are essentially different valuations of the basic propositions in φ. Clearly, given the number of proposition letters k, the maximal size of the canonical model is 2^k.

Bisimulation

The standard bisimulation for the basic multi-modal language is not adequate, as the languages have different expressivity.

For the Zig and Zag conditions, they should be designed to match the operator $\mathcal{K}$hs.
One might be tempted to require that, if Z is a bisimulation and $(w, w^{'}) \in Z$, then these states should have matching successors.
However, the actual evaluation point does not play any role in the semantic interpretation of $\mathcal{K}$hs.

Zig and Zag

Here are some notions:

$$U \xrightarrow{\sigma}_s V \text{ whenever } V = \bigcup_{u_i \in U} \text{ArrSta}(u_i, \sigma),$$

$$U \to_s V \text{ whenever there is a } \sigma \text{ such that } U \xrightarrow{\sigma}_s V.$$

$\mathcal{M}, w \vDash \mathcal{K}\text{hs}(\psi, \phi) : [\![\psi]\!]^{\mathcal{M}} \to_s V$ and $V \subseteq [\![\phi]\!]^{\mathcal{M}}$
Khs-**Zig**: for any $\mathbf{L_{Khs}}$-definable $U \subseteq \mathcal{S}$, if $U \to_s V$ for some $V \subseteq \mathcal{S}$, then there is $V' \subseteq \mathcal{S}'$ such that (i) $Z[U] \to_s V'$ and (ii) for each $v' \in V'$ there is $v \in V$ such that vZv'.

As the global modality is definable in $\mathbf{L_{Khs}}$, every world in one model should have a matching world in another model, and vice-versa.
A-**Zig**: for all v in $\mathcal{S}$ there is v' in $\mathcal{S}'$ such that vZv'.

Bisimulation

Definition (Bisimulation)

Let $\mathcal{M} = \langle \mathcal{S}, \Sigma, \mathcal{R}, \mathcal{V} \rangle$ and $\mathcal{M}' = \langle \mathcal{S}', \Sigma', \mathcal{R}', \mathcal{V}' \rangle$ be two models. A non-empty relation $Z \subseteq \mathcal{S} \times \mathcal{S}'$ is called an $\mathbf{L_{Khs}}$-bisimulation between $\mathcal{M}$ and $\mathcal{M}'$ iff wZw' implies:

Atom: $V(w) = V(w')$.

Khs-**Zig**: for any $\mathbf{L_{Khs}}$-definable $U \subseteq \mathcal{S}$, if $U \to_s V$ for some $V \subseteq \mathcal{S}$, then there is $V' \subseteq \mathcal{S}'$ such that (i) $Z[U] \to_s V'$ and (ii) for each $v' \in V'$ there is $v \in V$ such that vZv'.

Khs-**Zag**: for any $\mathbf{L_{Khs}}$-definable $U' \subseteq \mathcal{S}'$, if $U' \to_s V'$ for some $V' \subseteq \mathcal{S}'$, then there is $V \subseteq \mathcal{S}$ such that (i) $Z^{-1}[U'] \to_s V$ and (ii) for each $v \in V$ there is $v' \in V'$ such that vZv'.

A-**Zig**: for all v in $\mathcal{S}$ there is v' in $\mathcal{S}'$ such that vZv'.

A-**Zag**: for all v' in $\mathcal{S}'$ there is v in $\mathcal{S}$ such that vZv'.

Theorem

If $\mathcal{M}, w \underline{\leftrightarrow}_{\mathsf{L_{Khs}}} \mathcal{M}', w'$, *then* $\mathcal{M}, w \equiv_{\mathsf{L_{Khs}}} \mathcal{M}', w'$.

Proof.

The proof is by structural inducition on $\mathbf{L_{Khs}}$-formulas. □

Now we prove the other direction. Here we focus on finite models rather than image-finite models. This is because the global modality is definable in $\mathbf{L_{Khs}}$, and thus a finite domain is required in order to ensure the image-finiteness property.

Theorem

Let $\mathcal{M} = \langle W, \Sigma, \mathcal{R}, \mathcal{V}\rangle$ and $\mathcal{M}' = \langle W', \Sigma', \mathcal{R}', \mathcal{V}'\rangle$ be two finite models. If $\mathcal{M}, w \equiv_{\mathsf{L_{Khs}}} \mathcal{M}', w'$, then $\mathcal{M}, w \leftrightarroweq_{\mathsf{L_{Khs}}} \mathcal{M}', w'$.

Proof.

A-Zig: Take $v \in W$. Towards a contradiction, suppose that there is no $v' \in W'$ such that vZv'. To get a contradiction, we just need to find a formula α such that $\mathcal{M}, w \vDash \alpha$ but $\mathcal{M}, w' \nvDash \alpha$.
For each $v'_i \in W' = \{v'_1, \cdots, v'_n\}$, there is a formula θ_i such that $\mathcal{M}, v \vDash \theta_i$ but $\mathcal{M}, v'_i \nvDash \theta_i$. Let $\theta = \theta_1 \wedge \cdots \wedge \theta_n$. Then $\mathcal{M}, v \vDash \theta$ but $\mathcal{M}, v'_i \vDash \neg\theta$ for each $w'_i \in W'$. It follows that $\mathcal{M}, w \vDash \neg\mathcal{U}\neg\theta$ but $\mathcal{M}, w' \vDash \mathcal{U}\neg\theta$, contradicting $\mathcal{M}, w \equiv_{\mathsf{L_{Khs}}} \mathcal{M}', w'$.

cont.

Khs-Zig: Suppose that $[\![\psi]\!]^{\mathcal{M}} \to_s V$. It suffices to find a $V' \subseteq W'$ such that $Z[[\![\psi]\!]^{\mathcal{M}}] \to_s V'$ and for each $v' \in V'$ there is $v \in V$ such that vZv'.

- $Z[[\![\psi]\!]^{\mathcal{M}}] = [\![\psi]\!]^{\mathcal{M}'}$.
- Then,we just need to find an appropriate V' for $[\![\psi]\!]^{\mathcal{M}'}$.

(Assume that $[\![\psi]\!]^{\mathcal{M}} \neq \emptyset$.) Towards a contradiction, suppose that for each $V' \subseteq W'$ such that $[\![\psi]\!]^{\mathcal{M}'} \to_s V'$, there is $s'_{V'} \in V'$ such that there is no $s \in V$ such that $sZs'_{V'}$. Then we need to find a formula β such that $\mathcal{M}, w \vDash \beta$ but $\mathcal{M}, w' \nvDash \beta$.
For each $s \in V$ we have a formula $\varphi^s_{V'}$ such that $\mathcal{M}, s \vDash \varphi^s_{V'}$ but $\mathcal{M}', s'_{V'} \nvDash \varphi^s_{V'}$. As the models are finite, define $\theta_{V'} = \bigvee_{s \in V} \varphi^s_{V'}$ and $\theta = \bigwedge_{\{V' \mid [\![\psi]\!]^{\mathcal{M}'} \to_s V'\}} \theta_{V'}$. Then $\mathcal{M}, s \vDash \theta$ for all $s \in V$, but there is $s'_{V'} \in V'$ such that $\mathcal{M}', s'_{V'} \nvDash \theta$ for each V' such that $[\![\psi]\!]^{\mathcal{M}'} \to_s V'$. It follows that $\mathcal{M}, w \vDash \mathcal{K}hs(\psi, \theta)$ but $\mathcal{M}', w' \nvDash \mathcal{K}hs(\psi, \theta)$. Contradiction.

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