Formalizing the Meaning of Questions

With a focus on generalized inquisitive semantics and its logic

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Goals

- Basic concepts of inquisitive semantics and related intuitions
- Completeness of generalized **InqL**
- Characterizing questions (conversations)

Preliminaries

Clarification: what does “semantics” mean?

In logic

- Syntax
- Semantics

In natural language?

Morris (1938) Foundation of the theory of signs

- Syntactics (syntax): the study of the syntactical relations of signs to one another in abstraction from the relations of signs to one another in abstraction from the relations of signs to objects or to interpreters
- Semantics: deals with the relation of signs to their designate and so to the objects which they may or do denote
- Pragmatics: the science of the relation of signs to their interpreters

The early days of formal semantics

Truth-conditional semantics

- Formal, or truth-conditional, or model-theoretical semantics
- Montague (1970) English as a Formal Language
- semantic content/ informative content

Stalnaker (1978) a conversational twist: its potential to update the common ground (muddy children)

Limits

- Non-informative sentences
- update v.s. propose an update

Illustrations

(a) animals

(b) 10-questions

Ivano Ciardelli & Floris Roelofsen's 2011 paper

Introduction

Goals

- a generalized version of inquisitive semantics
- a complete axiomization of the associated logic
- the connection with intuitive logic and several intermediate logics
- advantages over other semantics

Intuition

Inquisitive semantics directly reflects that a primary use of language lies in the exchange of information in a cooperative dynamic process of raising and resolving issues.

Generalized inquisitive semantics

Definition (Language)

The language $\mathcal{L}_{\mathcal{P}}$ is defined inductively as follows:

$$\varphi ::= \ p \mid \bot \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \to \psi$$

where $p \in \mathcal{P}$ and $\mathcal{P}$ is in a finite or countably infinite set of proposition letters.

We write $\neg\varphi$ for $\varphi \to \bot$, $!\varphi$ for $\neg\neg\varphi$, and $?\varphi$ for $\varphi \vee \neg\varphi$.

Indices and states

Definition 2.1 (Indices)

A $\mathcal{P}$-index is a subset of $\mathcal{P}$. The set of all indices, $\wp(\mathcal{P})$, will be denoted by $\mathcal{I}_{\mathcal{P}}$. We will simply write $\mathcal{I}$ and talk of indices in case $\mathcal{P}$ is clear from the context.

Definition 2.2 (States)

A $\mathcal{P}$-state is a set of $\mathcal{P}$-indices. The set of all states, $\wp\wp(\mathcal{P})$, will be denoted by $\mathcal{S}_{\mathcal{P}}$. We will write $\mathcal{S}$ in case $\mathcal{P}$ is clear from the context.

Support

Definition 2.3 (Support)

Support is a relation between states and formulas. (written as $s \vDash \varphi$)

1. $s \vDash p$ iff $\forall w \in s : p \in w$.
2. $s \vDash \bot$ iff $s = \varnothing$. (We may think of $\varnothing$ as the inconsistent state.)
3. $s \vDash \varphi \wedge \psi$ iff $s \vDash \varphi$ and $s \vDash \psi$.
4. $s \vDash \varphi \vee \psi$ iff $s \vDash \varphi$ or $s \vDash \psi$.
5. $s \vDash \varphi \to \psi$ iff $\forall t \subseteq s$: if $t \vDash \varphi$ then $t \vDash \psi$.

Proposition 2.4 (Persistence)

If $s \vDash \varphi$ then for every $t \subseteq s$: $t \vDash \varphi$.

Proposition 2.5 (Singleton states behave classically)

For any index w and formula φ: $\{w\} \vDash \varphi \iff w \vDash \varphi$ where $w \vDash \varphi$ means φ is classically true under the valuation w. In particular, $\{w\} \vDash \varphi$ or $\{w\} \vDash \neg\varphi$ for any formula φ.

Support

Proposition 2.6 (Support for negation)

1. $s \vDash \neg\varphi$ iff $\forall w \in s : w \vDash \neg\varphi$
2. $s \vDash !\varphi$ iff $\forall w \in s : w \vDash \varphi$.

Definition 2.7 (Restriction of s)

Let $\mathcal{P} \subseteq \mathcal{P}'$ be two sets of propositional letters. Then for any $\mathcal{P}'$-state s, the restriction of s to $\mathcal{P}$ is defined as $s \restriction_{\mathcal{P}} := \{w \cap \mathcal{P} | w \in s\}$

Proposition 2.8 (Restriction invariance)

Let $\mathcal{P} \subseteq \mathcal{P}'$ be two sets of propositional letters. Then for any $\mathcal{P}'$-state s and any formula φ whose propositional letters are in $\mathcal{P}$:

$$s \vDash \varphi \iff s \restriction_{\mathcal{P}} \vDash \varphi$$

Possibilities, propositions, and truth-sets

1. A possibility for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.
2. The proposition expressed by φ, denoted by $[\varphi]$, is the set of possibilities for φ.
3. The truth set of φ, denoted by $|\varphi|$, is the set of indices where φ is classically true.

It may be expected, then, that the proposition expressed by φ would be defined as the set of all states supporting φ. Rather, though, it is defined as the set of all maximal states supporting φ, that is, the set of all possibilities for φ. This is motivated by the fact that propositions are viewed as proposals, consisting of one or more alternative possibilities.

Proposition 2.10 (Support and possibilities)
For any state s and any formula φ: $s \vDash \varphi \iff s \subseteq t$ and t is a possibility for φ.

Proof. $\Longleftarrow$ If $s \subseteq t$ and t is a possibility for φ, then by persistence $s \vDash \varphi$.

$\Longrightarrow$ First consider the case in which the set P of propositional letters is finite. Then there are only finitely many states, and therefore if s supports φ, then obviously s must be contained in a maximal state supporting φ, i.e. in a possibility.

If $\mathcal{P}$ is infinite, given a $\mathcal{P}$-state $s \vDash \varphi$, consider its restriction $s \restriction_{P_\varphi}$ to the (finite!) set P_φ of propositional letters occurring in φ. By Proposition 2.8, $s \restriction_{\mathcal{P}_\varphi} \vDash \varphi$, and thus $s \restriction_{\mathcal{P}_\varphi} \subseteq t$ for some $\mathcal{P}_\varphi$-state t which is a possibility for φ.

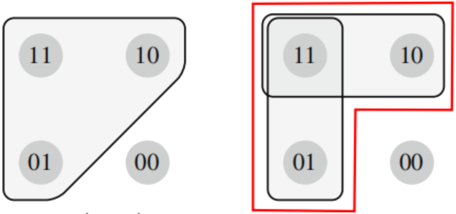
Now, consider $t^+ := \{w \in \mathcal{I}_\mathcal{P} | w \cap \mathcal{P}_\varphi \in t\}$. For any $w \in s$ we have $w \cap \mathcal{P}_\varphi \in s \restriction_{\mathcal{P}_\varphi} \subseteq t$ so $w \in t^+$ by definition of t^+; this proves that $s \in t^+$.

Moreover, we claim that t^+ is a possibility for φ. First, since $s \restriction_{\mathcal{P}_\varphi} = t$ and $t \vDash \varphi$, it follows from Proposition 2.8 that $t^+ \vDash \varphi$. Now, consider a state $w \supseteq t^+$ with $u \vDash \varphi$: then $u \restriction_{\mathcal{P}_\varphi} \supseteq t^+ \restriction_{\mathcal{P}_\varphi} = t$ and moreover, again by Proposition 2.8, $u \restriction_{\mathcal{P}_\varphi} \vdash \varphi$; but then, by the maximality of t it must be that $u \restriction_{\mathcal{P}_\varphi} = t$. Now, for any $w \in u$, $w \cap \mathcal{P}_\varphi \in u \restriction_{\mathcal{P}_\varphi} = t$, so $w \in t^+$ by definition of t^+: hence, $u = t^+$. This proves that t^+ is indeed a possibility for φ. $\square$

Example (Disjunction)

These figures assume that $\mathcal{P} = \{p, q\}$

Fig. 1 The truth-set of $p \vee q$, and the proposition it expresses



(a) $|p \vee q|$ (b) $[p \vee q]$

Proposition 2.12 (Negation)

1. $[\neg\varphi] = \{|\neg\varphi|\}$
2. $[!\varphi] = \{|\varphi|\}$

Inquisitiveness and Informativeness

- φ is inquisitive iff $[\varphi]$ contains at least two possibilities;
- φ is informative iff $[\varphi]$ proposes to elliminate certain indices: $\bigcup[\varphi] \neq \mathcal{I}$;
- φ is a question iff it is not informative;
- φ is an assertion iff it is not inquisitive;
- φ is a contradiction iff it is only supported by the inconsistent state, i.e. iff $[\varphi] = \{\varnothing\}$;
- φ is a tautology iff it is supported by all states, i.e. iff $[\varphi] = \{\mathcal{I}\}$.

Note: Classically, a formula is tautological iff it is not informative. In the present framework, a formula is tautological iff it is neither informative nor inquisitive.

Example (Questions)

These figures assume that $\mathcal{P} = \{p, q\}$

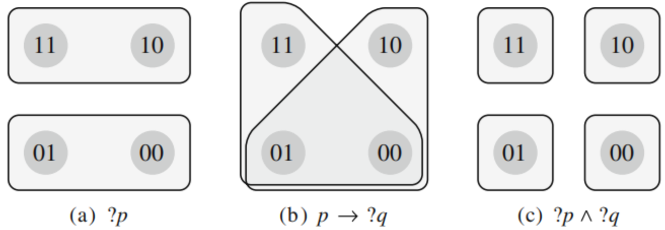


Fig. 2 A polar question, a conditional question, and a conjoined question

Proposition 2.18

For any two formulas φ, ψ :

1. $?\varphi$ and $?\psi$ are questions;
2. if φ and ψ are questions, then $\varphi \wedge \psi$ is a question;
3. if φ or ψ is a question, then $\varphi \vee \psi$ is a question;
4. if ψ is a question, then $\varphi \rightarrow \psi$ is a question.

Proposition 2.19

For any propositional letter p and formulas $\varphi, \ \psi$:

Proposition 2.19

1. p is an assertion;
2. $\bot$ is an assertion;
3. if φ or ψ are assertions, then $\varphi \wedge \psi$ is an assertion;
4. if ψ is an assertion, then $\varphi \to \psi$ is an assertion.

Note that items 2 and 4 of Proposition 2.19 imply that any negation is an assertion, which we already knew from Proposition 2.12. Of course, $!\varphi$ is also always an assertion.

Corollary 2.20 (Disjunction is the only source of inquisitiveness)
Any disjunction-free formula is an assertion.

Proposition 2.21 (Informative content remains the same)
For any formula $\varphi : \bigcup[\varphi] = |\varphi|$.

Proof. According to Proposition 2.5, if $w \in |\varphi|$, then $\{w\} \vDash \varphi$. But then, by Proposition 2.10, $\{w\}$ must be included in some $t \in [\varphi]$, whence $w \in \bigcup[\varphi]$.

Conversely, any $w \in \bigcup[\varphi]$ belongs to a possibility for φ, so by persistence and the classical behaviour of singletons we must have that $w \in |\varphi|$. □

Definition 2.22 (Equivalence)
Two formulas φ and ψ are equivalent, $\varphi \equiv \psi$, iff $[\varphi] = [\psi]$.

Proposition 2.23 (Characterization of questions)

For any formula φ, the following are equivalent:

1. φ is a question
2. φ is a classical tautology
3. $\neg\varphi$ is a contradiction
4. $\varphi \equiv ?\varphi$

Proof. Equivalence ($1 \iff 2$) follows from the definition of questions and Proposition 2.21. ($2 \iff 3$) and ($2 \Rightarrow 3$) are immediate from the fact that a formula is a contradiction in the inquisitive setting just in case it is a classical contradiction. For ($3 \Rightarrow 4$), note that for any state s, $s \models ?\varphi$ iff $s \models \varphi$ or $s \models \neg\varphi$. This means that, if $\neg\varphi$ is a contradiction, $s \models ?\varphi$ iff $s \models \varphi$. In other words, $\varphi \equiv ?\varphi$. $\square$

Note that an interrogative $?\varphi$ is always a question, thus ? is idempotent.

Proposition 2.24 (Characterization of assertions)

For any formula φ, the following are equivalent:

1. φ is an assertion
2. if $s_j \vDash \varphi$ for all $j \in J$, then $\bigcup_{j\in J} s_j \vDash \varphi$
3. $|\varphi| \vDash \varphi$
4. $\varphi \equiv !\varphi$
5. $[\varphi] = \{|\varphi|\}$

Note that: assertions behaves classically;

$!\varphi$ is always an assertion, thus ! is idempotent.

Proposition 2.25

The operators ! and ? work in a sense like projections on the ‘planes’ of assertions and questions, respectively. Moreover, the following proposition shows that the inquisitive meaning of a formula φ is completely determined by its ‘purely informative component’ $!\varphi$ and its ‘purely inquisitive component’ $?\varphi$

Proposition 2.25 (Division in theme and rheme)
For any formula φ, $\varphi \equiv ?\varphi \wedge !\varphi$.

Proof. We must show that for any state s, $s \vDash \varphi$ iff $s \vDash ?\varphi \wedge !\varphi$. Suppose $s \vDash ?\varphi \wedge !\varphi$. Then, since $s \vDash ?\varphi$, s must support one of φ and $\neg\varphi$; but since $s \vDash !\varphi$, s cannot support $\neg\varphi$. Thus, we have that $s \vDash \varphi$. The converse is immediate by the definitions of ! and ? and Proposition 2.6. $\square$

Intuition of support

It is important to emphasize that support should not be thought of as specifying conditions under which an agent with information state s can truthfully utter a sentence φ (this is a common interpretation of the notion of support in dynamic semantics). Rather, in the present setting support should be thought of as specifying conditions under which a sentence φ is insignif icant or redundant in a state s, in the sense that, given the information available in s, φ is neither informative nor inquisitive. This intuition can be made precise by defining notions of inquisitiveness and informativeness relative to a state.

Relative semantic notions

Let φ be a formula, and s a state. Then:

Definition 2.26

For any formula φ, $\varphi \equiv ?\varphi \wedge !\varphi$.

- a possibility for φ in s is a maximal substate of s supporting φ;
- φ is inquisitive in s iff there are at least two possibilities for φ in s;
- φ is informative in s iff there is at least one index in s that is not included in any possibility for φ in s.

Proposition 2.27 (Support, inquisitiveness, and informativeness)

A state s supports a formula φ iff φ is neither inquisitive in s nor informative in s.

Inquisitive Logic

Entailment and validity

Entailment

$\Theta \models_{InqL} \varphi$ iff any state that supports all formulas in Θ also supports φ.

Validity

$\models_{InqL} \varphi$ iff φ is supported by all states.

Note: if no confusion arises, we will simply write $\models$ instead of $\models_{InqL}$.

$\varphi \equiv \psi$ iff $\varphi \models \psi$ and $\psi \models \varphi$.

Intuition: whenever we are in a state where the information provided by φ has been accommodated and the issue raised by φ has been resolved, ψ does not provide any new information and does not raise any new issue.

The most informative assertion

Proposition 3.2

If ψ is an assertion, $\varphi \vDash \psi$ iff $|\varphi| \subseteq |\psi|$.

Note: for any formula φ, $!\varphi$ is the most informative assertion entailed by φ.

Proposition 3.3

For any formula φ and any assertion χ, $\varphi \vDash \chi \iff !\varphi \vDash \chi$.

Proof. Fix a formula φ and an assertion χ. The right-to-left implication is obvious, since it is clear from Proposition 2.6 that $\varphi \vDash !\varphi$. For the converse direction, suppose $\varphi \vDash \psi$. Any possibility $s \in [\varphi]$ supports φ and therefore also χ, whence by Proposition 2.10 it must be included in a possibility for χ, which must be $|\chi|$ by Proposition 2.24 on assertions. But then also $|\varphi| = [\varphi] \subseteq |\chi|$ whence $!\varphi \vDash \chi$ by Proposition 3.2. □

Questions do not entail informative contents

Proposition 3.4

If φ is a question and $\varphi \vDash \psi$, then ψ must be a question as well.

Proof. If φ is a question, it must be supported by every singleton state. If moreover $\varphi \vDash \psi$, then ψ must also be supported by every singleton state. But then, since singletons behave like indices, ψ must be a classical tautology, that is, a question. □

Proposition 3.5 (Logic)

Inquisitive logic, **InqL**, is the set of formulas that are valid in inquisitive semantics.

Proposition 3.6

A formula φ is in **InqL** iff $\mathcal{I} \vDash \varphi$.

Compared with classical logic

Proposition 3.7

A formula is in **InqL** iff it is both a classical tautology and an assertion.

Proof. If $\varphi \in$ **InqL**, it is supported by all states. In particular, it is supported by $\mathcal{I}$, which means that it is an assertion, and it is supported by all singleton states, which means, by Proposition 2.5, that it is a classical tautology. Conversely, if φ is an assertion, there is only one possibility for φ. If, moreover, φ is a classical tautology, this possibility must be $\mathcal{I}$. But then, by persistence, φ must be supported by all states. □

Note: **InqL** coincides with classical logic as far as assertions are concerned: in particular, it agrees with classical logic on the whole disjunction-free fragment of the language.

Remark 3.8 (Closed under MP but not USUB)

$\neg\neg p \to p \in$ **InqL**

Proposition 3.9 (Disjunction property)

InqL has the disjunction property. That is, whenever a disjunction $\varphi \vee \psi$ is in **InqL**, at least one of φ and ψ is in **InqL** as well.

Proposition 3.10 (Deduction theorem)

For any formulae $\theta_1, \ldots, \theta_n, \varphi$:

$$\theta_1, \ldots, \theta_n \vDash \varphi \iff \vDash \theta_1 \wedge \cdots \wedge \theta_n \to \varphi$$

Proof. $\theta_1, \ldots, \theta_n \vDash \varphi \iff$ for any $s \in S$, if $s \vDash \theta_i$ for $1 \leq i \leq n$, then $s \vDash \varphi \iff$ for any $s \in S$, if $s \vDash \theta_1 \wedge \cdots \wedge \theta_n$, then $s \vDash \varphi$ $\iff \mathcal{I} \vDash \theta_1 \wedge \cdots \wedge \theta_n \to \varphi \iff \vDash \theta_1 \wedge \cdots \wedge \theta_n \to \varphi$ □

Theorem 3.11 (Compactness)

For any set Θ and any formula φ, if $\vDash \varphi$ then there is a finite set $\Theta_0 \subseteq \Theta$ such that $\Theta_0 \vDash \varphi$.

Proof. Since $\mathcal{P}$ is countable, so must be Θ. Let $\Theta = \{\theta_k | k \in \omega\}$. For any $k \in \omega$, let $\gamma_k = \theta_0 \wedge \cdots \wedge \theta_k$, and define $\Gamma = \{\gamma_k | k \in \omega\}$. Clearly for any state s, $s \vDash \Gamma \iff s \vDash \Theta$, so $\Gamma \vDash \varphi$. For $k \geq k'$ we have $\gamma_k \vDash \gamma_{k'}$.

For any $k \in \omega$, let $\mathcal{P}$ be the set of propositional letters in φ or γ_k.

Towards a contradiction, suppose there is no $k \in \omega$ such that $\gamma_k \vDash \varphi$. Let $L_k = \{t | t \text{ is a } \mathcal{P}_k\text{-state with } t \vDash \gamma_k \text{ but } t \nvDash \varphi\}$. Thus for all k, $L_k \neq \varnothing$. Let $L = \{\varphi\} \cup \biguplus_{k \in \omega} L_k$. We define a relation R on L by putting:

- $\varnothing R t$ iff $t \in L_0$;
- sRt iff $s \in L_k$, $s \in L_k$ and $t \restriction_{\mathcal{P}_k} = s$.

cont. Compactness

Consider $t \in L_{k+1}$. Since $t \vDash \gamma_{k+1}$ and $\gamma_{k+1} \vDash \gamma_k$, we have $t \vDash \gamma_k$. By proposition 2.8, $t \restriction_{\mathcal{P}_k} \vDash \gamma_k$. And since $t \nvDash \varphi$, $t \restriction_{\mathcal{P}_k} \vDash \varphi$. Therefore $t \restriction_{\mathcal{P}_k} \in L_k$.

It follows that (L, R) is a tree with root $\varnothing$. Since there are infinitely many L_k, L is infinite. Since all the successors of a state $s \in L_k$ are $\mathcal{P}_{k+1}$-states, and there are only finitely many of those as P_{k+1} is finite. Therefore, the tree (L, R) is finitely branching.

By Kön̈ig's lemma, a tree that is infinite and finitely branching must have an infinite branch $< t_k | k \in \omega >$. Let its "limit" be t, such that

$$t = \{w \in \wp(\mathcal{P}) | \text{there are } w_k \in t_k \text{ with } w_{k+1} \restriction_{\mathcal{P}_k} = w_k \text{ and } w = \bigcup_{k \in \omega} w_k\}$$

For any k, $t \restriction_{\mathcal{P}_k} = t_k \vDash \gamma_k$. By Proposition 2.8 we have $t \vDash \gamma_k$; hence, $t \vDash \Gamma$. On the other hand, since $t \restriction_{\mathcal{P}_0} = t_0 \nvDash \varphi$, also $t \nvDash \varphi$. Yet $Gamma \vDash \varphi$. So for some k we must have $\gamma_k \vDash \varphi$. □

Remark 3.12 (Decidability)

InqL is clearly decidable: to determine whether a formula φ is in **InqL**, by Propositions 2.8 and 3.6 we only have to test whether $\mathcal{I}_{\{p_1,\ldots,p_n\}}$ supports φ, where $p_1, \ldots, p_n$ are the propositional letters in φ. This is a finite procedure since $\mathcal{I}_{\{p_1,\ldots,p_n\}}$ is finite and has only finitely many substates which have to be checked to determine support for implications.

3.2 Disjuctive Negative Translation

- A formula can always be rewritten as a disjunction of negations
- A number of expressive completeness results and completeness;
- **InqL** is isomorphic to the disjunctive-negative fragment of **IPL**.

Definition 3.13 (Disjunctive negative translation)

- $\mathrm{DNT}(p) = \neg\neg p$
- $\mathrm{DNT}(\bot) = \neg\neg\bot$
- $\mathrm{DNT}(\psi \vee \chi) = \mathrm{DNT}(\psi) \vee \mathrm{DNT}(\chi)$
- $\mathrm{DNT}(\psi \wedge \chi) = \bigvee\{\neg(\psi_i \vee \chi_j) | 1 \leq i \leq n, 1 \leq j \leq m\}$
- $\mathrm{DNT}(\psi \to \chi) = \bigvee_{k_1,\ldots,k_n}\{\neg\neg \bigwedge_{1 \leq i \leq n}(\chi_{k_i} \to \psi_i) | 1 \leq k_j \leq m\}$

where $\mathrm{DNT}(\psi) = \neg\psi_1 \vee \cdots \vee \neg\psi_n$, $\mathrm{DNT}(\chi) = \neg\chi_1 \vee \cdots \vee \neg\chi_m$

Proposition 3.14

For any φ, $\varphi \equiv_{\mathbf{InqL}}$ DNT(φ).

Proof. We proof by induction on φ.

p Since φ is an assertion, $\varphi \equiv_{\mathbf{InqL}}$ by 2.24.

$\psi \vee \chi$ Trivial.

$\psi \wedge \chi$ For any state s, $s \vDash \varphi \iff s \vDash \psi$ or $s \vDash \chi \iff \vDash DNT(\psi)$ and $s \vDash DNT(\chi) \iff s \vDash \neg\psi_i$ and $s \vDash \neg\chi_j$ for some i and j $\iff \vDash \neg(\psi_i \vee \chi_j) \iff \vDash DNT(\varphi)$.

$\psi \to \chi$ For any state s, $s \vDash \varphi \iff$ for any $t \subseteq s$, if $t \vDash \psi$, then $t \vDash \chi$ $\iff$ for any $t \subseteq s$, $t \vDash DNT(\psi)$ implies $t \vDash DNT(\chi) \iff$ for any $t \subseteq s$, $t \vDash \neg\psi_i$ for any i implies $t \vDash \neg\chi_j$ for some $j = k_i \iff$ for any $t \subseteq s$, for any $1 \leq k_i \leq m$, $t \vDash \chi_{k_i}$ implies $t \vDash \psi_i \iff$ for any $t \subseteq s$, $t \vDash \bigwedge_{1 \leq i \leq n}(\chi_{k_i} \to \psi_i)$ for some $\{< i, k_i > | 1 \leq i \leq n\} \iff$ $s \vDash DNT(\varphi) = \bigvee_{k_1,\ldots,k_n}\{\neg\neg \bigwedge_{1 \leq i \leq n}(\chi_{k_i} \to \psi_i) | 1 \leq k_j \leq m\}$. □

3.15-3.17

Corollary 3.15

Any formula is equivalent to a disjunction of negations.

Corollary 3.16 (Expressive completeness of $\{\neg, \vee\}$)

Any formula is equivalent to a formula containing only disjunctions and negations.

Corollary 3.17 (Expressive completeness of $\{\neg, \wedge\}$ for assertions)

A formula is an assertion if and only if it is equivalent to a formula containing only conjunctions and negations.

3.3 Inquisitive Semantics and Intuitionistic Kripke Semantics

In inquisitive semantics, formulas are evaluated with respect to information states. Whether a certain state s supports a formula φ may depend not only on the information available in s, but also on the information that may become available. Formally, support is partly defined in terms of subsets of s which can be seen as possible future information states.

Similarly, in intuitionistic semantics, formulas are evaluated with respect to points in a Kripke models, which can also be thought of as information states. Whether a point u in a model M satisfies a formula φ may depend not only on the information available at u, but also on the information that may become available. Formally, satisfaction at u is partly defined in terms of points in M that are accessible from u.

This informal analogy can be made precise: in fact, inquisitive semantics amounts to intuitionistic semantics on a suitable Kripke model.

3.18 Kripke model for inquisitive semantics

The Kripke model for inquisitive semantics is the model $M_I =< W_I, \supseteq, V_I >$ where $W_I := S - \{\varnothing\}$ is the set of all non-empty states and the valuation VI is defined as follows: for any letter p, $V_I(p) = \{s \in I | s \vDash p\}$.

Observe that MI is a Kripke model for intuitionistic logic. For, the relation $\supseteq$ is clearly a partial order. Moreover, suppose $s \supseteq t$ and $s \in V_I(p)$: this means that $s \vDash p$, and so by persistence $t \vDash p$, which amounts to $t \in V_I(p)$. So the valuation V_I is persistent.

3.19 (Support coincides with Kripke satisfaction on M_I)

For every formula φ and every non-empty state s:

$$s \vDash \varphi \iff M_I, s \Vdash \varphi$$

Proof. We prove by induction on φ.

p Since $V_I(p) = \{s \in I | s \vDash p\}$, $s \vDash p \iff M_I, s \Vdash p$.

$\psi_1 \vee \psi_2$ $s \vDash \varphi \iff s \vDash \psi_1$ or $s \vDash \psi_2 \iff M_I, s \Vdash \psi_1$ or $M_I, s \Vdash \psi_2$ $\iff M_I, s \Vdash \varphi$.

$\psi_1 \wedge \psi_2$ $s \vDash \varphi \iff s \vDash \psi_1$ and $s \vDash \psi_2 \iff M_I, s \Vdash \psi_1$ and $M_I, s \Vdash \psi_2$ $\iff M_I, s \Vdash \varphi$.

$\psi_1 \to \psi_2$ Since $\varphi \vDash \chi$ trivially holds for any formula *chi*. $s \vDash \varphi \iff$ for every $t \subseteq s$, if $t \vDash \psi_1$ then $t \vDash \psi_2 \iff$ for every $t \subseteq s$ other than φ, of $M_I, s \Vdash \psi_1$ then $M_I, s \Vdash \psi_2 \iff M_I, s \Vdash \varphi$. □

IPL ⊆ InqL ⊆ CPL

The logic **InqL** contains intuitionistic propositional logic **IPL**. For suppose that $\varphi \notin$ **InqL**. Then there must be a non-empty state s such that $s \not\models \varphi$. But then we also have that $W_I, s \not\Vdash \varphi$, which means that $\varphi \notin$ **IPL**.

On the other hand, **InqL** is contained in classical propositional logic **CPL**, because any formula that is not a classical tautology is falsified by a singleton state in inquisitive semantics. So we have:

IPL ⊆ **InqL** ⊆ **CPL**

Moreover, both inclusions are strict: for instance, $p \vee \neg p$ is in **CPL** but not in **InqL**, while $\neg\neg p \to p$ is in **InqL** but not in **IPL**.

ND (preserving logical equivalence under DNT)

- intuitionistic validities
- all substitution instances of
  $ND_k = (\neg p \to \bigvee_{1 \leq i \leq k} \neg q_i) \to \bigvee_{1 \leq i \leq k} (\neg p \to \neg q_i)$, $k \in \omega$
- Modus ponens

Proposition 3.33

For any logic $\Lambda \supseteq ND$ and any formula φ, $\varphi \equiv_{\Lambda^n} DNT(\varphi)$.

Where Λ^n is a system with $\Lambda \cup \{\neg\neg p \to p | p \in \mathcal{P}\}$ as axioms and MP as the derivation rule.

Theorem 3.32 (Disjunction Property + DNT = InqL)

Let L be a weak intermediate logic. If $\varphi \equiv_L DNT(\varphi)$ for all φ, then InqL$\subset L$. If, additionally, L has disjunction property, then InqL$= L$.

Proof. Suppose $\varphi \in$**InqL**. Then $DNT(\varphi) \in$ **InqL**. Write $DNT(\varphi) = \neg\mu_1 \vee \cdots \vee \nu_k$: since **InqL** has the disjunction property, we must have $\neg\nu_i \in$ **InqL** for some $1 \leq i \leq k$. Now, we know that **IPL** coincides with **CPL** as far as negations are concerned and it follows from this that every two weak intermediate logics coincide as far as negations are concerned. So if $\neg\nu_i \in$ **InqL**, then also $\neg\nu_i \in L$. Hence, $DNT(\varphi) \in L$ and since $\varphi \equiv_L DNT(\varphi)$, also $\varphi \in L$. This shows that **InqL**$\subset L$. . .

Proposition 3.34 (Completeness Theorem)
$\Lambda^n = \mathit{InqL}$ for any logic $\Lambda \supseteq \mathit{ND}$ with disjunction property.

Proof. Let Λ be an extension of ND with the disjunction property. Then according to Proposition 3.33 we have $\varphi \equiv_{\Lambda^n} \mathit{DNT}(\varphi)$ for all φ; moreover, since Λ has the disjunction property, it can be shown that Λ^n also has the disjunction property. Hence by Theorem 3.32 we have $\Lambda^n = \mathit{InqL}$. □

A brief hsitory: Roelofsen (2018) and others

How to characterize the meaning of a question?

Early thoughts of Frege(1918)

- Is David coming?

  David is coming.
- semantic content (same) v.s. force (different)

Limits

- Only for polar questions
- Against compositionality (in cases of embedded clauese)

But can we build upon Frege's ideas?

Proposition-set theories

Alternative semantics (Hamblin, 1973; Karttunen, 1977)
Propositions representing the possible answers to the question

(overgeneration)

Partition semantics (Groenendijk & Stokhof, 1984)
Propositions representing exhaustive answers

(undergeneration)

Inquisitive semantics (Ciardelli, Groenendijk, & Roelofsen, 2018)
Propositions as pieces of information that resolve the issue that the question expresses.

Alternative semantics

"questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it"(Hamblin, 1973, p.48)

Problems

- what are possible answers?

⋆ What's Alice's phone number?

a. It's 055-9090231.

b. It's 055-9090231 but she prefers to be contacted by email.

c. It's either 055-9090231 or 055-9090233.

d. It starts with 055-9090.

- entaiment

⋆⋆ Does Alice's phone number end with a 4?

- overgeneralization

∗ Is John American or is he Californian?

Partition semantics

A question denotes, in each world in which its presuppositions are satisfied, a single proposition embodying the true exhaustive answer to the question in that world. (The propositions form a partition of such possible worlds.)

⋆ Who is coming for dinner?

a. Only Paul and Nina are coming.

Problem

- undergeneralization

a. (after Belnap, 1982) Where in Pittsburgh can I get gas on a Sunday? (mention-some question)

b. (Yablo, 2014) How many stars are there in the Milky way, give or take 10? (approximate value question)

c. (Ciardelli et al., 2018) Where can we rent a car or who has one we could borrow? (disjunctive question)

Inquisitive semantics

The semantic content of a question is intended to capture its resolution conditions. (downward closed)

Problems

- Beyond resolution conditions

a. Is the door open? (polar question)

b. Is the door open or closed? (alternative question)

c. The door is open, isn't it? (tag question)

- Contextual parameters

d. Which students passed the exam? (domain of quantification)

e. What is the winning card? (method of identification)

f. Who is driving to the party tonight? (goal)

g. Where is Mary? (level of granularity)

Question embedding

responsive predicates

a. Mary knows/predicted/forgot that John left.

b. Mary knows/predicted/forgot who left.

rogative predicates

a. Mary wonders/investigated that John left.

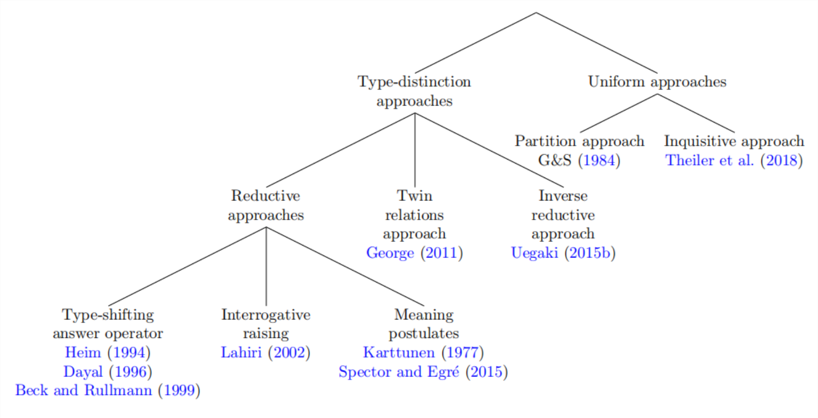
b. Mary wonders/investigated who left.

anti-rogative predicates

a. Mary believes/hopes that John left.

b. Mary believes/hopes who left.

Various approaches



The original inquisitive semantics

Definition 7.1 (Pair semantics)

1. $< v, w > \models p$ iff $p \in v$ and $p \in w$
2. $< v, w > \not\models \bot$
3. $< v, w > \models \varphi \wedge \psi$ iff $< v, w > \models \varphi$ and $< v, w > \models \psi$
4. $< v, w > \models \varphi \vee \psi$ iff $< v, w > \models \varphi$ or $< v, w > \models \psi$
5. $< v, w > \models \varphi \rightarrow \psi$ iff for all pairs $\pi \in \{v, w\}^2$: if $\pi \models \varphi$, then $\pi \models \psi$

Support as "knowing how"

Definition 9.1 (Realization)

1. $\mathcal{R}(p) = \{p\}$ for $p \in \mathcal{P}$
2. $\mathcal{R}(\bot) = \{\bot\}$
3. $\mathcal{R}(\varphi \vee \psi) = \mathcal{R}(\varphi) \cup \mathcal{R}(\psi)$
4. $\mathcal{R}(\varphi \wedge \psi) = \{(\rho \wedge \sigma)_{nf} | \rho \in \mathcal{R}(\varphi) \text{ and } \sigma \in \mathcal{R}(\psi)\}$
5. $\mathcal{R}(\varphi \to \psi) = \{(\bigwedge_{\rho \in \mathcal{R}(\varphi)}(\rho \to f(\rho)))_{nf} | f : \mathcal{R}(\varphi) \to \mathcal{R}(\psi)\}$

Where φ_{nf} is the disjunction-free normal form of φ.

Definition 9.3 For any state s and formula φ,

$$s \vDash \varphi \iff s \subseteq |\rho| \text{ for some } \rho \in \mathcal{R}(\varphi)$$

References

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