Update Semantics 2

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Identity and Identification

The languages we're interested in

I will **sketch** a semantics for languages of first order predicate logic with

- identity: $=$
- epistemic possibility: *might*
- logical necessity: $\Box$

Modeling Information about the world

Definition 1 Fix a domain $\mathcal{D}$ of objects. Then w is an element of the set of possible worlds $\mathcal{W}$ iff $w = \langle \nu, \iota \rangle$, where

(a) the *naming function* ν assigns to every individual constant c an element $\nu(c) \in \mathcal{D}$;

(b) the *interpretation function* ι assigns to every n-place predicate P an n-ary relation $\iota(P) \subseteq \mathcal{D}^n$.

Example

This example involves two objects d_1, and d_2 . One is called a and one (possibly the same one) is called c. One – you don't know which one – is blue, and the other is red.

Now what are the relevant possible worlds?

Set $\mathcal{D} = \{d_1, d_2\}$. Then $\mathcal{W}$ is the set of pairs $\langle \nu, \iota \rangle$ such that

(a) ν is a function from $\{a, c\}$ into $\{d_1, d_2\}$;

(b) ι is a function from $\{blue, red\}$ into the powerset of $\{d_1, d_2\}$ such that either ι(*blue*) $= \{d_1\}$ and ι(*red*) $= \{d_2\}$, or ι(*blue*) $= \{d_2\}$ and ι(*red*) $= \{d_1\}$

Note that there are four naming functions, and two interpretation functions

$\nu_1 = \{\langle a, d_1 \rangle, \langle c, d_1 \rangle\}$
$\nu_2 = \{\langle a, d_1 \rangle, \langle c, d_2 \rangle\}$
$\nu_3 = \{\langle a, d_2 \rangle, \langle c, d_1 \rangle\}$
$\nu_4 = \{\langle a, d_2 \rangle, \langle c, d_2 \rangle\}$

$\iota_1(\textit{blue}) = \{d_1\}$, $\iota_1(\textit{red}) = \{d_2\}$
$\iota_2(\textit{blue}) = \{d_2\}$, $\iota_2(\textit{red}) = \{d_1\}$

So, there are eight possible worlds, pictured on next slide. In these pictures, d_1 is the left object, and d_2 the right object.

The table also shows which of these possible worlds are left if you learn that

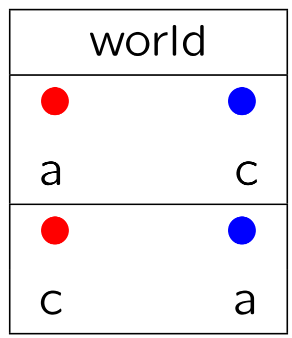
- *the blue one* $= a$
- $c = a$
- $this_{d_1} = a$

world	*the blue one* $= a$	$c = a$	$this_{d_1} = a$
$\color{red}{\bullet}$ a $\color{blue}{\bullet}$ c			yes
$\color{red}{\bullet}$ c $\color{blue}{\bullet}$ a	yes		
$\color{red}{\bullet}$ ac $\color{blue}{\bullet}$		yes	yes
$\color{red}{\bullet}$ $\color{blue}{\bullet}$ ac	yes	yes	
$\color{blue}{\bullet}$ a $\color{red}{\bullet}$ c	yes		yes
$\color{blue}{\bullet}$ c $\color{red}{\bullet}$ a			
$\color{blue}{\bullet}$ ac $\color{red}{\bullet}$	yes	yes	yes
$\color{blue}{\bullet}$ $\color{red}{\bullet}$ ac		yes	

The puzzle

Now let's look at set of worlds left if all you know about the case is this:$Red(this_{d_1})$; $Blue(this_{d_2})$; $c \neq a$.

Given this information the possibilities left are



Given these two epistemic possibilities, do you accept $\forall x\, \textit{might}(x = a)$? How about $\forall x\, \textit{might}(x = this_{d_1})$*?

*Read '$\forall x$' as 'for all objects x in the domain $\mathcal{D}$'

I expect that most of you accept $\forall x\, \textit{might}(x = a)$. I also expect that most of you, knowing that $c \neq a$ will reject $c = a$. But then, it looks like *universal instantiation* is not always valid.

Right!!

In the resulting system *universal instantiation* is not always valid. You can always instantiate with a demonstrative, but with an individual constant only if you know which object it denotes. And here you don't .

Likewise, existential generalization sometimes fails:

$\forall y\ \textit{might}(y \neq a) \not\models \exists x \forall y\ \textit{might}(y \neq x)$

Here, too, generalization is not allowed because the constant a is not – not yet – *epistemically* rigid.

Analytic aposteriori

In *Naming and Necessity* Saul Kripke claims that true identity statements like *The Morning Star* $=$ *The Evening Star* express necessary truths, but that they are not epistemically *a priori*. In many cases it's a truth that has to be discovered.

We already saw that in the logical system I am describing identity statements can be *a posteriori*. One can at first be in a state in which one accepts *might*$(a \neq c)$, and next find out that in fact $a = c$. Now we want to get on top of this that

$$a = c \models \Box(a = c)$$

Sketch

A *cognitive state** S is function that assigns to every naming function ν a pair $\langle U, F\rangle$, where U and F are sets of interpretation functions such that (a) $F \subseteq U$, and (b) If $F = \emptyset$, then $U = \emptyset$.

- If $\langle U, F\rangle = S(\nu)$, and $\iota \in U$, then the world $\langle \nu, \iota\rangle$ is a world that the agent considers *logically possible*.
- If $\langle U, F\rangle = S(\nu)$, and $\iota \in F$, then, given the agent's information, the world $\langle \nu, \iota\rangle$ *might be the actual world*.
- If $S(\nu) = \langle \emptyset, \emptyset\rangle$, this means that the possibility is excluded that the objects in $\mathcal{D}$ are named as ν describes.

*I am leaving out everything needed for the quantifiers

- The *minimal state* is the state **1** which assigns to every ν the pair $\langle I, I \rangle$ where I is the set of all interpretation functions.

- The *absurd state*, is the state **0** which assigns to every ν the pair $\langle \emptyset, \emptyset \rangle$.

Before we can define the update clauses for the full language we first define what it means to update a set of worlds that all have the same naming function with a descriptive sentence.

Definition 2

- Let X be a set of interpretation functions. Set ${}^{\nu}X = \{\nu\} \times X$
- ${}^{\nu}X[Ra_1 \dots a_n] = \{\iota \in X \mid \langle \nu(a_1), \dots, \nu(a_n) \rangle \in \iota(R)\}$.
- ${}^{\nu}X[c = a] = X$ if $\nu(c) = \nu(a)$, and ${}^{\nu}X[c = a] = \emptyset$ if $\nu(c) \neq \nu(a)$.
- etcera

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Definition 3

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- ${}^{\nu}X[c{=}a] = X$ if $\nu(c) = \nu(a)$, and ${}^{\nu}X[c{=}a] = \emptyset$ if $\nu(c) \neq \nu(a)$.
- etcera

Now the update clause for descriptive sentences φ is given by

- $S[\varphi]$ is determined as follows.
  Suppose $S(\nu) = \langle U, F \rangle$. Then $S[\varphi](\nu) = \langle U', F' \rangle$, where
  (a) $F' = {}^{\nu}F[\varphi]$
  (b) $U' = U$ if $F' \neq \emptyset$, and $U' = \emptyset$ if $F' = \emptyset$.

The clause for *might*φ remains the same.

- $S[\textit{might}\,\varphi] = S$ if $S[\varphi] \neq \mathbf{0}$
  $S[\textit{might}\,\varphi] = \emptyset$ if $S[\varphi] = \mathbf{0}$

- $S[\Box\varphi]$ is determined as follows.
  Suppose $S(\nu) = \langle U, F \rangle$. Then $S[\Box\varphi](\nu) = \langle {}^{\nu}U[\varphi], {}^{\nu}F[\varphi] \rangle$

Exercise 1

(a) Check that $\mathbf{1}[\mathit{might}(c \neq a)][c = a] \neq \mathbf{0}$

(b) Prove that $c = a \models \Box(c = a)$.

There is a lot more to say. . .

But now it's time for comments and questions.